General Economics

THE COSTS OF EUROPEAN INTEGRATION

Professor Romeo Victor IONESCU, PhD
„Danubius" University from Galaţi,

Abstract: *First and foremost, impact studies pointed out that the adhering of Romania to the European Union in 2007 is the most advantageous prospect to consider both financially and in terms of economic development, as the increase of the living standards and of modernizing processes in Romania is far more evident in this case than in a postponed adhering or in a seclusion scenario. Consequently, if we fail the European Union adhering in 2007, we would have to face higher costs.*

Keywords: *European Union, Community Funds, impact*
Jel classification: *F15 - Economic Integration;*

The costs of integration are, in fact, Romanian modernizing costs, that are to be made ether way, even without any Community support in case of having missed the integration. We may say that we invest in highways for the very need to get integrated, while we had better say we want better roads, that would contribute to the economic development and, implicitly, creating new jobs. Could we simply say that investments meant to reduce pollution or to improve the quality of drinking water have been made for the sake of the European Union?

I do think a proper statement would be that we are, ourselves, concerned in having better water and a cleaner environment, to the benefit of all of us. Furthermore, I think that we all want slaughter-houses and dairy shops to comply with all the rules of hygiene so as to ensure the safety of our food. Thus, most of the costs considered as adhesion-connected are actually meant to improve our living standards. Such costs should have been made under any events; now they are

supported by the Community Funds. For instance, environmental costs for the next 20 years were estimated to approximately EUR 29 billion. They are to be covered with money that comes from the central public budget and the local budgets, but also from company's budget.

Between 2005 and 2025, Romania will allot, yearly, approximately 1% of the G.D.P. to environmental and transport activities, while the European Union is to earmark 1.7% out of Romania's G.D.P., for the same needs. It is true that our co-financing has to be added to the European Union funds. This doesn't point out any contribution to the Community budget, but an investment in Romania's development projects. If such investments supposed to receive a 100% Romanian funding, this would not have been a bearable effort for the population.

Subsequently, Romania's contribution to the Community budget is about EUR 2.5 billion, between 2007 and 2009 (i.e. EUR 800 million in 2007 and about EUR 900 million, twice in 2008 and respectively in 2009) and not about EUR 5.6 billion as published in media. Co-financing does not stand for any contribution to the Community budget. The money does not leave the country, but are invested here in Romanian modernizing projects (roads, sewerages, farming developments, firms, jobs etc.). However, Romania is to receive EUR 11 billion from the Community budget in 2007-2009 out of which some EUR 6 billion payments, other remaining payments being scheduled all along the unfolding projects.

The only **costs that are likely to associate in their entirety with the integrating process** are those connected to setting up several specific institutions e.g. for administrating Community funds, for agricultural purposes and so on. However, the benefits brought by this management system of European financial inflows are exponentially higher as related to costs.[1]

And it would also be something else: the monetary costs that are to be taken - as previously mentioned- for economic growth, development or modernizing ones are liable to have some extremely amplified medium and long-term effects for each Euro spent in this process. Nevertheless, the benefits of integration must also be assumed as against the background of non-monetary effects: the altering of economic behaviour now highly based on moral hazard; the increase of

[1] Ciupangea Constantin, *Theoretical and Methodological Approach of the Costs and Benefits related to the E.U. Integration of Romania, Economic Problems*, Weekly Economic Magazine, 116th issue, 2004

technological, economic and commercial competitiveness; the improvement of the business environment, as considered predictable and uncertain risk reducing the terms.

Maybe the most significant benefit is Romania's possibility to actively participate in working out the new *acquis communautaire* so that the interests of our country should best be served.

I think it is high time we got over the demagogical approach and **asses the adhering impact at a global scale.** The European Union does not impose arbitrary standards to cause the effecting of additional costs, but it establishes certain standards in order to increase the living standards of each individual and to shape up a dynamic and competitive economic environment. Obviously, there are costs to be used in modernizing Romania, costs that now relate to the European Union adhering process. However these costs have definitely existed in the last years and have been meant to increase the efficiency of the Romanian economy. Now we do benefit of the European Union support and we are to integrate into a *milieu* that ought to ensure a better living for each of us, other state's example is very specific in this respect.

I think that, instead of demagogically approaching this topic, we would rather inform companies and citizens on how they should better get ready for the status of the European Union members.

I do also think that consciousness, honesty and responsibility in making the assessment of the European Union adhering of Romania must stand for the indispensable intellectual ingredients.

The bill of integration will probably amount to at least EUR 17 billion in the next five years.

The four million wage-earners in Romania will bear an expenditure of EUR 4200 besides the present tax contributions. In the following five years, Romania will have to spend EUR 17 billion in order to reach European Union standards and most of this money is to be paid by Romanian citizens and firms.

The adhering costs, that Romanian population will bear, are of three kinds: public costs (having state budget support), private costs (made by companies) and

individual costs (that are to be paid by each Romanian citizen in order to: reshape the retirement system, help the professional redeployment, and increase the living standards). The first two types of costs are liable to be assessed, even if roughly, but individual costs can be, by no means, previously estimated.

Scientific studies emphasize the important aspects of the European Union adhering, which are: the degree of conformity of Romanian legislation with the Community legal framework; monetary strategies and exchange course issues; the consequences of adopting the *acquis communautaire* on the financial control; the European security and defence policy; emigration and immigration phenomena; structural funds - the candidate states' experience; the plan of action aiming at storing the industrial waste; the comparative analysis of member states insolvency and bankruptcy; financial management options and solutions in ensuring a real increase in the value of the retirement pays; the state aids in tender competition sectors; agricultural and rural development priorities; the costs and benefits of the adhering process.[1]

[1] *The European Institute in Romania,* Impact Studies, PAIS II

DEGENERATE FOLIATIONS IN SEMI-RIEMANNIAN MANIFOLDS

Professor Cătălin Angelo IOAN, PhD
„Danubius" University from Galaţi

Abstract*: The main notions and results concerning the linear spaces, semi-Riemannian manifolds and submanifolds have a direct link with the subject. Because the Gram-Schmidt orthogonalization is fundamental we have to proceed at his resumption in the intention to do it applicable for our demarches.*

Keywords: *foliations, Semi-Riemannian geometry, Semi-Riemannian manifolds*
Jel Classification**:** *C - Mathematical and Quantitative Methods, C0 - General, C00 - General*

Introduction

The theory of Riemannian foliations has been treated during the time under various aspects.

We can cite references like [15], [24], [27], [28] or [34]. Also has been treated particular foliations like totally geodesics ([5], [9], [20], minimals ([14]) or of other types. All these results have been obtained under the generous foundation of the Riemannian geometry. Once with the development of the researches in the field of the Semi-Riemannian geometry ([1], [2], [3], [6], [17], [30], [31]) it is natural to search how we can extend all these results. It is born a new problem that concerns the study of degenerate foliations.

The main notions and results concerning the linear spaces, semi-riemannian manifolds and submanifolds have a direct link with the subject. Because the Gram-Schmidt orthogonalization is fundamental, we have proceeded at a resumption of his

in the intention to do it applicable for our demarches. Many works of Semi-Riemannian geometry remind us that this procedure it is applicable also in the case of Semi-Riemannian metrics ([11], [26]). In [11] it is presented the concrete manner of orthonormal vectors construction, but the author ignores the fact that if a Gram determinant is nul all this construction stops even if we try to change the basis. In the sequel we present some aspects concerning Semi-Riemannian manifolds and fibre bundles. Also, we introduce the notions of spacelike, timelike and lightlike vectors following in this direction the paper [26].

The notion of degenerate foliation builds the transversal distribution of a foliation, notion which will substitute that of classical orthogonal distribution. We proceed also at a decomposition of these foliations following [2] in four categories: r-degenerate foliations, coisotropic, isotropic and totally degenerates. On account of specific aspects we shall work permanently with some distributions like the screen distribution, transversal screen distribution and degenerate transversal distribution. After the description of various geometric objects we shall study its behaviour at the change of the screen distribution and the change of the coordinates' neighbourhood of an arbitrary point.

We generalize the tensors presented in [27] and we clarify some problems like the integrability and the totally geodesibility of the null and screen distributions. Moreover, we shall build the Gauss-Weingarten formulae together with all geometrical objects concerned. After this demarche we shall obtain a number of characterisation theorems for the distributions or various introduced geometrical objects.

In this paper it will be defined the total geodesic degenerate foliations and totally umbilical degenerate foliations and we shall obtain some characterisation theorems. The discussion is made on the r-degenerate foliations, the results were modulated for the other types.

The final chapter gives some examples of degenerate foliations on a class of 4-manifolds endowed with a relativistic metric, which generalises the exterior Schwarzschild, Reissner-Weil, de Sitter and Minkowski metrics. There are presented four concrete examples and the last proves that on this type of manifolds does not exist totally degenerate foliations.

1. Preliminaries

Let V a linear space and $g:V\times V\to \mathbf{R}$ a symmetric bilinear form. The form g is called non-degenerate if $g(x,y)=0\ \forall y\in V\Rightarrow x=0$ and degenerate if $\exists x\neq 0$ such that $\forall y\in V\Rightarrow g(x,y)=0$. g is called positive definite (negative definite) if $g(x,x)\geq 0$

$(g(x,x)\leq 0)\ \forall x\in V$ and $g(x,x)=0\Rightarrow x=0$ and semi-definite if $\exists x,y\in V$ such that $g(x,x)>0$ and $g(y,y)<0$.

We note (V,g) a linear space V provided with a bilinear, symmetric, non-degenerate form g. We note also $W<V$ the fact that W is a subspace of V. The set $W^{\perp}=\{y\in V \mid g(y,x)=0\ \forall x\in W\}$ is called the orthogonal subspace of W. In general $W^{\perp}$ is not a complementary subspace of W.

Theorem 1.1 [26] Let $W<(V,g)$. Then:

$$\dim W+\dim W^{\perp}=\dim V \tag{1.1}$$
$$(W^{\perp})^{\perp}=W \tag{1.2}$$

If g is non-degenerate on V it is not obligatory that she is non-degenerate on any subspace of V.

A subspace $W<(V,g)$ is called non-degenerate (degenerate) subspace if the restriction $g \mid W$ is non-degenerate (degenerate).

Theorem 1.2 [26] A subspace W of (V,g) is non-degenerate if and only if $V=W\oplus W^{\perp}$.

W is non-degenerate if and only if $W\cap W^{\perp}=\{0\}$. By (1.2) and the theorem 1.2 follows that W is non-degenerate if and only if $W^{\perp}$ is non-degenerate.

A basis $B=\{e_1,...,e_n\}$ of a linear space (V,g) is called orthonormal basis if $g(e_i,e_j)=\pm\delta_{ij}$, i,j=1,...,n where δ_{ij} is the Kronecker symbol.

The Gram-Schmidt orthogonalization process.

Let (V,g) a linear space provided with a bilinear, symmetric, non-degenerate form g. Let also $B=\{v_1,...,v_n\}$ an arbitrary basis of V, composed by non-null vectors $(g(v_i,v_i)\neq 0$, i=1,...,n). We shall determine by depart of B an orthonormal basis of V.

Let therefore $w_1=\frac{v_1}{\sqrt{|g(v_1,v_1)|}}$. We have $g(w_1,w_1)=\frac{g(v_1,v_1)}{|g(v_1,v_1)|}=\varepsilon_1=\pm 1$. Let suppose that we have determined the vectors $w_1,...,w_{p-1}$ such that $g(w_i,w_j)=0$, i,j=1,...,p-1, $i\neq j$ and $g(w_i,w_i)=\varepsilon_i\in\{-1,1\}$, i=1,...,p-1. Let:

$$w_p = \varepsilon_1 ... \varepsilon_{p-1} \sqrt{\frac{\varepsilon_p}{g(v_p, v_p) - \sum_{i=1}^{p-1} \varepsilon_i g^2(w_i, v_p)}} \left(v_p - \sum_{j=1}^{p-1} \varepsilon_j g(w_j, v_p) w_j \right) \tag{1.3}$$

if $g(v_p,v_p) \neq \sum_{i=1}^{p-1} \varepsilon_i g^2(w_i, v_p)$ and $\varepsilon_p \in \{-1,1\}$ such that the square root be definite. We have $g(w_p,w_p)=\varepsilon_p$ and $g(w_p,w_i)=0$, i=1,...,p-1. Let now $W_{p-1}^{\perp}=Span(w_1,...,w_{p-1})^{\perp}$ where Span(...) is the subspace generate by the respective vectors. The subspace $W_{p-1}=Span(w_1,...,w_{p-1})$ is non-degenerate. Indeed, let suppose that there is $x \in W_{p-1}$, $x \neq 0$ such that $g(x,y)=0 \quad \forall y \in W_{p-1}$. Let $x=\sum_{i=1}^{p-1} \alpha_i w_i \neq 0$. We have $g(x,w_k)= g(\sum_{i=1}^{p-1} \alpha_i w_i, w_k)=\alpha_k \varepsilon_k$ therefore $\alpha_k \varepsilon_k=0$ that is $\alpha_k=0$, k=1,...,p-1. Accordingly x=0 therefore contradiction. By the theorem 1.2 we have that $V=W_{p-1} \oplus W_{p-1}^{\perp}$. Let now $v_p=\sum_{i=1}^{p-1} c_i w_i + z_p$ where $z_p \in W_{p-1}^{\perp}$ (the decomposition being unique by the direct sum). We have:

$$g(v_p,v_p) - \sum_{i=1}^{p-1} \varepsilon_i g^2(v_p, w_i) = g(z_p,z_p)$$

If $g(z_p,z_p) \neq 0$ then (1.3) is applicable. If $g(z_p,z_p)=0$ we do a parmutation of the vectors $\{v_p,...,v_n\}$. If $\exists k \in \{p,...,n\}$ such that $g(z_k,z_k) \neq 0$ after a possible renumbering we can apply (1.3). If $\forall k \in \{p,...,n\} \Rightarrow g(z_k,z_k)=0$ where $z_k = pr_{W_{p-1}^{\perp}} v_k$, k=p,...,n (the projection of v_k on $W_{p-1}^{\perp}$) then $\exists k,r=p,...,n$ with $k \neq r$ such that $g(z_k,z_r) \neq 0$. Indeed, if $\forall k,r=p,...,n \Rightarrow g(z_k,z_r)=0$ then how $\{z_p,...,z_n\}$ constitutes a basis of $W_{p-1}^{\perp}$ follows that $W_{p-1}^{\perp}$ is degenerate therefore contradiction. Let therefore, after a possible renumbering, z_p and z_{p+1} such that $g(z_p,z_{p+1}) \neq 0$. Let now:

$\overline{v_p} = av_p + bv_{p+1}$ with $a,b \neq 0$

We have $g(\overline{v_p}, \overline{v_p}) - \sum_{i=1}^{p-1} \varepsilon_i g^2(\overline{v_p}, w_p) = 2abg(z_p,z_{p+1}) \neq 0$ therefore we can apply (1.3) for $v_p \rightarrow \overline{v_p}$.

Finally, for p=n follows trivial $g(z_n,z_n)\neq 0$ because in the opposite case $W_{n-1}^{\perp}=\mathrm{Span}(z_n)$ is degenerate therefore contradiction.

If we consider now the orthonormal basis $B=\{e_1,...,e_n\}$ of (V,g) and we note $\varepsilon_i=g(e_i,e_i)$, i=1,...,n follows:

$$x=\sum_{i=1}^{n}\varepsilon_i g(x,e_i)e_i \quad \forall x\in V \tag{1.4}$$

Let (V,g) a linear spece. We call the index of g: $\nu=\mathrm{ind}\ g=\max\{\dim W \mid W<V,\ g\mid W \text{ is negative definite}\}$. We shall write sometimes $\nu=\mathrm{ind}\ V$.

Lemma 1.1 [26] Let (V,g) a linear space and W a non-degenerate subspace of V. Then

$$\mathrm{ind}\ V=\mathrm{ind}\ W+\mathrm{ind}\ W^{\perp} \tag{1.5}$$

Remark In general the inequality holds: $\mathrm{ind}\ V\geq \mathrm{ind}\ W+\mathrm{ind}\ W^{\perp}\ \forall W<V$.

Lemma 1.2 [26] Let (V,g) a linear space. Then there is a subspace W<V of maximal dimension=min {ind g, dim V-ind g} such that $g\mid W=0$.

In what follows we suppose that all the differentiable manifolds have the metrics with constant index on them and all the geometrical objects are of infinite class.

Let a Semi-Riemannian manifold (M,g). A tangent vector $X\in T_pM$, $p\in M$ is called spacelike vector if $g(X,X)>0$ or X=0, lightlike vector if $g(X,X)=0$ and $X\neq 0$ and timelike vector if $g(X,X)<0$. The collection of lightlike vectors of T_pM is called the null cone in $p\in M$.

2. Degenerate foliations of the Semi-Riemannian manifolds

Let (M,g) a Semi-Riemannian manifold, (m+n)-dimensional, m,n≥1, g being the semi-riemannian metric on M.

Let q the index of the metric g which we shall suppose constant on M. If q=0 or q=m+n then the metric is riemannian. How in this case the induced metric on any leaf of the foliation is also riemannian follows that if we want to talk about

degeneration we shall suppose that $1\leq q\leq m+n-1$. Therefore M is not a Riemannian manifold.

Definition 2.1 A degenerate foliation of codimension m of M is a decomposition of M into a disjoint union of conected, degenerate submanifolds of codimension m of M, called leafs of the foliation such that for any $p\in M$ there is a neighbourhood U of p in M and a submersion $f_U:U\rightarrow \mathbf{R}^m$ with the property: $\forall\ x\in\mathbf{R}^m$, $f_U^{-1}(x)$ is a leaf of the restriction of foliation at U, $F\mid_U$.

We shall consider in what follows like coordinates neighbourhoods of any point $p\in M$ the collections U by the upper definition.

Considering now a degenerate foliation of codimension m of M, let:

$$T(F)=\bigcup_{\substack{p\in M\\ L \text{ the leaf of } F\\ \text{which contains } p}} T_pL$$

We shall show now that T(F) is a fibre bundle of rank n on M.

Let $p\in M$ and U a neighbourhood of p in M such there is a submersion $f_U:U\rightarrow \mathbf{R}^m$ with the property that $\forall\ x\in\mathbf{R}^m$, $f_U^{-1}(x)$ is a leaf of the restriction of the foliation on U, $F\mid_U$.

Considering now the leaf L passing through $p\in M$ we define:

$\pi:T(F)\rightarrow \mathbf{R}^m$, $\pi(T_pL)=f_U(p)\ \forall p\in M$

The map π is correct defined because by any leaf L of F corresponds an unique $x\in\mathbf{R}^m$ such that $L=f_U^{-1}(x)$. Indeed, if we suppose that $\exists x\neq y\in\mathbf{R}^m$ such that $L=f_U^{-1}(x)=f_U^{-1}(y)$ then $x=f_U(f_U^{-1}(x))=f_U(L)=f_U(f_U^{-1}(y))=y$ from where follows contradiction. On the other hand $f_U(p)\in f_U(L)=f_U(f_U^{-1}(x))=\{x\}$ otherwise: $f_U(p)=x$. We have also that the map π does not depend on the coordinates neighbourhood U. Indeed, if we shall consider U and V neighbourhoods of $p\in M$ satifying the definition conditions and the submersions $f_U:U\rightarrow \mathbf{R}^m$, $f_V:V\rightarrow \mathbf{R}^m$ then $\forall x\in\mathbf{R}^m$ follows that $f_U^{-1}(x)$ is a leaf of the restriction of the foliation on U and $f_V^{-1}(x)$ is a leaf of the restriction of the foliation on V. How through $p\in M$ pass a unique leaf follows that $f_U^{-1}(x)\cap f_V^{-1}(x)$ is a restriction of the foliation on $U\cap V$. But $p\in U\cap V\subset U,V$ follows $f_U(p)=f_V(p)=x$.

We have now that for any $x\in\mathbf{R}^m$: $\pi^{-1}(x)=T_pL$, $p\in M$ such that $f_U(p)=x$ is a real linear space of dimension n.

Let $p\in M$ and L the leaf passing through p. Let consider also a neighbourhood U of p and the submersion $f_U:U\to\mathbf{R}^m$ with the property that $L=f_U^{-1}(x)$ for a fixed point $x\in\mathbf{R}^m$. Let also a basis $\{e_1(p),...,e_n(p)\}$ of T_pL.

We define the diffeomorphism:

$$\phi:\pi^{-1}(f_U(U))\to f_U(U)\times\mathbf{R}^n,\qquad \phi(v)=(f_U(q),v_1,...,v_n)\qquad \forall q\in U$$
$$\forall v=v^1e_1(q)+...+v^ne_n(q)\in T_qL$$

If we note with pr_1 the projection on the first component we have $pr_1(\phi(v))=f_U(q)=\pi(v)$ $\forall v\in T_qL$ $\forall q\in U$ and the map $\phi_q:T_qL\to f_U(q)\times\mathbf{R}^n$, $\phi(v)=(v_1,...,v_n)$ $\forall q\in U$ $\forall v=v^1e_1(q)+...+ v^ne_n(q)\in T_qL$ is simply an **R**-isomorphism.

We have therefore proved that $(T(F),\pi,\mathbf{R}^n)$ is a fibre bundle of rank n.

Definition 2.2 Considering the vector bundle which we have build we shall say that T(F) is the fibre bundle tangent to the foliation F on M.

From the definition of F, follows that T(F) is an integrable distribution.

Let now L a leaf of F passing through $p\in M$. Considering

$$T_p(L)^{\perp}=\{X_p\in T_pM \mid g(X_p,Y_p)=0,\ \forall Y_p\in T_pL\}$$

we have that $T_p(L)^{\perp}$ is also degenerate. Let $T(F)^{\perp}=\bigcup_{L\in F}T_p(L)^{\perp}$. Like in the preceding construction we can show that $T(F)^{\perp}$ is a fibre bundle on $\mathbf{R}^m$ called the normal fibre bundle of the foliation F .

Considering now the fibre bundle T(F) of a degenerate foliation F and join to any point $p\in M$ the tangent space $T_p(L)$ at the leaf L passing through p we shall obtain an n-dimensional integrable distribution on M noted in what follows with D_F and called the distribution asociated to the degenerate foliation. Because the distribution D_F is integrable follows that she is involutive that is $\forall X,Y\in D_F \Rightarrow[X,Y]\in D_F$. Considering now D_F let $D_F{}^{\perp}$ the orthogonal distribution of D_F in TM. Is obvious that $D_F{}^{\perp}$ is obtained also by the association at any point $p\in M$ of the orthogonal space $T_p(L)^{\perp}$ of $T_p(L)$ relative of the leaf L passing through p.

Let now $p\in M$ and U a coordinates neighbourhood of p in M. Considering a coordinates system in $p\in M$: $(x^1,...,x^{n+m})$ follows by the definition that there is a submersion $f_U:U\to\mathbf{R}^m$ with the property that for any $x=(a^1,...,a^m)\in\mathbf{R}^m$, the leaf of the restriction of the foliation at U is given by the equations:

$$x^{n+1}=a^1,...,x^{n+m}=a^m$$

If we consider another coordinates system $(y^1,...,y^{n+m})$ in U follows:

$$y^i=\frac{\partial y^i}{\partial x^k}x^k,\ i=1,...,n+m$$

How y^{n+i}=constant, i=1,...,m follows $\frac{\partial y^{n+i}}{\partial x^k}=0$, i=1,...,m, k=1,...,n.

The structural group consists by the matrices of the form:

$$\begin{pmatrix} A & B \\ 0 & C \end{pmatrix}$$

where $A\in M_n(\mathbf{R})$ and $C\in M_m(\mathbf{R})$ are non-singular and $B\in M_{nm}(\mathbf{R})$.

Let now $T(M)\big|_U$ the restriction on U of the tangent bundle of the manifold M and $\{X_1,...,X_n,Y_{n+1},...,Y_{n+m}\}$ a basis for the local sections of $T(M)\big|_U$. If V is another neighbourhood of p in M such that $U\cap V\neq\varnothing$ and $\{X'_1,...,X'_n,Y'_{n+1},...,Y'_{n+m}\}$ is a basis for the local sections of $T(M)\big|_V$ then according to the structural group we have:

(2.1) $$X'_i=\sum_{k=1}^{n}A_i^kX_k+\sum_{\beta=n+1}^{n+m}B_i^\beta Y_\beta$$

(2.2) $$Y'_\alpha=\sum_{\beta=n+1}^{+m}C_\alpha^\beta Y_\beta$$

$\forall$ i=1,...,n $\forall\alpha$=n+1,...,n+m, A_i^k, B_i^β, C_α^β being arbitrary maps, indefinite differentiable on $U\cap V$ satisfying in addition the condition that the matrices $A=(A_i^k)$ and $C=(C_\alpha^\beta)$ being non-singular.

From now on, if we shall introduce new geometrical objects we shall verify the invariability of them at the transforming (2.1) and (2.2).

From the degeneration of the foliation F follows that the intersection of the distributions D_F and $D_F^{\perp}$ is non-null therefore T_pL and $T_pL^{\perp}$ are orthogonal non-complementary degenerate subspaces in T_pM $\forall p \in M$, L being the leaf passing through $p \in M$.

We define now: $N = D_F \cap D_F^{\perp}$ named accordingly with [2] the null distribution of M appropriate the foliation F .

Let r=dim N . By the lemma 1.2 follows that $r \leq \min\{q, m+n-q\}$ and how $N \subset D_F$, $N \subset D_F^{\perp}$ follows that $r \leq \min\{q,n,m,m+n-q\}$. We can consider always (taking possible -g like metric on M) that we have: $q \leq \left[\frac{m+n}{2}\right]$ (where [a] is the bigest integer less then a). Because $q \leq m+n-q$ follows that $1 \leq r \leq \min\{q,n,m\} \leq \min\{n,m\}$ from where:

$$1 \leq r \leq \min\{n,m\}$$

Definition 2.3 The foliation F of M is called r-degenerate foliation (or degenerate foliation if the rank r is undercurrent from context) if the null distribution is of dimension r.

If we consider the bracket [X,Y] $\forall X,Y \in N$ follows that the null distribution is not necessary integrable.

From this reason we shall distinguish in what follows two important cases: N is an integrable distribution or N is not integrable.

Let suppose now that N is an integrable distribution. If we consider in M an open neighbourhood U and an adapted basis for the null distribution: $\{\xi_1,...,\xi_r,X_{r+1},...,X_n\}$ where ξ_i are vector fields defined on U hwo generates N and X_j vector fileds defined on U which complete the basis for D_F follows that N_p $\forall p \in U$ is the tangent space for a submanifold of the leaf L passing through $p \in U$.

The problem is now what is happend at the intersection of two coordinates neighbourhoods of an arbitrary point $p \in M$. If U and V are two such neighbourhoods such that $U \cap V \neq \varnothing$ let consider $\{\xi_1,...,\xi_r,X_{r+1},...,X_m\}$ a basis for $D_F \big|_U$ and $\{\xi'_1,...,\xi'_r,X'_{r+1},...,X'_m\}$ a basis for $D_F \big|_V$.

That $\{\xi'_1,...,\xi'_r\}$ be a basis for $N\,|_V$ it must that $\xi'_i=\sum_{j=1}^{r}\alpha_i^j\xi_j$. In that case $\left[\xi'_i,\xi'_j\right]=\sum_{k,p=1}^{r}\left\{\alpha_i^k\alpha_j^k\left[\xi_k,\xi_p\right]+\alpha_i^k\xi_k(\alpha_j^p)\xi_p-\alpha_j^p\xi_p(\alpha_i^k)\xi_k\right\}$ and with the integrability of $N\,|_U$ follows that $N\,|_V$ is also integrable. Therefore the integrability of N in a point $p\in M$ does not depend of its coordinates neighbourhood.

We see upper that for a leaf L passing through $p\in M$ the subspaces T_pL and $T_p(L)^{\perp}$ are not complementary. In order that we can introduce similar notions to the geometry of the nondegenerate foliations it is necessary the construction of a distribution complementary to those of the foliation, called the transversal distribution, hwo is different from that orthogonal.

In order that we can build now the transversal fibre bundle of a degenerate foliation it is necessary to distinguish between four cases: **I.**$1\le r<\min\{m,n\}$; **II.**$1\le r=m<n$; **III.**$1\le r=n<m$; **IV.**$1\le r=m=n$.

Case I. $1\le r<\min\{m,n\}$ In that case the foliation is called r-degenerate foliation the danger of confusion being discarded because we shall specify always if it is the general case or those particular.

Let consider now $S(F)$ the complementary distribution orthogonal to N in D_F. We call it, in agreement with [2] $S(F)$-the screen distribution of the foliation F. We have therefore the direct orthogonal sum:

$$D_F=N\perp S(F) \tag{2.3}$$

The screen distribution $S(F)$ is nondegenerate relative to g. Indeed, if $\exists Z\in S(F)$ such that $g(Z,Y)=0\ \forall Y\in S(F)$ then like $Z\in D_F$ we have also $g(Z,\xi)=0\ \forall\xi\in N$. It follows therefore that $g(Z,X)=0\ \forall X\in D_F$ hwo imply the fact that $Z\in N$. But this fact comes into contradiction with $N\cap S(F)=\{0\}$.

We shall suppose in what follows that ind(g) is constant on $S(F)$.

Remark The screen distribution $S(F)$ is not unique determined by N in D_F therefore from this reason every time we shall obtain a result we shall examine the relationship of this from $S(F)$.

Let now the complementary distribution of N in the orthogonal distribution $D_{F^{\perp}}$ marked with $S(F^{\perp})$ and called the transversal screen distribution of the foliation F .

Like in the case of the screen distribution this is nondegenerate relative to g. We have therefore the orthogonal decomposition:

(2.4) $$D_{F^{\perp}} = N \perp S(F^{\perp})$$

Because S (F) is nondegenerate in TM we consider the decomposition:

(2.5) $$TM = S(F) \perp S(F)^{\perp}$$

where $S(F)^{\perp}$ is the complementary distribution, orthogonal to S (F) in TM. We have therefore, finally the following decomposition:

(2.6) $$S(F)^{\perp} = S(F^{\perp}) \perp S(F^{\perp})^{\perp}$$

where $S(F^{\perp})^{\perp}$ is the complementary distribution, orthogonal to $S(F^{\perp})$ in $S(F)^{\perp}$.

Considering now $\xi \in N$ follows from (2.3) $\xi \perp S(F)$ therefore from (2.5) we have: $\xi \in S(F)^{\perp}$. From (2.4) follows $\xi \perp S(F^{\perp})$. Finally, from the decomposition (2.6) follows that $\xi \in S(F^{\perp})^{\perp}$. We have therefore $N \subset S(F^{\perp})^{\perp}$.

We shall note from now on, a r-degenerate foliation with:

$(F, g, S(F), S(F^{\perp}))$.

Remark From the fact that dim N =r we have:

$\dim S(F) = n-r$, $\dim S(F)^{\perp} = m+r$, $\dim S(F^{\perp}) = m-r$, $\dim S(F^{\perp})^{\perp} = 2r$

Lemma 2.1 Let $(F, g, S(F), S(F^{\perp}))$ a r-degenerate foliation of a Semi-Riemannian manifold (M,g). If U is an open set of M and $\{\xi_1,...,\xi_r\}$ is a basis of $N|_U$ then there are vector fields $\{N_1,...,N_r\}$ from $S(F^{\perp})^{\perp}|_U$) such that:

(2.7) $$g(N_i, \xi_j) = \delta_{ij}$$

(2.8) $$g(N_i, N_j) = 0$$

$\forall$ i,j=1,...,r.

Proof. Let consider the distribution H complementary to N in S $(F^{\perp})^{\perp}$ and a basis $\{V_1,...,V_r\}$ of $H\big|_U$. Relative to the decomposition $S(F^{\perp})^{\perp}=N\perp H$ the vector fileds N_i have the expressions:

$$N_i=\sum_{k=1}^{r}\left(\alpha_i^k\xi_k+\beta_i^kV_k\right),\ i=1,...,r \tag{2.9}$$

where α_i^k and β_i^k are smooth mapings on U. We shall define the matrices of r-order: $A=(\alpha_i^j)$, $B=(\beta_i^j)$, $C=(g(V_i,\xi_j))$, $D=(g(V_i,V_j))$. In order that N_i satisfy the relations (2.7), (2.8) it must that:

$$\delta_{ij}=g(N_i,\xi_j)=g(\sum_{k=1}^{r}\alpha_i^k\xi_k+\sum_{k=1}^{r}\beta_i^kV_k,\xi_j)=\sum_{k=1}^{r}\alpha_i^kg(\xi_k,\xi_j)+\sum_{k=1}^{r}\beta_i^kg(V_k,\xi_j)=$$

$$\sum_{k=1}^{r}\beta_i^kg(V_k,\xi_j)$$

$$0=g(N_i,N_j)=g(\sum_{k=1}^{r}\alpha_i^k\xi_k+\sum_{k=1}^{r}\beta_i^kV_k,\sum_{s=1}^{r}\alpha_j^s\xi_s+\sum_{s=1}^{r}\beta_j^sV_s)=\sum_{k=1}^{r}\sum_{s=1}^{r}\alpha_i^k\alpha_j^sg(\xi_k,\xi_s)+$$

$$\sum_{k=1}^{r}\sum_{s=1}^{r}\alpha_i^k\beta_j^sg(\xi_k,V_s)+\sum_{k=1}^{r}\sum_{s=1}^{r}\beta_i^k\alpha_j^sg(V_k,\xi_s)+\sum_{k=1}^{r}\sum_{s=1}^{r}\beta_i^k\beta_j^sg(V_k,V_s)=$$

$$\sum_{k=1}^{r}\sum_{s=1}^{r}\alpha_i^k\beta_j^sg(\xi_k,V_s)+\sum_{k=1}^{r}\sum_{s=1}^{r}\beta_i^k\alpha_j^sg(V_k,\xi_s)+\sum_{k=1}^{r}\sum_{s=1}^{r}\beta_i^k\beta_j^sg(V_k,V_s)$$

With the matrices upper introduced, these relations become:

$$BC=I \tag{2.10}$$

$$AC^tB^t+BCA^t+BDB^t=0 \tag{2.11}$$

where I is the identity of $M_r(\mathbf{R})$ and ^{t} describes the transpose of a matrix. Because $S(F^{\perp})^{\perp}$ is nondegenerate follows that C is invertible therefore from (2.10):

$$B=C^{-1} \tag{2.12}$$

From (2.11), (2.12) follows:

$$A+A^t=-C^{-1}D(C^{-1})^t \tag{2.13}$$

therefore:

(2.14) $$A = -\frac{1}{2}C^{-1}D(C^{-1})^t + S$$

for any skew-symmetrical matrix S of r-order. From (2.9) we have:

(2.15) $$N = \left(-\frac{1}{2}C^{-1}D(C^{-1})^t + S\right)\xi + C^{-1}V$$

where we note $N=(N_1,...,N_r)^t$, $\xi=(\xi_1,...,\xi_r)^t$, $V=(V_1,...,V_r)^t$.

From (2.7) and (2.8) follows easy that $\{\xi_1,...,\xi_r,N_1,...,N_r\}$ is a basis of $S(F^{\perp})^{\perp}\big|_U$.

Remark From (2.15) follows that the vector fields N_i, i=1,...,r are not unique determined, they depending by the arbitrary choice of the matrix S.

Theorem 2.1 Let (F, g, S (F), S ($F^{\perp}$)) a r-degenerate foliation of a Semi-Riemannian manifold (M,g). There is a complementary distribution of N in S ($F^{\perp}$)$^{\perp}$ marked with deg(F) and called the degenerate transversal distribution of the foliation F relative to S (F) and S ($F^{\perp}$) such that the vector fields $\{N_1,...,N_r\}$ defined in lemma 2.1 is a basis for deg(F)).

Proof. From lemma 2.1 let $\{N_1,...,N_r\}$ be definite through (2.15). Considering another open set U' of M such that $U\cap U'\neq\emptyset$ let $\{\xi'_1,...,\xi'_r\}$ and $\{V'_1,...,V'_r\}$ basis of $N\big|_{U'}$ respectively $H\big|_{U'}$. If we note like in lemma 2.1: $N'=(N_1',...,N_r')^t$, $\xi'=(\xi_1',...,\xi_r')^t$, $V'=(V_1',...,V_r')^t$ we have $\xi'=E\xi$ and $V'=FV$ where E and V are nonsingular matrices. With the notations $C'=(g(V_i',\xi_j'))$, $D'=(g(V_i',V_j'))$ we have on $U\cap U'$:

(2.16) $$C'=FCE^t$$

(2.17) $$D'=FDF^t$$

From (2.15) we have on U':

(2.18) $$N'=\left(-\frac{1}{2}C'^{-1}D'(C'^{-1})^t + S'\right)\xi'+C'^{-1}V'$$

with S' skew-symmetrical matrix. Using (2.16) and (2.17) in (2.18) we have:

(2.19) $$N' = E^{-1}\left[\left(-\frac{1}{2}C^{-1}D\left(C^{-1}\right)^{t} + E^{t}S'E\right)\xi + C^{-1}V\right]$$

If we choose $S'=(E^{-1})^{t}SE^{-1}$ which is also skew-symmetric, we have

(2.20) $$N'=E^{-1}N$$

From (2.20) follows that there is the distribution deg(F) generated by $\{N_1,...,N_r\}$ from lemma 2.1.

Let now show that deg(F) is complementary to N in S $(F^{\perp})^{\perp}$. If we suppose the reverse, let $0\neq X\in N \cap \deg(F)$. Considering a basis like upper we have: $X=\sum_{i=1}^{r} a^{i}N_i = \sum_{j=1}^{r} b^{j}\xi_j$. Using (2.7) and (2.8) we have that:

$$0 = g(\sum_{j=1}^{r} b^{j}\xi_j, \xi_i) = g(X, \xi_i) = g(\sum_{j=1}^{r} a^{j}N_j, \xi_i) = a^{j}$$

therefore X=0-contradiction with our suppose. How dim deg(F)=r and ind$\{N_1,...,N_r\}$ follows that the set of vector fields $\{N_1,...,N_r\}$ is a basis of (deg(F)).

Remark From the theorem 2.1 we conclude that dim (deg(F))=r.

If we return now to the beginning problem, that is the replacement of the classical orthogonal distribution with a complementary distribution to D_F in TM, let therefore the ortogonal direct sum:

(2.21) $$tr(F)=\deg(F)\perp S(F^{\perp})$$

where deg(F) is an arbitrary degenerate transversal distribution of F . From (2.21) follows that tr(F) is a distribution on M named the transversal distribution of the foliation F .

The dimension of this distribution is therefore:

dim tr(F)=dim deg(F)+dim S $(F^{\perp})$=r+n-r=n

Finally, we have the decomposition:

(2.22) $$TM=D_F \oplus tr(F)=S(F)\perp S(F^{\perp})\perp(N \oplus deg(F))$$

From the upper considerations we have on TM a local quasi-orthonormal basis along to F in an open neighbourhood U: $\{X_{r+1},...,X_n, W_{r+1},...,W_m,\xi_1,...,\xi_r,N_1,...,N_r\}$ where $X_\alpha \in S(F)|_U$, $\alpha=r+1,...,n$, $W_a \in S(F^{\perp})|_U$, $a=r+1,...,m$, $\xi_i \in N|_U$, $i=1,...,r$, $N_i \in deg(F)|_U$, $i=1,...,r$.

From now on we shall make the understanding about the indexes: $\alpha,\beta,...=r+1,...,n$; $a,b,...=r+1,...,m$; $i,j,...= 1,...,r$.

On $(N \oplus deg(F))|_U$ we have an orthonormal basis:

$$\left\{u_i = \frac{\xi_i - N_i}{\sqrt{2}}, v_i = \frac{\xi_i + N_i}{\sqrt{2}}\right\}_{i=1,...,r}$$ and how $g(u_i,u_i)=-1$, $g(v_i,v_i)=1$, $i=1,...,n$ follows that the index of $(N \oplus deg(F))|_U=r$. Because $N \oplus deg(F)$ is nondegenerate, by the lemma 1.1 and (2.22) we have:

(2.23) $$q=ind(S(F))+ind(S(F^{\perp}))+r$$

Theorem 2.2 Let $(F, g, S(F), S(F^{\perp}))$ a r-degenerate foliation of a Semi-Riemannian manifold (M,g). If the index of the manifold M and those of the null distribution N are equals nule, then S (F) and $S(F^{\perp})$ are Riemannian distributions.

Proof. We consider in (2.23) q=r from where $ind(S(F))+ind(S(F^{\perp}))=0$ therefore $ind(S(F))=ind(S(F^{\perp}))=0$.

Corollary 2.1 Let $(F, g, S(F), S(F^{\perp}))$ a r-degenerate foliation of a Lorentz manifold (M,g). Then S (F) and $S(F^{\perp})$ are Riemannian distributions.

Proof. On a Lorentz manifold we have q=1 and how $1 \le r \le q$ follows r=1. The assertion reduce to the check of the theorem 2.2.

Case II. $1 \le r=m<n$ In this case the foliation is called coisotropic foliation. How $N \subset D_F^{\perp}$ and $\dim N = r = m = \dim D_F^{\perp}$ follows that $N = D_F^{\perp}$ therefore $S(F^{\perp})=\{0\}$. Considering now the screen distribution S (F) we have: $D_F = S(F) \perp D_F^{\perp}$.

We note from now on a coisotropic foliation with $(F, g, S(F))$.

Remark From the fact that dim N =r=m we have:

dim S (F)=n-m, dim S (F)$^{\perp}$=dim S (F $^{\perp}$)$^{\perp}$=2m

Similar to the proofs of lemma 2.1 and of theorem 2.1 follows:

Lemma 2.2 Let (F, g, S (F)) a coisotropic foliation of a Semi-Riemannian manifold (M,g). If U is an open set from M and $\{\xi_1,...,\xi_m\}$ a basis of $D_F{}^{\perp}|_U$ then there is a system of vector fields $\{N_1,...,N_m\}$ of $S(F)^{\perp}|_U$ such that

(2.24) $$g(N_i,\xi_j)=\delta_{ij}$$

(2.25) $$g(N_i,N_j)=0$$

$\forall$ i,j=1,...,m.

Theorem 2.3 Let (F, g, S (F)) a coisotropic foliation of a Semi-Riemannian manifold (M,g). Then there is a complementary distribution of $D_F{}^{\perp}$ in $S(F^{\perp})^{\perp}$ noted with deg(F) and called the degenerate transversal distribution of the foliation F relative to S (F) such that the system of vector fields $\{N_1,...,N_m\}$ introduced in lemma 2.2 is a basis of deg(F).

The transversal distribution of F becomes:

(2.26) $$tr(F)=deg(F)$$

and the decomposition of TM is:

(2.27) $$TM=S(F)\perp D_F{}^{\perp}\oplus deg(F))$$

The local quasi-orthonormal basis along F in an open neighbourhood U is: $\{X_{m+1},...,X_n,\xi_1,...,\xi_m,N_1,...,N_m\}$ where $X_\alpha\in S(F)|_U$, α=m+1,...,n, $\xi_i\in D_F{}^{\perp}|_U$, i=1,...,m and $N_i\in deg(F)|_U$, i=1,...,m.

Relative to the index follows with the same remark like those preceding the theorem 2.2:

(2.28) $$q=m+ind(S(F))$$

From (2.28) follows:

Theorem 2.4 Any screen distribution of a coisotropic foliation in a Semi-Riemannian manifold has constant index q-m.

Corollary 2.2 In a coisotropic foliation of a Semi-Riemannian manifold with the index equal of those of the orthogonal distribution, the screen distribution becomes Riemannian.

Case III. 1≤r=n<m In this case the foliation is called isotropic foliation. How $N \subset D_F^{\perp}$ and dim N =r=n=dim D_F follows that $N = D_F$ therefore $S(F) = \{0\}$. Considering the transversal screen distribution $S(F^{\perp})$ we have:

$D_F^{\perp} = D_F \perp S(F^{\perp})$.

We shall note from now on an isotropic foliation with $(F, g, S(F^{\perp}))$.

Remark From the fact that dim N =r=n we have:

$\dim S(F)^{\perp} = m+n$, $\dim S(F^{\perp}) = m-n$, $\dim S(F^{\perp})^{\perp} = 2n$

Remark In the case of an isotropic foliation $N = D_F$ therefore N is an integrable distribution.

Similar to the proofs of lemma 2.1 and of theorem 2.1 follows:

Lemma 2.3 Let $(F, g, S(F^{\perp}))$ an isotropic foliation of a Semi-Riemannian manifold (M,g). If U is an open set of M and $\{\xi_1,...,\xi_n\}$ is a basis of $D_F|_U$ then there is a system of vector fields $\{N_1,...,N_n\}$ of $S(F^{\perp})^{\perp}|_U$ such that

(2.29) $$g(N_i,\xi_j)=\delta_{ij}$$

(2.30) $$g(N_i,N_j)=0$$

$\forall$ i,j=1,...,n.

Theorem 2.5 Let $(F, g, S(F^{\perp}))$ an isotropic foliation of a Semi-Riemannian manifold (M,g). Then there is a complementary distribution to D_F in $S(F^{\perp})^{\perp}$ noted with deg(F) and called the degenerate transversal distribution of the foliation F

relative to S ($F^{\perp}$) such that the system of vector fields $\{N_1,...,N_n\}$ defined in lemma 2.3 is a basis of deg(F)).

The transversal distribution of F is now:

(2.31) $$tr(F)=deg(F)\perp S(F^{\perp})$$

and the decomposition of TM becomes:

(2.32) $$TM=D_F \oplus deg(F))\perp S(F^{\perp})$$

The local quasi-orthonormal basis along F in an open neighbourhood U is: $\{\xi_1,...,\xi_n,N_1,...,N_n,W_{n+1},...,W_m\}$ where $W_a \in S(F^{\perp})|_U$, a=n+1,...,m, $\xi_i \in D_F|_U$, i=1,...,n and $N_i \in deg(F)|_U$, i=1,...,n.

With the same remark like those preceding the theorem 2.2 we have:

(2.33) $$q=n+ind(S(F^{\perp}))$$

From (2.33) follows:

Theorem 2.6 Any transversal screen distribution of an isotropic foliation in a Semi-Riemannian manifold has constant index q-n.

Corollary 2.3 In an isotropic foliation of a Semi-Riemannian manifold with index equal with those of the foliation's distribution, the transversal screen distribution becomes Riemannian.

Case IV. 1≤r=m=n In this case, the foliation is called totally degenerate foliation. How $N \subset D_F$ and $N \subset D_F^{\perp}$ and dim N =r=m=n=dim D_F = dim $D_F^{\perp}$ follows that $N=D_F=D_F^{\perp}$ therefore $S(F)=S(F^{\perp})=\{0\}$.

We note from now on a totally degenerate foliation with (F, g).

Remark From the fact that dim N =r=n=m we have:

dim $S(F)^{\perp}$=2m, dim $S(F^{\perp})^{\perp}$=2m

Remark In the case of a totally degenerate foliation $N=D_F$ therefore N is an integrable distribution.

We have now analogously with lemma 2.1 and theorem 2.1:

Lemma 2.4 Let (F, g) a totally degenerate foliation of a Semi-Riemannian manifold (M,g). If U is an open set of M and $\{\xi_1,...,\xi_m\}$ is a basis of $D_F|_U$ then there is a system of vector fields $\{N_1,...,N_m\}$ of $TM|_U$ such that

(2.34) $$g(N_i,\xi_j)=\delta_{ij}$$

(2.35) $$g(N_i,N_j)=0$$

$\forall$ i,j=1,...,m.

Theorem 2.7 Let (F, g) a totally degenerate foliation of a Semi-Riemannian manifold (M,g). Then there is a complementary distribution of D_F in TM noted with deg(F) and called the degenerate transversal distribution of the foliation F such that the system of vector fields $\{N_1,...,N_m\}$ defined in lemma 2.4 is a basis of deg(F).

The transversal distribution of F is now:

(2.36) $$tr(F)=deg(F)$$

and the decomposition of TM becomes:

(2.37) $$TM=D_F\oplus deg(F)$$

The local quasi-orthonormal basis along F in an open neighbourhood U of M is: $\{\xi_1,...,\xi_m,N_1,...,N_m\}$ where $\xi_i\in D_F|_U$), i=1,...,m and $N_i\in deg(F)|_U$), i=1,...,m.

With the same remark like those preceding the theorem 2.2 we have:

(2.38) $$q=m$$

From (2.38) we have:

Theorem 2.8 A degenerate foliation of a Semi-Riemannian manifold can be totally degenerate only if the codimension of the foliation is equal with the index of the manifold.

Finally in this section we shall investigate two problems:

- Which are the conversion formulae of a local quasi-orthonormal basis along F in a coordinate neighbourhood U when we change the screen distribution?
- Which are the conversion formulae of a local quasi-orthonormal basis along F at the change of the coordinate neighbourhood?

Before the beginning we make the following:

Remark Let $X=(X_1,...,X_n)^t$ and $Y=(Y_1,...,Y_m)^t$, $n,m\geq 1$ two systems of vector fields where X_i, $Y_j\in TM$, $i=1,...,n$, $j=1,...,m$. Let consider also $X'=(X_1',...,X_n')^t$ and $Y'=(Y_1',...,Y_m')^t$ another two systems of vector fields with X_i', $Y_j'\in TM$, $i=1,...,n$, $j=1,...,m$. Let $A=(a_{ij})\in M_n(\mathbf{R})$ and $B=(b_{ij})\in M_m(\mathbf{R})$ the passing matrices from X at X' respectively from Y at Y'. We have therefore X'=AX and Y'=BY. Let consider now the matrices $G(X,Y)=(g(X_i,Y_j))\in M_{nm}(\mathbf{R})$ and $G(X',Y')=(g(X_i',Y_j'))\in M_{nm}(\mathbf{R})$. We have:

$$g(X_i', Y_j') = g(\sum_{k=1}^{n} a_{ik}X_k, \sum_{p=1}^{m} b_{jp}Y_p) = \sum_{k=1}^{n}\sum_{p=1}^{m} a_{ik}g(X_k, Y_p)b_{jp}\ ,\ i=1,...,n,j=1,...,m$$

from where we obtain the relation:

$$G(X',Y')=AG(X,Y)B^t \qquad (2.39)$$

where G(X,Y) is the Gram determinat of X and Y.

For the first question, let consider for the beginning the case of r-degenerate foliations with $1\leq r<\min\{m,n\}$. Let U a coordinates neighbourhood of M and $\{\xi_1,...,\xi_r,X_{r+1},...,X_n,W_{r+1},...,W_m,N_1,...,N_r\}$ a local quasi-orthonormal basis along F in U and $\{\xi_1,...,\xi_r,X'_{r+1},...,X'_n,W'_{r+1},...,W'_m, N'_1,...,N'_r\}$ a local quasi-orthonormal basis along F in U relative to the decompositions $TM=S(F)\perp S(F^{\perp})\perp(N\oplus deg(F))$ respectively
$TM=S'(F)\perp S'(F^{\perp})\perp(N\oplus deg'(F))$.

Let therefore:

(2.40)
$$\begin{pmatrix} \xi \\ X' \\ W' \\ N' \end{pmatrix} = \begin{pmatrix} I & 0 & 0 & 0 \\ A_1 & A_2 & 0 & 0 \\ B_1 & B_2 & B_3 & B_4 \\ C_1 & C_2 & C_3 & C_4 \end{pmatrix} \begin{pmatrix} \xi \\ X \\ W \\ N \end{pmatrix}$$

where we note with ξ,X,W,N,X',W',N' the matrices who have like components the vector fields with the same name and A_i,B_j,C_j, i=1,2, j=1,2,3,4 are matrices of corresponding dimensions, A_2 and B_3 being nonsingular.

The conditions for the first basis are:

(2.41) $G(\xi,\xi)=0$, $G(\xi,X)=0$, $G(\xi,W)=0$, $G(\xi,N)=I$

$G(X,X)=G_{X,}$ $G(X,W)=0$, $G(X,N)=0$

$G(W,W)=G_{W,}$ $G(W,N)=0$

$G(N,N)=0$

and for the second:

(2.42) $G(\xi,\xi)=0$, $G(\xi,X')=0$, $G(\xi,W')=0$, $G(\xi,N')=I$

$G(X',X')=G'_{X,}$ $G(X',W')=0$, $G(X',N')=0$

$G(W',W')=G'_{W,}$ $G(W',N')=0$

$G(N',N')=0$

where we have note: $G(X,X)=G_X \in M_{n-r}(\mathbf{R})$, $G(W,W)=G_W \in M_{m-r}(\mathbf{R})$, $G(X',X')=G'_X \in M_{n-r}(\mathbf{R})$, $G(W',W')=G'_W \in M_{m-r}(\mathbf{R})$. Is obvious that G_X,G_W,G'_X and G'_W are nonsingular matrices.

From (2.40)-(2.42) we have after the notation: $A_2=A$, $B_3=B$, $C_1=E$, $C_2=C$, $C_3=D$:

(2.43)
$$\begin{pmatrix} \xi \\ X' \\ W' \\ N' \end{pmatrix} = \begin{pmatrix} I & 0 & 0 & 0 \\ -AG_X C^t & A & 0 & 0 \\ -BG_W D^t & 0 & B & 0 \\ E & C & D & I \end{pmatrix} \begin{pmatrix} \xi \\ X \\ W \\ N \end{pmatrix}$$

where

(2.44)
$$G_X A^t = A^{-1} G'_X$$

$$G_W B^t = B^{-1} G'_W$$

$$E+E^t+CG_X C^t+DG_W D^t=0$$

If we coinsider now the case of coisotropic foliations, let U a coordinates neighbourhood of M and $\{\xi_1,...,\xi_m,X_{m+1},...,X_n,N_1,...,N_m\}$ a local quasi-orthonormal basis along F in U and $\{\xi_1,...,\xi_m,X'_{m+1},...,X'_n,N'_1,...,N'_m\}$ a local quasi-orthonormal basis along F in U relative to the decompositions $TM=S(F)\perp D_F^{\perp}\oplus \deg(F)$ and $TM=S'(F)\perp D_F^{\perp}\oplus \deg'(F)$ respectively. Let therefore:

(2.45)
$$\begin{pmatrix} \xi \\ X' \\ N' \end{pmatrix} = \begin{pmatrix} I & 0 & 0 \\ A_1 & A_2 & 0 \\ B_1 & B_2 & B_3 \end{pmatrix} \begin{pmatrix} \xi \\ X \\ N \end{pmatrix}$$

with the same notations like upper.

The conditions for the first basis are:

(2.46) $G(\xi,\xi)=0$, $G(\xi,X)=0$, $G(\xi,N)=I$

$G(X,X)=G_X$, $G(X,N)=0$

$G(N,N)=0$

and for the second:

(2.47) $G(\xi,\xi)=0,\ G(\xi,X')=0,\ G(\xi,N')=I$

$G(X',X')=G'_X,\ G(X',N')=0$

$G(N',N')=0$

where we have note: $G(X,X)=G_X\in M_{n-m}(\mathbf{R})$, $G(X',X')=G'_X\in M_{n-m}(\mathbf{R})$. It is obvious that G_X and G'_X are nonsingular matrices.

From (2.45)-(2.47) we have with the notations: $A_2=A$, $B_1=E$, $B_2=C$:

(2.48) $$\begin{pmatrix}\xi\\ X'\\ N'\end{pmatrix}=\begin{pmatrix}I & 0 & 0\\ -AG_XC^t & A & 0\\ E & C & I\end{pmatrix}\begin{pmatrix}\xi\\ X\\ N\end{pmatrix}$$

where:

(2.49) $G_XA^t=A^{-1}G'_X$

$E+E^t+CG_XC^t=0$

Let now the case of the isotropic foliations and U a coordinates neighbourhood of M, $\{\xi_1,...,\xi_n,W_{n+1},...,W_m,N_1,...,N_n\}$ a local quasi-orthonormal basis along F in U and $\{\xi_1,...,\xi_n,W'_{n+1},...,W'_m,N'_1,...,N'_n\}$ a local quasi-orthonormal basis along F in U relative to the decompositions $TM=(D_F\oplus deg(F))\perp S\ (F^{\perp})$ and $TM=(D_F\oplus deg'(F))\perp S\ '(F^{\perp})$ respectively. Let therefore:

(2.50) $$\begin{pmatrix}\xi\\ W'\\ N'\end{pmatrix}=\begin{pmatrix}I & 0 & 0\\ A_1 & A_2 & A_3\\ B_1 & B_2 & B_3\end{pmatrix}\begin{pmatrix}\xi\\ W\\ N\end{pmatrix}$$

with the same notations like upper.

The conditions for the first basis are:

(2.51) $G(\xi,\xi)=0,\ G(\xi,W)=0,\ G(\xi,N)=I$

$G(W,W)=G_W,\ G(W,N)=0$

$G(N,N)=0$

and for the second:

(2.52) $G(\xi,\xi)=0,\ G(\xi,W')=0,\ G(\xi,N')=I$

$G(W',W')=G'_W,\ G(W',N')=0$

$G(N',N')=0$

where we have note: $G(W,W)=G_W\in M_{m-n}(\mathbf{R})$, $G(W',W')=G'_W\in M_{m-n}(\mathbf{R})$.

Is obvious that G_W and G'_W are nonsingular matrices.

From (2.50)-(2.52) we have with the notations: $A_2=B$, $B_1=E$, $B_2=D$:

(2.53)
$$\begin{pmatrix} \xi \\ W' \\ N' \end{pmatrix} = \begin{pmatrix} I & 0 & 0 \\ -BG_W D^t & B & 0 \\ E & D & I \end{pmatrix} \begin{pmatrix} \xi \\ W \\ N \end{pmatrix}$$

where:

(2.54) $G_W B^t=B^{-1}G'_W$

$E+E^t+DG_WD^t=0$

Finally, let consider now the case of totally degenerate foliations and let U a coordinates neighbourhood of M, $\{\xi_1,...,\xi_m,N_1,...,N_m\}$ a local quasi-orthonormal basis along F in U and $\{\xi_1,...,\xi_m, N'_1,...,N'_m\}$ a local quasi-orthonormal basis along F in U relative to the decompositions $TM=D_F \oplus deg(F)$ and $TM=D_F \oplus deg'(F)$ respectively.

Let therefore:

(2.55) $$\begin{pmatrix} \xi \\ N' \end{pmatrix} = \begin{pmatrix} I & 0 \\ A_1 & A_2 \end{pmatrix} \begin{pmatrix} \xi \\ N \end{pmatrix}$$

with the same notations like upper.

The conditions for the first basis are:

(2.56) $$G(\xi,\xi)=0,\ G(\xi,N)=I$$

$G(N,N)=0$

and for the second:

(2.57) $$G(\xi,\xi)=0,\ G(\xi,N')=I$$

$G(N',N')=0$

From (2.55)-(2.57) we have with the notation: $A_1=E$:

(2.58) $$\begin{pmatrix} \xi \\ N' \end{pmatrix} = \begin{pmatrix} I & 0 \\ E & I \end{pmatrix} \begin{pmatrix} \xi \\ N \end{pmatrix}$$

where E is a matrix of n-order satisfying the relation:

(2.59) $$E+E^t=0$$

being therefore skew-symmetric.

Let consider now the second question. We treat this in the case of the integrability of the null distribution N.

For the beginning we shall analyse the case of r-degenerate foliations with $1\leq r<\min\{m,n\}$.

If we have U and V two coordinates neighbourhoods such that $U\cap V\neq\varnothing$ let consider $\{\xi_1,...,\xi_r,X_{r+1},...,X_n,W_{r+1},...,W_m,N_1,...,N_r\}$ a local quasi-orthonormal basis along F in U and $\{\xi'_1,...,\xi'_r,X'_{r+1},...,X'_n,W'_{r+1},...,W'_m,\ N'_1,...,N'_r\}$ a local quasi-

orthonormal basis along F in V. From (2.1), (2.2) and the integrability of N on $U\cap V$ we have:

$$(2.60)\qquad \begin{pmatrix} \xi' \\ X' \\ W' \\ N' \end{pmatrix} = \begin{pmatrix} A & 0 & 0 & 0 \\ B & C & D & E \\ 0 & 0 & F & G \\ 0 & 0 & H & J \end{pmatrix} \begin{pmatrix} \xi \\ X \\ W \\ N \end{pmatrix}$$

where A,B,C,D,E,F,G,H,J are matrices of appropriate dimensions. Let note also that A,C and $\begin{pmatrix} F & G \\ H & J \end{pmatrix}$ are nonsingular matrices. If we proceed similar like in the first problem we have finally:

Theorem 2.9 Let F a r-degenerate foliation with integrable null distribution of a Semi-Riemannian manifold (M,g). If we have U and V two coordinates neighbourhoods in an arbitrary point $p\in M$ such that $U\cap V\neq\varnothing$ and if we shall consider $\{\xi_1,...,\xi_r,X_{r+1},\ ...,X_n,W_{r+1},...,W_m,N_1,...,N_r\}$ a local quasi-orthonormal basis along F in U and $\{\xi'_1,...,\xi'_r,X'_{r+1},...,X'_n,W'_{r+1},...,W'_m,\ N'_1,...,N'_r\}$ a local quasi-orthonormal basis along F in V, follows:

$$(2.61)\qquad \begin{pmatrix} \xi' \\ X' \\ W' \\ N' \end{pmatrix} = \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & C & 0 \\ 0 & 0 & 0 & (A^{-1})^t \end{pmatrix} \begin{pmatrix} \xi \\ X \\ W \\ N \end{pmatrix}$$

where A is a nonsingular matrix of r-order and B and C orthogonal matrices of n-r respectively m-r orders satisfying in addition the conditions:

$$(2.62)\qquad BG_XB^t=G'_X$$

$$CG_WC^t=G'_W$$

In the cases of coisotropic, isotropic and totally degenerate foliations we have analogously:

Theorem 2.10 Let F a coisotropic foliation with integrable null distribution of a Semi-Riemannian manifold (M,g). If we have U and V two coordinates

neighbourhoods in an arbitrary point $p\in M$ such that $U\cap V\neq\varnothing$ and if we shall consider $\{\xi_1,...,\xi_m,X_{m+1},...,X_n,N_1,...,N_m\}$ a local quasi-orthonormal basis along F in U and $\{\xi'_1,...,\xi'_m,X'_{m+1}, ...,X'_n,N'_1,...,N'_m\}$ a local quasi-orthonormal basis along F in V, follows:

$$\begin{pmatrix}\xi'\\X'\\N'\end{pmatrix}=\begin{pmatrix}A&0&0\\0&B&0\\0&0&(A^{-1})^t\end{pmatrix}\begin{pmatrix}\xi\\X\\N\end{pmatrix} \tag{2.63}$$

where A is a nonsingular matrix of m-order and B an orthogonal matrix of n-m-order satisfying in addition the condition:

$$BG_XB^t=G'_X \tag{2.64}$$

Theorem 2.11 Let F an isotropic foliation of a Semi-Riemannian manifold (M,g). If we have U and V two coordinates neighbourhoods in an arbitrary point $p\in M$ such that $U\cap V\neq\varnothing$ and if we shall consider $\{\xi_1,...,\xi_n,W_{n+1},...,W_m, N_1,...,N_n\}$ a local quasi-orthonormal basis along F in U and $\{\xi'_1,...,\xi'_n, W'_{n+1},...,W'_m,N'_1,...,N'_n\}$ a local quasi-orthonormal basis along F in V, follows:

$$\begin{pmatrix}\xi'\\W'\\N'\end{pmatrix}=\begin{pmatrix}A&0&0\\0&B&0\\0&0&(A^{-1})^t\end{pmatrix}\begin{pmatrix}\xi\\W\\N\end{pmatrix} \tag{2.65}$$

where A is a nonsingular matrix of n-order and B an orthogonal matrix of m-n-order satisfying in addition the condition:

$$BG_WB^t=G'_W \tag{2.66}$$

Theorem 2.12 Let F a totally degenerate foliation of a Semi-Riemannian manifold (M,g). If we have U and V two coordinates neighbourhoods in an arbitrary point $p\in M$ such that $U\cap V\neq\varnothing$ and if we shall consider $\{\xi_1,...,\xi_m, N_1,...,N_m\}$ a local quasi-orthonormal basis along F in U and $\{\xi'_1,...,\xi'_m, N'_1,...,N'_m\}$ a local quasi-orthonormal basis along F in V, follows:

$$\begin{pmatrix}\xi'\\N'\end{pmatrix}=\begin{pmatrix}A&0\\0&(A^{-1})^t\end{pmatrix}\begin{pmatrix}\xi\\N\end{pmatrix} \tag{2.67}$$

where A is a nonsingular matrix of m-order.

The end of this section consists in five examples of various kind of degenerate foliations with integrable null distribution. We shall see in the next chapters that particular types of foliations come into this hypotehesis.

In what follows in examples on $\mathbf{R}^n_\mu(\underbrace{--...-}_{\mu \text{ times}}\underbrace{+...+}_{n-\mu \text{ times}})$ with the coordinates $(x^1,...,x^n)$ we shall note $\partial_i = \frac{\partial}{\partial x^i}$, i=1,...,n. We shall note also the Semi-Riemannian metric on $\mathbf{R}^n_\mu$ with g.

2.1. Let the smooth map $f:\mathbf{R}\to\mathbf{R}$, $f^2(x)>1$ $\forall x\in\mathbf{R}$. Such an example is $f(x)=x^n+2$, $x\in\mathbf{R}$, $n\in\mathbf{N}$. Let also $\alpha\in\mathbf{R}-\left\{\frac{\pi}{2}+n\pi \middle| n\in\mathbf{Z}\right\}$. Let consider now the map $\varphi:\mathbf{R}^2\to\mathbf{R}$, $\varphi(x^1,x^2)=f(x^1\cos\alpha - x^2\sin\alpha)$ $\forall(x^1,x^2)\in\mathbf{R}^2$.

On the Semi-Riemannian manifold $M=\mathbf{R}^4_2(-,-,+,+)$ let the vector fields:

$$\xi=\sin\alpha\cos\alpha\,(\varphi^2(x^1,x^2)-1)\partial_1+\cos^2\alpha(\varphi^2(x^1,x^2)-1)\partial_2+\cos\alpha(\varphi^2(x^1,x^2)-1)\partial_3$$

$$X=\frac{1}{\cos\alpha\sqrt{\varphi^2(x^1,x^2)-1}}\partial_1+\frac{\sin\alpha}{\cos\alpha\sqrt{\varphi^2(x^1,x^2)-1}}\partial_2+\frac{\varphi(x^1,x^2)}{\sqrt{\varphi^2(x^1,x^2)-1}}\partial_4$$

We have now $g(\xi,\xi)=g(\xi,X)=0$ and $g(X,X)=1$. On the other hand:

$$[\xi,X]=-\frac{2\varphi(x^1,x^2)\varphi'(x^1,x^2)}{\sqrt{\varphi^2(x^1,x^2)-1}^3}\xi$$

therefore ξ and X defined a 1-degenerate foliation F on M.

A local quasi-orthonormal basis along the 1-degenerate foliation is given by: $\{X,W,\xi,N\}$ where:

$$W=\frac{\varphi(x^1,x^2)\cos\alpha}{\sqrt{\varphi^2(x^1,x^2)-1}}\partial_1-\frac{\varphi(x^1,x^2)\sin\alpha}{\sqrt{\varphi^2(x^1,x^2)-1}}\partial_2+\frac{1}{\sqrt{\varphi^2(x^1,x^2)-1}}\partial_4,$$

$$N=\frac{1}{2\cos^3\alpha\left(\varphi^2(x^1,x^2)-1\right)^2}[-\sin\alpha\,(1+\varphi^2(x^1,x^2)\cos^2\alpha)\partial_1+$$

$\cos\alpha\,(1-\varphi^2(x^1,x^2)\cos^2\alpha)\partial_2+(\varphi^2(x^1,x^2)\cos^2\alpha-1)\partial_3-2\sin\alpha\cos\alpha\,\varphi(x^1,x^2)\partial_4]$

2.2. Let $M=\mathbf{R}^4_2-\mathbf{R}\times\mathbf{R}\times\{-1,0,1\}\times\mathbf{R}$ and the vector fields:

$\xi=\sin x^4\,\partial_1+\cos x^4\,\partial_2+\partial_3$, $X=\cos x^4\,\partial_1-\sin x^4\,\partial_2+\frac{1}{x^3}\,\partial_4$

We have now $g(\xi,\xi)=g(\xi,X)=0$ and $g(X,X)=\frac{1}{(x^3)^2}-1\neq 0$ because $x^3\neq\pm 1$. On the other hand: $[\xi,X]=-\frac{1}{x^3}X$ therefore ξ and X defined a 1-degenerate foliation F on M.

A local quasi-orthonormal basis along the 1-degenerate foliation is given by: $\{X,W,\xi,N\}$ where:

$W=\frac{1}{\sqrt{\left|(x^3)^2-1\right|}}(\cos x^4\,\partial_1-\sin x^4\,\partial_2+x^3\partial_4)$,

$N=\frac{1}{2}(-\sin x^4\,\partial_1-\cos x^4\,\partial_2+\partial_3)$

2.3. Let the Semi-Riemannian manifold $M=\{(x,y,z)\in\mathbf{R}^3 \mid y\neq 0, z\neq 2k\pi, z\neq\frac{\pi}{2}+2k\pi, k\in\mathbf{Z}\}$ endowed with the metric g defined through:

$ds^2=dx^2+y^2dz^2+2(\sin z+\cos z-1)dxdy+2y(\cos z-\sin z)dxdz$

We have $\det g=-y^2(\sin z+\cos z-1)^2=-8y^2\sin^2\frac{z}{2}\sin^2\left(\frac{\pi}{4}-\frac{z}{2}\right)<0$.

If we apply the Jacobi theorem we have that g is a Semi-Riemannian metric of index 1.

We note in what follows $\partial_x=\frac{\partial}{\partial x},\partial_y=\frac{\partial}{\partial y},\partial_z=\frac{\partial}{\partial z}$. Let now the vector fields:

$\xi=-2(\sin z+\cos z-1)\partial_x+\partial_y$ and $X=-2y(\cos z-\sin z)\partial_x+\partial_z$

We have $g(\xi,\xi)=0$, $g(\xi,X)=0$ and $g(X,X)=y^2>0$. On the other hand: $[\xi,\xi]=[X,X]=0$ and $[\xi,X]=[\partial_y-2(\sin z+\cos z-1)\partial_x,-2y(\cos z-\sin z)\partial_x+\partial_z]=0$.

We have therefore a foliation F generated by the vector fields ξ and X. We have $D_F=\mathrm{Span}(\xi,X)$, $N=\mathrm{Span}(\xi)$, $S(F)=\mathrm{Span}(X)$. In order that the foliation be coisotropic it is necessary that $N=D_F^{\perp}$. We have:

$D_F^{\perp}=\{4\lambda(1+\cos z\sin z-\cos z-\sin z)\partial_x+\lambda(1-\cos z-\sin z)\partial_y \mid \forall\lambda\in F(M)\}=\mathrm{Span}(\xi)=N$ therefore the foliation F is coisotropic.

A local quasi-orthonormal basis along the coisotropic foliation is given by: $\{X,\xi,N\}$ where:

$$N=\frac{1}{9y(\sin z-\cos z)^2(\sin z+\cos z-1)^2}\left[y(\sin z+\cos z-1)\partial_x+\right.$$
$$\left.y(3\sin z\cos z-2)\partial_y+3(\sin z-\cos z)(\sin z+\cos z-1)\partial_z\right]$$

2.4. Let $M=\mathbf{R}^4{}_2(-,-,+,+)$ and the vector field: $\xi=\sin u\,\partial_1+\cos u\,\partial_2+\partial_3$ where u is an arbitrary smooth map on M. We have now: $g(\xi,\xi)=0$ and how $[\xi,\xi]=0$ follows that ξ defined an isotropic foliation.

A local quasi-orthonormal basis along the isotropic foliation is given by: $\{\xi,N,W_1,W_2\}$ where:

$N=\frac{1}{2}(-\sin u\,\partial_1-\cos u\,\partial_2+\partial_3)$,

$W_1=\partial_4$, $W_2=\cos u\,\partial_1-\sin u\,\partial_2$

2.5. Let $M=\mathbf{R}^4{}_2(-,-,+,+)$ and the vector fields: $\xi_1=f\partial_1+f\partial_3$, $\xi_2=h\partial_2+h\partial_4$ where f and h are smooth mapings on M, everywhere non-null. We have: $g(\xi_1,\xi_1)=0$, $g(\xi_2,\xi_2)=0$, $g(\xi_1,\xi_2)=0$ and $[\xi_1,\xi_2]=-\frac{h}{f}\left(\frac{\partial f}{\partial x^2}+\frac{\partial f}{\partial x^4}\right)\xi_1+\frac{f}{h}\left(\frac{\partial h}{\partial x^1}+\frac{\partial h}{\partial x^3}\right)\xi_2\in\mathrm{Span}(\xi_1,\xi_2)$, $[\xi_1,\xi_1]=[\xi_2,\xi_2]=0$ therefore they generate a totally degenerate foliation.

A local quasi-orthonormal basis along the isotropic foliation is given by: $\{\xi_1,\xi_2,N_1,N_2\}$ where:

$$N_1 = \frac{1}{2f}(-\partial_1 + \partial_3), N_2 = \frac{1}{2h}(\partial_2 + \partial_4)$$

3. Fundamental tensors of a degenerate foliation

Let F a degenerate foliation of a Semi-Riemannian manifold (M,g). We shall note with ∇ the Levi-Civita connection on M corresponding to g. For the sake of simplicity we shall consider the decomposition of TM given by (2.22):

(3.1) $$TM=S(F)\perp S(F^{\perp})\perp(N \oplus \deg(F))$$

where in the case of a coisotropic foliation we have that $S(F^{\perp})=\{0\}$ and $N=D_F{}^{\perp}$, in the case of an isotropic foliation having $S(F)=\{0\}$ and $N=D_F$ and in the case of totally degenerate foliations having: $S(F)=S(F^{\perp})=\{0\}$ and $N=D_F=D_F{}^{\perp}$.

We shall define four projectors relative to the decomposition (3.1):

(3.2) $P_1:TM\to N$, $P_2:TM\to S(F)$, $P_3:TM\to S(F^{\perp})$, $P_4:TM\to \deg(F)$

We have:

(3.3) $$P_1+P_2+P_3+P_4=I,\ P_iP_j=\delta_{ij}P_i$$

$\forall$ i,j=1,...,4, I being the identity.

From (3.1), (3.2) follows:

(3.4) $g(P_iX,P_jY)=0\ \forall(i,j)\in(\{1,2,3,4\}\times\{1,2,3,4\})-\{(1,4),(2,2),(3,3)\}\ \forall\ X,Y\in TM$

In what follows we shall note also:

$S_1=N$, $S_2=S(F)$, $S_3=S(F^{\perp})$, $S_4=\deg(F)$

We shall define a tensors family of type (1,2):

(3.5) $$A^k:TM \times TM\to TM,\ A^k_X Y=\nabla_{P_kX}Y-\sum_{i=1}^{4}P_i\nabla_{P_kX}P_iY=\sum_{\substack{i,j=1\\ i\neq j}}^{4}P_i\nabla_{P_kX}P_jY$$

$\forall$ k=1,2,3,4 $\forall$ X,Y$\in$TM.

From (3.5) follows that if $Y\in S_q$: $A^k_X Y = \sum_{\substack{i=1\\ i\neq q}}^{4} P_i \nabla_{P_k X} P_q Y \quad \forall X\in TM \ \forall k,q=1,2,3,4$

and how in the upper sum i≠q follows:

(3.6) $g(A^k{}_X Y,Z)=0 \ \forall Y,Z\in S_q \ \forall X\in TM \ \forall k,q=1,2,3,4$

From the definition we have also:

(3.7) $A^k_{P_k X} Y = A^k_X Y \quad \forall X,Y\in TM \ \forall k=1,2,3,4$

Theorem 3.1 The tensors A^k, k=1,2,3,4 are skew-symmetric with g that is:

(3.8) $g(A^k{}_X Y,Z)+g(Y,A^k{}_X Z)=0 \ \forall \ X,Y,Z\in TM$

Proof. Let X,Y,Z$\in$TM and 1≤k≤4 fixed. We have:

$$g(A^k_X Y, Z) + g(Y, A^k_X Z) = g(\nabla_{P_k X} Y - \sum_{i=1}^{4} P_i \nabla_{P_k X} P_i Y, Z) + g(Y, \nabla_{P_k X} Z - \sum_{j=1}^{4} P_j \nabla_{P_k X} P_j Z) =$$

$$\nabla_{P_k X} g(P_4 Y, P_1 Z) + \nabla_{P_k X} g(P_2 Y, P_2 Z) + \nabla_{P_k X} g(P_3 Y, P_3 Z) + \nabla_{P_k X} g(P_1 Y, P_4 Z) -$$

$$g(\nabla_{P_k X} P_4 Y, P_1 Z) - g(\nabla_{P_k X} P_2 Y, P_2 Z) - g(\nabla_{P_k X} P_3 Y, P_3 Z) - g(\nabla_{P_k X} P_1 Y, P_4 Z) -$$
$$g(P_4 Y, \nabla_{P_k X} P_1 Z) - g(P_2 Y, \nabla_{P_k X} P_2 Z) - g(P_3 Y, \nabla_{P_k X} P_3 Z) - g(P_1 Y, \nabla_{P_k X} P_4 Z) =$$
$$(\nabla_{P_k X} g)(P_4 Y, P_1 Z) + (\nabla_{P_k X} g)(P_2 Y, P_2 Z) + (\nabla_{P_k X} g)(P_3 Y, P_3 Z) + (\nabla_{P_k X} g)(P_1 Y, P_4 Z) = 0$$

Theorem 3.2 The distribution S_k, k=1,2 is integrable if and only if $A^k{}_X Y = A^k{}_Y X$, $\forall X,Y\in S_k$.

Proof. Let k=1,2-fixed. We have for any $X,Y\in S_k$:

$$A^k_X Y - A^k_Y X = \sum_{\substack{i=1\\ i\neq k}}^{4} P_i \nabla_{P_k X} P_k Y - \sum_{\substack{i=1\\ i\neq k}}^{4} P_i \nabla_{P_k Y} P_k X = \sum_{\substack{i=1\\ i\neq k}}^{4} P_i (\nabla_{P_k X} P_k Y - \nabla_{P_k Y} P_k X) =$$

$$\sum_{\substack{i=1\\ i\neq k}}^{4} P_i [P_k X, P_k Y] = [P_k X, P_k Y] - P_k [P_k X, P_k Y]$$

If $A^k_X Y = A^k_Y X$ follows $[P_k X, P_k Y] = P_k[P_k X, P_k Y] \in S_k$ therefore S_k is integrable. Reciprocal, if S_k is integrable then $[P_k X, P_k Y] \in S_k$ therefore $[P_k X, P_k Y] = P_k[P_k X, P_k Y]$. But this means that $A^k_X Y = A^k_Y X \ \forall \ X, Y \in S_k$.

Remarks

Let a degenerate foliation F of a Semi-Riemannian manifold (M,g).

1. The null distribution N is integrable if and only if the tensor A^1 is symmetric (k=1 in the theorem 3.2);
2. The screen distribution S (F) is integrable if and only if the tensor A^2 is symmetric (k=2 in the theorem 3.2);
3. If the foliation F is isotropic or totally degenerate then the tensors A^1 and A^2 are symmetrics. Indeed, in these cases we have: $N = D_F$ and $S(F) = \{0\}$. We have therefore that the null distribution N is integrable from where follows that the tensor A^1 is symmetric. Also on the screen distribution S (F) the bracket identically vanishes and therefore the tensor A^2 is symmetric.

In the case of the integrability of S_k, k=1,2 we have the following:

Theorem 3.3 The integral manifold of the distribution S_k, k=1,2 is totally geodesic if and only if $A^k_X Y = 0 \ \forall \ X, Y \in S_k$.

Proof. For any $X, Y \in S_k$ we have that $A^k_X Y = \sum_{\substack{i=1 \\ i \neq k}}^{4} P_i \nabla_{P_k X} P_k Y$. Let S the integral manifold of S_k for a fixed k. S is totally geodesic if and only if $\nabla_{P_k X} P_k Y \in S_k$. But this is equivalent with $A^k_X Y = 0$.

In what follows we shall determine the Gauss-Weingarten formulae for the degenerate foliations.

Considering $X, Y \in D_F$ we have: $P_3 X = P_4 X = 0$, $P_3 Y = P_4 Y = 0$. By the fact that $X = P_1 X + P_2 X$, $Y = P_1 Y + P_2 Y$ follows from (3.5):

(3.9) $$A^1_{P_1 X} P_1 Y = \nabla_{P_1 X} P_1 Y - P_1 \nabla_{P_1 X} P_1 Y$$

(3.10) $$A^1_{P_1 X} P_2 Y = \nabla_{P_1 X} P_2 Y - P_2 \nabla_{P_1 X} P_2 Y$$

(3.11) $$A^2_{P_2 X} P_1 Y = \nabla_{P_2 X} P_1 Y - P_1 \nabla_{P_2 X} P_1 Y$$

(3.12) $A^2_{P_2X}P_2Y = \nabla_{P_2X}P_2Y - P_2\nabla_{P_2X}P_2Y$

The Levi-Civita connection becomes:

(3.13) $\nabla_X Y = \nabla_{P_1X}P_1Y + \nabla_{P_1X}P_2Y + \nabla_{P_2X}P_1Y + \nabla_{P_2X}P_2Y = A^1_{P_1X}P_1Y + P_1\nabla_{P_1X}P_1Y +$
$A^1_{P_1X}P_2Y + P_2\nabla_{P_1X}P_2Y + A^2_{P_2X}P_1Y + P_1\nabla_{P_2X}P_1Y + A^2_{P_2X}P_2Y + P_2\nabla_{P_2X}P_2Y$

From (3.13) decomposing after $D_F = N \perp S(F), S(F^{\perp})$ and deg(F) we have:

(3.14) $\nabla^F_x Y = P_1A^1_{P_1X}P_1Y + P_1A^1_{P_1X}P_2Y + P_1A^2_{P_2X}P_1Y + P_1A^2_{P_2X}P_2Y + P_2A^1_{P_1X}P_1Y +$
$P_2A^1_{P_1X}P_2Y + P_2A^2_{P_2X}P_1Y + P_2A^2_{P_2X}P_2Y + P_1\nabla_{P_1X}P_1Y + P_1\nabla_{P_2X}P_1Y +$
$P_2\nabla_{P_1X}P_2Y + P_2\nabla_{P_2X}P_2Y$

(3.15) $h^S(X,Y) = P_3A^1_{P_1X}P_1Y + P_3A^1_{P_1X}P_2Y + P_3A^2_{P_2X}P_1Y + P_3A^2_{P_2X}P_2Y$

(3.16) $h^L(X,Y) = P_4A^1_{P_1X}P_1Y + P_4A^1_{P_1X}P_2Y + P_4A^2_{P_2X}P_1Y + P_4A^2_{P_2X}P_2Y$

Considering now $X \in D_F$ and $V \in tr(F)$ we have: $P_3X=P_4X=0$, $P_1V=P_2V=0$. By the fact that $X=P_1X+P_2X$, $V=P_3V+P_4V$ follows from (3.5):

(3.17) $A^1_{P_1X}P_3V = \nabla_{P_1X}P_3V - P_3\nabla_{P_1X}P_3V$

(3.18) $A^1_{P_1X}P_4V = \nabla_{P_1X}P_4V - P_4\nabla_{P_1X}P_4V$

(3.19) $A^1_{P_2X}P_3V = \nabla_{P_2X}P_3V - P_3\nabla_{P_2X}P_3V$

(3.20) $A^1_{P_2X}P_4V = \nabla_{P_2X}P_4V - P_4\nabla_{P_2X}P_4V$

The Levi-Civita connection becomes:

(3.21) $\nabla_X V = \nabla_{P_1X}P_3V + \nabla_{P_1X}P_4V + \nabla_{P_2X}P_3V + \nabla_{P_2X}P_4V = A^1_{P_1X}P_3V + P_3\nabla_{P_1X}P_3V +$
$A^1_{P_1X}P_4V + P_4\nabla_{P_1X}P_4V + A^1_{P_2X}P_3V + P_3\nabla_{P_2X}P_3V + A^1_{P_2X}P_4V + P_4\nabla_{P_2X}P_4V$

From (3.21) decomposing after $D_F = N \perp S(F), S(F^{\perp})$ and $\deg(F)$ we have:

$$(3.22)\ -A_V X = P_1 A^1_{P_1 X} P_3 V + P_1 A^1_{P_1 X} P_4 V + P_1 A^1_{P_2 X} P_3 V + P_1 A^1_{P_2 X} P_4 V + P_2 A^1_{P_1 X} P_3 V + P_2 A^1_{P_1 X} P_4 V + P_2 A^1_{P_2 X} P_3 V + P_2 A^1_{P_2 X} P_4 V$$

$$(3.23)\ D^S_X V = P_3 A^1_{P_1 X} P_3 V + P_3 A^1_{P_1 X} P_4 V + P_3 A^1_{P_2 X} P_3 V + P_3 A^1_{P_2 X} P_4 V + P_3 \nabla_{P_1 X} P_3 V + P_3 \nabla_{P_2 X} P_3 V$$

$$(3.24)\ D^L_X V = P_4 A^1_{P_1 X} P_3 V + P_4 A^1_{P_1 X} P_4 V + P_4 A^1_{P_2 X} P_3 V + P_4 A^1_{P_2 X} P_4 V + P_4 \nabla_{P_1 X} P_4 V + P_4 \nabla_{P_2 X} P_4 V$$

From the tensorial character of A^1 respectively A^2 and from the fact that ∇ is **R**-bilinear in both terms and is F (M)-linear in the first term follows that all the geometrical objects introduced through (3.14), (3.15), (3.16), (3.22), (3.23) and (3.24) are **R**-bilinear and F (M)-linear in the first term. The fact that ∇^F is linear connection on D_F is easy to proven therefore we shall name ∇^F the linear connection induced on F . From (3.15), (3.16) and the tensorial character of A^1 respectively A^2 follows that h^S and h^L are tensors of type (1,2) defined by: $h^L: D_F \times D_F \to \deg(F)$, $h^S: D_F \times D_F \to S(F^{\perp})$. We shall name h^L the second degenerate fundamental form of F and h^S the second screen fundamental form of F . From (3.22) follows from the same tensorial character of A^1, A^2 that A is a tensor of type (1,2) defined by: $A: D_F \times \mathrm{tr}(F) \to D_F$. We shall name A_V the Weingarten operator of F relative to V.

From (3.23), (3.24) follows:

$$(3.25)\ D^S_X fV = fD^S_X V + X(f)P_3 V$$

$$(3.26)\ D^L_X fV = fD^L_X V + X(f)P_4 V$$

$\forall f \in F(M)$.

Remarks

1. From (3.25) follows: $D^S_X fP_3 V = f\nabla^S_X P_3 V + X(f)P_3 V$, $D^S_X fP_4 V = f\nabla^S_X P_4 V$
2. From (3.26) follows: $D^L_X fP_3 V = fD^L_X P_3 V$, $D^L_X fP_4 V = fD^L_X P_4 V + X(f)P_4 V$
3. From (3.25) and (3.26) follows:

$$D^S_X fV + D^L_X fV = fD^S_X V + X(f)P_3V + fD^L_X V + X(f)P_4V = f(D^S_X V + D^L_X V) + X(f)V$$

Because D^S and D^L are not linear connections, we shall consider theirs restrictions at $S(F^\perp)$ respectively at $\deg(F)$. Let therefore:

(3.27) $$\nabla^S : D_F \times S(F^\perp) \to S(F^\perp),\ \nabla^S_X(P_3V) = D^S_X P_3 V$$

(3.28) $$\nabla^L : D_F \times \deg(F) \to \deg(F),\ \nabla^L_X(P_4V) = D^L_X P_4 V$$

(3.29) $$D^S : D_F \times \deg(F) \to S(F^\perp),\ D^S(X, P_4V) = D^S_X P_4 V$$

(3.30) $$D^L : D_F \times S(F^\perp) \to \deg(F),\ D^L(X, P_3V) = D^L_X P_3 V$$

$\forall X \in D_F \ \forall V \in tr(F)$.

From the first remark and the preceding considerations, follows that ∇^S is a linear connection on $D_F \times S(F^\perp)$ and D^S is a tensor of type (1,2) on $D_F \times \deg(F)$. Also from the second remark follows that ∇^L is a linear connection on $D_F \times \deg(F)$ and D^L is a tensor of type (1,2) on $D_F \times S(F^\perp)$. We have therefore from (3.27)-(3.30):

(3.31) $$D^S_X V = D^S_X P_3 V + D^S_X P_4 V = \nabla^S_X P_3 V + D^S(X, P_4 V)$$

(3.32) $$D^L_X V = D^L_X P_3 V + D^L_X P_4 V = D^L(X, P_3 V) + \nabla^L_X P_4 V$$

We define now:

(3.33) $$h : D_F \times D_F \to tr(F),\ h(X,Y) = h^S(X,Y) + h^L(X,Y)\ \forall X, Y \in D_F$$

and we shall call the second fundamental form of F relative to $tr(F)$.

Let also:

(3.34) $$\nabla^t : D_F \times tr(F) \to tr(F),\ \nabla^t_X V = D^S_X V + D^L_X V\ \ \forall X \in D_F\ \forall V \in tr(F)$$

By the third remark follows that ∇^t is a linear connection on $D_F \times tr(F)$ named the transversal linear connection of F .

We can write now:

(3.35) $$\nabla_X Y=\nabla^F{}_X Y+h(X,Y)$$

(3.36) $$\nabla_X V= -A_V X+\nabla^t{}_X V$$

$\forall\ X,Y\in D_F\ \forall V\in tr(F)$.

Because the distribution D_F is integrable follows that ∇^F is a linear connection whitout torsion on D_F.

From (3.33), (3.34) follows that the formulae (3.35), (3.36) become:

(3.37) $$\nabla_X Y=\nabla^F{}_X Y+h^L(X,Y)+h^S(X,Y)$$

(3.38) $$\nabla_X V= -A_V X+D^L{}_X V+ D^S{}_X V$$

$\forall\ X,Y\in D_F\ \forall V\in tr(F)$.

Analogously, using (3.31), (3.32) follows:

(3.39) $$\nabla_X V= -A_V X+\nabla^L{}_X P_4 V+D^L(X,P_3 V)+\nabla^S{}_X P_3 V+D^S(X,P_4 V)$$

$\forall\ X\in D_F\ \forall V\in tr(F)$.

From (3.39) follows the particular cases:

(3.40) $$\nabla_X W= -A_W X+D^L(X,W)+\nabla^S{}_X W$$

(3.41) $$\nabla_X N= -A_N X+\nabla^L{}_X N+D^S(X,N)$$

$\forall\ X\in D_F\ \forall W\in S(F^{\perp})\ \forall N\in deg(F)$.

Remark In the cases of coisotropic or totally degenerate foliations we have: $S(F^{\perp})=\{0\}$ therefore $P_3=0$ from where $h^S=0$, $\nabla^S=0$, $D^S=0$ and $D^L=0$. The formulae (3.37) and (3.41) become:

(3.42) $$\nabla_X Y=\nabla^F{}_X Y+h^L(X,Y)$$

(3.43) $$\nabla_X N= -A_N X+\nabla^L{}_X N$$

$\forall X,Y \in D_F \forall N \in \deg(F)$.

We shall call the formulae (3.35), (3.37), (3.42) the Gauss formulae and (3.36), (3.38), (3.39), (3.40), (3.41), (3.43) the Weingarten formulae for the degenerate foliation F.

Theorem 3.4 Let a degenerate foliation F of a Semi-Riemannian manifold (M,g). We have:

(3.44) $g(h^S(X,Y),W)+g(Y,D^L(X,W))=g(A_WX,Y)$

(3.45) $g(h^L(X,Y),\xi)+g(h^L(X,\xi),Y)+g(Y,\nabla^F_X\xi)=0$

(3.46) $g(D^S(X,N),W)=g(A_WX,N)$

(3.47) $g(A_NX,N')+g(A_{N'}X,N)=0$

(3.48) $g(A_NX,P_2Y)=g(N,\nabla_XP_2Y)$

$\forall X,Y \in D_F \forall \xi \in N \ \forall W \in S(F^{\perp})) \forall N,N' \in \deg(F))$.

Proof. Let $X,Y \in D_F$, $\xi \in N$, $W \in S(F^{\perp}))$, $N,N' \in \deg(F))$. Then:

- $g(A_WX,Y)=g(-\nabla_XW+D^L(X,W),Y)=-g(\nabla_XW,Y)+g(D^L(X,W),Y)=g(W,\nabla_XY)+g(D^L(X,W),Y)=g(W,h^S(X,Y))+g(D^L(X,W),Y)$
- $0=\nabla_Xg(Y,\xi)=g(\nabla_XY,\xi)+g(Y,\nabla_X\xi)=g(h^L(X,Y),\xi)+g(Y,\nabla^F_X\xi+h^L(X,\xi))=g(h^L(X,Y),\xi)+g(h^L(X,\xi),Y)+g(Y,\nabla^F_X\xi)$
- $g(A_WX,N)=g(-\nabla_XW,N)=g(W,\nabla_XN)=g(W,D^S(X,N))$
- $g(A_NX,N')+g(A_{N'}X,N)=g(-\nabla_XN,N')+g(-\nabla_XN',N)=-\nabla_Xg(N,N')=0$
- $g(A_NX,P_2Y)=g(-\nabla_XN,P_2Y)=g(N,\nabla_XP_2Y)$

If we have now $\{N_1,...,N_r\}$ a basis for $\deg(F)$ and $\{W_{r+1},...,W_m\}$ a basis for $S(F^{\perp}))$ (the last in the case of r-degenerate foliations or of those isotropic) for a given screen distribution $S(F)$ we define:

(3.49) $$h^L(X,Y)=\sum_{i=1}^{r} h_i^L(X,Y)N_i$$

(3.50) $$h^S(X,Y)= \sum_{a=r+1}^{m} h^S_a(X,Y)W_a$$

in the case of coisotropic or totally degenerate foliations defining $h^S_a=0$ $\forall$ a=r+1,..,m.

We call h^L_i the degenerate local second fundamental forms and h^S_a the screen local second fundamental forms of F .

Theorem 3.5 In a degenerate foliation (F, g, S (F),S ($F^{\perp}$)) of a Semi-Riemannian manifold (M,g) the degenerate local second fundamental forms are independent by the screen distribution and by the transversal distribution.

Proof. From (3.37) follows:

(3.51) $$h^L_i(X,Y)=g(h^L(X,Y),\xi_i)=g(\nabla_X Y,\xi_i)$$

$\forall$ $X,Y\in D_F$ $\forall i=1,...,r$.

From (2.61), (2.63), (2.65), (2.67), (3.51) follows that at a change of coordinates neighbourhood of a point $p\in F$ where F is a foliation with integrable null distribution we have:

(3.52) $$h^L{}'(X,Y)=Ah^L(X,Y)$$

$\forall$ $X,Y\in D_F$, $h^L{}'$, h^L being column vectors with the components $h^L_i{}'$ respectively h^L_i relative to the two bases. After this remark we have immediately:

Theorem 3.6 In a degenerate foliation (F, g, S (F),S ($F^{\perp}$)) with integrable null distribution of a Semi-Riemannian manifold (M,g) the vanishing of the degenerate local second fundamental forms does not depend by the coordinates neighbourhood of an arbitrary point p of F .

From (3.51) we have now $h^L_i(X,\xi_j)=g(\nabla_X\xi_j,\xi_i)=-g(\nabla_X\xi_i,\xi_j)=-h^L_j(X,\xi_i)$ therefore:

(3.53) $$h^L_i(X,\xi_j)+h^L_j(X,\xi_i)=0$$

$\forall X \in D_F \forall i,j=1,...,r$ and for $i=j$:

(3.54) $$h^L_i(X,\xi_i)=0$$

$\forall X \in D_F \forall i=1,...,r.$

If we make a circular permutation in (3.53) we have:

(3.55) $$h^L_i(\xi_j,\xi_k)=0$$

$\forall i,j,k=1,...,r.$

From (3.54) and (3.55) we have:

Theorem 3.7 In a degenerate foliation (F, g, S (F),S ($F^{\perp}$)) of a Semi-Riemannian manifold (M,g) the degenerate local second fundamental forms are degenerate and they identically vanish on the null distribution N of F .

In the cases of isotropic or totally degenerate foliations we have that $N = D_F$ therefore:

Corrolary 3.1 In an isotropic or totally degenerate foliation (F, g, S ($F^{\perp}$)) of a Semi-Riemannian manifold (M,g) the degenerate local second fundamental forms identically vanish on D_F.

The problem is now how the induced connection ∇^F will transform on F at a change of the screen distribution?

For the beginning we shall analyse the case of r-degenerate foliation with $0<r<\min\{m,n\}$.

Let U a coordinates neighbourhood of M and $\{\xi_1,...,\xi_r,X_{r+1},...,X_n, W_{r+1},...,W_m,N_1,...,N_r\}$ a local quasi-orthonormal basis along F in U and $\{\xi_1,...,\xi_r,X'_{r+1},...,X'_n,W'_{r+1},...,W'_m,N'_1,...,N'_r\}$ a local quasi-orthonormal basis along F in U relative to the decompositions $TM=S(F)\perp S(F^{\perp})\perp(N \oplus \deg(F))$ respectively $TM=S'(F)\perp S'(F^{\perp})\perp(N \oplus \deg'(F))$. From (2.43), (2.44), (3.37), (3.49)-(3.51) we have:

$$(3.56)\ \nabla^F_X Y = \nabla'^F_X Y + \sum_{i=1}^{r}\left(E^t h^L(X,Y) - DG_W{}^t B^t h'^S(X,Y)\right)^i \xi_i + \sum_{a=r+1}^{n}\left(C^t h^L(X,Y)\right)^a X_a$$

$\forall\ X,Y \in D_F$, $(\ )^i$ and $(\ )^a$ being the coordinates in corresponding bases.

Analogously, we have:

$$(3.57)\qquad \nabla^F_X Y = \nabla'^F_X Y + \sum_{i=1}^{m}\left(E^t h^L(X,Y)\right)^i \xi_i + \sum_{a=m+1}^{n}\left(C^t h^L(X,Y)\right)^a X_a$$

for o coisotropic foliation,

$$(3.58)\qquad \nabla^F_X Y = \nabla'^F_X Y - \sum_{i=1}^{n}\left(DG_W{}^t B^t h'^S(X,Y)\right)^i \xi_i$$

for an isotropic foliation and

$$(3.59)\qquad \nabla^F_X Y = \nabla'^F_X Y$$

for a totally degenerate foliation.

Theorem 3.8 In a r-degenerate foliation (F, g, S (F), S ($F^{\perp}$)) of a Semi-Riemannian manifold (M,g) the induced connection ∇^F on F is independent by the screen distribution if and only if

$$(3.60)\qquad E^t h^L(X,Y) = DG_W{}^t B^t h'^S(X,Y)$$

$$(3.61)\qquad C^t h^L(X,Y) = 0$$

$\forall\ X,Y \in D_F$ and $\forall$ B a non-singular matrix of m-r-order, C,D,E being arbitrary matrices of types r×(n-r), r×(m-r) respectively r×r which satisfy in addition the relations (2.44).

Theorem 3.9 In a coisotropic foliation (F, g,S (F)) of a Semi-Riemannian manifold (M,g) the induced connection ∇^F on F is independent by the screen distribution if and only if

$$(3.62)\qquad E^t h^L(X,Y) = 0$$

$$(3.63)\qquad C^t h^L(X,Y) = 0$$

$\forall$ $X,Y \in D_F$ and $\forall$ C,E m×(n-m) and m×m-orders matrices, which satisfy in addition, the relations (2.49).

Theorem 3.10 In an isotropic foliation (F,g,S ($F^{\perp}$)) of a Semi-Riemannian manifold (M,g) the induced connection ∇^F on F is independent by the screen distribution if and only if

(3.64) $$DG_W{}^t B^t h'^S(X,Y)=0$$

$\forall$ $X,Y \in D_F$ and $\forall$ B a non-singular matrix of m-n-order, D an arbitrary matrix of n×(m-n)-order, satisfying in addition the relations (2.54).

Theorem 3.11 In a totally degenerate foliation (F, g) of a Semi-Riemannian manifold (M,g) the induced connection ∇^F on F is independent by the screen distribution.

We shall study in what follows the manner in which the induced connection ∇^F depend on the coordinates neigbourhood. From (2.61), (2.63), (2.65), (2.67) follows:

Theorem 3.12 In a degenerate foliation F, with integrable null distribution, of a Semi-Riemannian manifold (M,g) the induced connection ∇^F on F is independent by the coordinates neighbourhood of an arbitrary point $p \in M$.

We define now a system of 1-local differential forms:

(3.65) $$\eta_i(X)=g(X,N_i),\ i=1,...,r$$

$\forall$ $X \in D_F$. We have from (3.65):

(3.66) $$X = P_2X + \sum_{i=1}^{r} \eta_i(X)\xi_i$$

$\forall$ $X \in D_F$. We remark from (3.66) that the screen distribution is defined locally by $\eta_i=0$, i=1,...,r.

We have define in (3.14) and (3.34) two linear connections ∇^F and ∇^t where the first is symmetrical. The problem is now is if these are metric connextions. From (3.37), (3.49), (3.66) and the condition that ∇ is metric we have:

(3.67) $(\nabla^F_X g)(Y,Z)=g(h^L(X,Y),Z)+g(h^L(X,Z),Y)=$

$$\sum_{i=1}^{r}\left[h_i^L(X,Y)\eta_i(Z)+h_i^L(X,Z)\eta_i(Y)\right]\ \forall\ X,Y,Z\in D_F.$$

From (3.36) we have also:

(3.68) $(\nabla^t_X g)(V,V')=-[g(A_V X,V')+g(A_{V'}X,V)]\ \forall\ X\in D_F\ \forall V,V'\in tr(F)$.

Theorem 3.13 In a degenerate foliation F of a Semi-Riemannian manifold (M,g) the induced connection ∇^F on F is metric if and only if the local degenerate second fundamental forms identically vanishes on D_F.

Proof. From (3.65) and (3.67) we have for any $X,Y,Z\in D_F$:

(3.69) $$(\nabla^F_X g)(P_2Y,P_2Z)=\sum_{i=1}^{r}\left[h_i^L(X,PY)\eta_i(PZ)+h_i^L(X,PZ)\eta_i(PY)\right]=0$$

From (3.53) follows:

(3.70) $$(\nabla^F_X g)(\xi_i,\xi_j)=\sum_{k=1}^{r}\left[h_k^L(X,\xi_i)\eta_k(\xi_j)+h_k^L(X,\xi_j)\eta_k(\xi_i)\right]=h_j^L(X,\xi_i)+h_i^L(X,\xi_j)=0$$

$\forall$ i,j=1,...,r and finally:

(3.71) $$(\nabla^F_X g)(P_2Y,\xi_i)=\sum_{k=1}^{r}\left[h_k^L(X,PY)\eta_k(\xi_i)+h_k^L(X,\xi_i)\eta_k(PY)\right]=h^L{}_i(X,PY)$$

$\forall$ i=1,...,r.

The vanishing of $\nabla^F g$ is therefore equivalent with $h^L{}_i(X,P_2Y)=0\ \forall X,Y\in D_F$. From the theorem 3.7 follows that it is equivalent with $h^L=0$.

From the corrolary 3.1 and the theorem 3.13 we have:

Corrolary 3.2 In an isotropic or totally degenerate foliation F of a Semi-Riemannian manifold (M,g) the induced connection ∇^F on F is metric.

Theorem 3.14 In a coisotropic or totally degenerate foliation F of a Semi-Riemannian manifold (M,g) the transversal connection ∇^t is metric.

Proof. Because A_V has values in D_F and in the case of coisotropic or totally degenerate foliations we have that tr(F)=deg(F) follows from (3.43) and (3.68):

$(\nabla^t_X g)(V,V')=-[g(A_V X,V')+g(A_{V'}X,V)]=g(\nabla_X V,V')+g(V,\nabla_X V')=\nabla_X g(V,V')=0$

$\forall X\in D_F \ \forall V,V'\in tr(F))$.

Theorem 3.15 In a r-degenerate or isotropic foliation F of a Semi-Riemannian manifold (M,g) the next statements are equivalents:

a) ∇^t is a linear metric connection;
b) The degenerate transversal distribution deg(F) is parallel with respect to ∇^t;
c) A_W takes values in S (F)) $\forall$ $W\in S(F^{\perp})$;
d) $D^S(X,N)=0$ $\forall$ $X\in D_F$ $\forall N\in deg(F)$.

Proof. From (3.47), (3.68) we have:

(3.72) $(\nabla^t_X g)(N,N')=-g(A_N X,N')-g(A_{N'}X,N)=0$

(3.73) $(\nabla^t_X g)(W,W')=-g(A_W X,W')-g(A_{W'}X,W)=0$

(3.74) $(\nabla^t_X g)(W,N)=-g(A_W X,N)-g(A_N X,W)=-g(A_W X,N)$

$\forall X\in D_F \ \forall W,W'\in S(F^{\perp})) \ \forall N,N'\in deg(F)$. In (3.74) we have use the fact that $A_N X\in D_F$ therefore $g(A_N X,W)=0$.

a)$\Rightarrow$c) From (3.74) follows that if ∇^t is metric connection then $g(A_W X,N)=0$ therefore $A_W X\in S(F))\ \forall X\in D_F$.

c)$\Rightarrow$a) From (3.74) follows that $(\nabla^t_X g)(W,N)=-g(A_W X,N)=0$ and together with (3.72) and (3.73) imply a).

a)$\Rightarrow$d) From (3.46) and (3.74) we have $0=(\nabla^t_X g)(W,N)=-g(A_W X,N)=-g(D^S(X,N),W)$ $\forall W\in S(F^{\perp})$ and how $S(F^{\perp})$ is nondegenerate follows $D^S(X,N)=0$.

d)$\Rightarrow$a) From (3.46) and (3.74) we have $(\nabla^t_X g)(W,N)=-g(A_W X,N)=-g(D^S(X,N),W)=0$ and with (3.62) and (3.73) imply a).

a)⇒b) From (3.36) we obtain now:

(3.75) $$g(\nabla^t_X N,W)=-(\nabla^t_X g)(W,N)$$

From a) and (3.75) follows therefore $g(\nabla^t_X N,W)=0$ and how $S(F^{\perp})$ is nondegenerate we have $\nabla^t_X N=0\ \forall X\in D_F\ \forall N\in \deg(F)$. But this nothing means else that $\deg(F)$ is parallel with respect to ∇^t.

b)⇒a) From (3.75) we have $(\nabla^t_X g)(W,N)=0$ and together with (3.72) and (3.73) imply a).

We have seen that the screen distribution is fundamental in the study of degenerate foliations. On the other hand all the introduced geometrical objects does not put in obviousness properties of this. This is the reason for we shall proceed at a refinement of the Gauss formula with respect to the decomposition $D_F=S(F)\perp N$. Let therefore $X,Y\in D_F$. From (3.14) we have:

(3.76) $$\nabla^F_x P_2Y=P_1A^1_{P_1X}P_2Y+P_1A^2_{P_2X}P_2Y+P_2A^1_{P_1X}P_2Y+P_2A^2_{P_2X}P_2Y+P_2\nabla_{P_1X}P_2Y+P_2\nabla_{P_2X}P_2Y$$

(3.77) $$\nabla^F_x P_1Y=P_1A^1_{P_1X}P_1Y+P_1A^2_{P_2X}P_1Y+P_1\nabla_{P_1X}P_1Y+P_1\nabla_{P_2X}P_1Y+P_2A^1_{P_1X}P_1Y+P_2A^2_{P_2X}P_1Y$$ We define:

(3.78) $$\nabla^*_x P_2Y=P_2A^1_{P_1X}P_2Y+P_2A^2_{P_2X}P_2Y+P_2\nabla_{P_1X}P_2Y+P_2\nabla_{P_2X}P_2Y$$

(3.79) $$\nabla^F_x P_2Y=P_1A^1_{P_1X}P_2Y+P_1A^2_{P_2X}P_2Y+P_2A^1_{P_1X}P_2Y+P_2A^2_{P_2X}P_2Y+P_2\nabla_{P_1X}P_2Y+P_2\nabla_{P_2X}P_2Y$$

(3.80) $$A^*_{P_1Y}X=-P_2A^1_{P_1X}P_1Y-P_2A^2_{P_2X}P_1Y$$

(3.81) $$\nabla^{*t}_x P_1Y=P_1A^1_{P_1X}P_1Y+P_1A^2_{P_2X}P_1Y+P_1\nabla_{P_1X}P_1Y+P_1\nabla_{P_2X}P_1Y$$

Like in the preceeding discussion, follows immediately that h^* and A are tensors of type (1,2) defined thus:

$$h^*:D_F\times S(F)\to N,\ A:D_F\times N\to S(F)$$

We shall call h^* the second fundamental form of $S(F)$ and A^*_ξ the Weingarten operator of $S(F)$ with respect to $\xi\ \forall\xi\in N$.

Also, ∇^* and ∇^{*t} are linear connections on $S(F)$ respectively N named the induced connection on $S(F)$ respectively the induced connection on N. From (3.76), (3.78) and (3.79) follows:

(3.82) $$\nabla^F_X P_2Y=\nabla^*_X P_2Y+h^*(X,P_2Y)\ \forall X,Y\in D_F$$

From (3.77), (3.80) and (3.81) follows:

(3.83) $$\nabla^F_X\xi = -A^*_\xi X + \nabla^{*t}_X\xi \quad \forall X \in D_F \; \forall \xi \in N$$

Remark In the case of isotropic or totally degenerate foliations we have $D_F = N$ therefore ∇^* and A^* vanish.

Theorem 3.16 Let a degenerate foliation F of a Semi-Riemannian manifold (M,g). The following relations hold:

(3.84) $$g(A^*_\xi X, P_2Y) = g(\xi, h^L(X, P_2Y))$$

(3.85) $$(\nabla^{*t}_X g)(\xi,\xi') = g(\xi, h^L(X,\xi')) + g(\xi', h^L(X,\xi))$$

(3.86) $$(\nabla^*_X g)(P_2Y, P_2Z) = 0$$

(3.87) $$g(h^*(X,P_2Y),N) = g(A_N X, P_2Y)$$

$\forall X,Y \in D_F \; \forall \xi,\xi' \in N \; \forall N \in \deg(F)$.

Proof. Let $X,Y \in D_F$, $\xi,\xi' \in N$, $N \in \deg(F)$. Using (3.37), (3.45), (3.69), (3.82), (3.83) we have:

- From (3.37), (3.83) follow: $g(A^*_\xi X, P_2Y) = -g(\nabla^F_X\xi, P_2Y) = -g(\nabla_X\xi, P_2Y) = g(\xi, \nabla_X P_2Y) = g(\xi, h^L(X, P_2Y))$;
- From (3.37), (3.83) follow: $(\nabla^{*t}_X g)(\xi,\xi') = X(g(\xi,\xi')) - g(\nabla^{*t}_X\xi,\xi') - g(\xi, \nabla^{*t}_X\xi') =$
$X(g(\xi,\xi')) - g(\nabla^F_X\xi,\xi') - g(\xi,\nabla^F_X\xi') = X(g(\xi,\xi')) - g(\nabla_X\xi,\xi') + g(h^L(X,\xi),\xi') -$
$g(\nabla_X\xi',\xi) +$
$g(h^L(X,\xi'),\xi) = (\nabla_X g)(\xi,\xi') + g(\xi, h^L(X,\xi')) + g(\xi', h^L(X,\xi)) = g(\xi, h^L(X,\xi')) + g(\xi', h^L(X,\xi))$;
- From (3.37), (3.82) follow: $(\nabla^*_X g)(P_2Y, P_2Z) = X(g(P_2Y,P_2Z)) - g(\nabla^*_X P_2Y, P_2Z) -$
$g(P_2Y, \nabla^*_X P_2Z) = X(g(P_2Y,P_2Z)) - g(\nabla^F_X P_2Y, P_2Z) - g(P_2Y, \nabla^F_X P_2Z) = X(g(P_2Y,P_2Z)) -$
$g(\nabla_X P_2Y, P_2Z) - g(P_2Y, \nabla_X P_2Z) = 0$;
- From (3.37), (3.41), (3.82) follow: $g(h^*(X,P_2Y),N) = g(\nabla^F_X P_2Y, N) = g(\nabla_X P_2Y, N) = -g(P_2Y, \nabla_X N) = g(P_2Y, A_N X)$.

Theorem 3.17 Let a degenerate foliation F of a Semi-Riemannian manifold (M,g). Then the operator A^*_ξ is self-adjoint on $S(F) \; \forall \xi \in N$.

Proof. From (3.84) using the fact that h^L is symmetric follows:

(3.88) $g(A^*_\xi P_2X,P_2Y)=g(\xi,h^L(P_2X,P_2Y))=g(\xi,h^L(P_2Y,P_2X))=g(P_2X,A^*_\xi P_2Y)$

Theorem 3.18 Let a degenerate foliation F with the null distribution of rank 1 of a Semi-Riemannian manifold (M,g). Then ∇^{*t} is metric connection on N .

Proof. If the null distribution is of rank 1 then from (3.54) and (3.85) we have: $(\nabla^{*t}{}_X g)(\xi,\xi)=2g(\xi,h^L(X,\xi))=2g(\xi,h^L{}_1(X,\xi)N)=2h^L{}_1(X,\xi)=0$.

Theorem 3.19 Let a degenerate foliation F of a Semi-Riemannian manifold (M,g). Then ∇^{*t} is metric connection on S (F).

Proof. Follows from (3.86).

From (3.45) when Y=ξ‘, $X\to P_2X$ and (3.84) we have that:

$0=g(h^L(P_2X,\xi'),\xi)+g(h^L(P_2X,\xi),\xi')+g(\xi',\nabla^F_{P_2X}\xi)=g(A^*_\xi\xi'+A^*_{\xi'}\xi,P_2X)$

How S (F) is nondegenerate, follows:

(3.89) $A^*_\xi\xi'+A^*_{\xi'}\xi=0\ \forall\ \xi,\xi'\in N$

We shall suppose now that the null distribution N is integrable.

Theorem 3.20 The Weingarten operator of the screen distribution S (F) corresponding to the degenerate foliation F, with integrable null distribution, in a Semi-Riemannian manifold (M,g) vanishes on the null distribution.

Proof. Because N is integrable we have: $\forall\xi,\xi'\in N : [\xi,\xi']\in N$. Let $X\in S(F)$, arbitrary. Then:

$0=g([\xi,\xi'],X)=g(\nabla_\xi\xi',X)-g(\nabla_{\xi'}\xi,X)=g(A^*_\xi\xi',X)-g(A^*_{\xi'}\xi,X)=g(A^*_\xi\xi'-A^*_{\xi'}\xi,X)$

How S (F) is nondegenerate follows:

(3.90) $A^*_\xi\xi'=A^*_{\xi'}\xi\ \forall\ \xi,\xi'\in N$

From (3.89) and (3.90) follow:

(3.91) $A^*_\xi\xi'=0\ \forall\ \xi,\xi'\in N$

Theorem 3.21 The second degenerate fundamental form of a degenerate foliation F, with integrable null distribution, in a Semi-Riemannian manifold (M,g) vanishes on $N\times D_F$.

Proof. From (3.84), (3.91) follow: $g(\xi',h^L(\xi,P_2X))=g(A^*_{\xi'}\xi,P_2X)=0\ \forall\xi'\in N$ from where:

(3.92) $h^L(\xi,P_2X)=0\ \forall\ \xi\in N\ \ \forall X\in D_F$

From (3.92) and the theorem 3.7 we have:

(3.93) $h^L(\xi,X)=0\ \forall\ \xi\in N\ \ \forall X\in D_F$

Before the next theorem let do the remark that from (3.69)-(3.71) and (3.92) follow:

(3.94) $\nabla^F_\xi g=0\ \forall\ \xi\in N$

Theorem 3.22 Let a degenerate foliation F, with integrable null distribution, in a Semi-Riemannian manifold (M,g). The next assertions are equivalent:

a) The induced connection ∇^F is metric;
b) A^*_ξ vanishes on S (F) $\forall\ \xi\in N$;
c) N is a Killing distribution;
d) N is a parallel distribution with respect to ∇^F.

Proof. From the corrolary 3.2 follows that for isotropic or totally degenerate foliations the connection ∇^F is metric. We shall consider therefore that F is r-degenerate or coisotropic. From the theorem 3.13 follows that ∇^F is metric if and only if the degenerate second fundamental forms vanish identically on F . On the other hand from the theorem 3.7 and (3.92) follow that ∇^F is metric if and only if $h^L(P_2X,P_2Y)=0\ \forall\ X,Y\in D_F$.

a)$\Rightarrow$b) From (3.84) and the nondegenerate character of S (F) we have: $g(A^*_\xi P_2X,P_2Y)=g(\xi,h^L(P_2X,P_2Y))=0$ therefore $A^*_\xi P_2X=0\ \forall X,Y\in D_F\ \ \forall\xi\in N$.

b)$\Rightarrow$a) From (3.84) follows: $g(\xi,h^L(P_2X,P_2Y))=g(A^*_\xi P_2X,P_2Y)=0$ therefore $h^L(P_2X,P_2Y)=0\ \forall X,Y\in D_F$ that is ∇^F is metric connection.

N is a Killing distribution if and only if $g(\nabla_X\xi,Y)+g(\nabla_Y\xi,X)=0\ \ \forall\xi\in N\ \forall X,Y\in D_F$. Using (3.37), (3.83) and (3.93) we have:

$$g(\nabla_X\xi,Y)+g(\nabla_Y\xi,X)=-g(A^*_\xi X,Y)-g(A^*_\xi Y,X)$$

From (3.91) follows that for $X,Y\in N$ the upper expresion vanishes. Also If $Y=\xi'$ then $-g(A^*_\xi X,\xi')-g(A^*_\xi\xi',X)=0$. It follows therefore that N is Killing distribution if and only if $g(A^*_\xi P_2X,P_2Y)+g(A^*_\xi P_2Y,P_2X)=0\ \forall X,Y\in D_F$. But from (3.88) we have that N is Killing if and only if $g(A^*_\xi P_2X,P_2Y)=0\ \forall X,Y\in D_F$.

a)$\Rightarrow$c) From (3.84) follows $g(A^*_\xi P_2X,P_2Y)=g(\xi,h^L(P_2X,P_2Y))=0$.

c)$\Rightarrow$a) From (3.84) follows $0=g(A^*_\xi P_2X,P_2Y)=g(\xi,h^L(P_2X,P_2Y))\ \ \forall\xi\in N$ therefore $h^L(P_2X,P_2Y)=0\ \forall\ X,Y\in D_F$.

b)$\Rightarrow$d) If $A^*_\xi P_2X=0\ \forall X\in D_F\ \ \forall\xi\in N$ then from (3.91) follows $\nabla^F_X\xi\in N\ \ \forall X\in D_F\ \forall\xi\in N$ therefore N is parallel with respect to ∇^F.

d)$\Rightarrow$b) If N is parallel with respect to ∇^F then $A^*_\xi X=0\ \forall X\in D_F\ \forall\xi\in N$.

If we consider now foliations with arbitrary null distribution we can proove other general results.

From (3.89) we have like a particular case:

(3.95) $$A^*_\xi\xi=0\ \forall\ \xi\in N$$

It is easy to show that in this case the theorem 3.22 becomes:

Theorem 3.23 Let a degenerate foliation F in a Semi-Riemannian manifold (M,g). The next assertions are equivalent:

a) The induced connection ∇^F is metric;
b) A^*_ξ vanishes on $D_F\ \forall\ \xi\in N$;
c) N is a Killing distribution;
d) N is a parallel distribution with respect to ∇^F.

Theorem 3.24 Let a degenerate foliation F in a Semi-Riemannian manifold (M,g). The next assertions are equivalent:

(i) The screen distribution S (F) is integrable;
(ii)The second fundamental form of S (F) h^* is symmetric on S (F);
(iii)The Weingarten operator A_N is self-adjoint on S (F)) with respect to g $\forall N \in \deg(F)$.

Proof. (i)$\Leftrightarrow[P_2X,P_2Y] \in S(F) \ \forall X,Y \in D_F \Leftrightarrow (\nabla^*_{P_2X} PY - \nabla^*_{P_2Y} PX)+(h^*(P_2X,P_2Y)-h^*(P_2Y,P_2X)) \in S(F)) \Leftrightarrow h^*(P_2X,P_2Y)=h^*(P_2Y,P_2X) \ \forall X,Y \in D_F \Leftrightarrow$(ii). From (3.86) we have that (ii)$\Leftrightarrow g(h^*(P_2X,P_2Y),N)=g(h^*(P_2X,P_2Y),N) \Leftrightarrow g(A_N P_2X,P_2Y)=$

$g(P_2X,A_N P_2Y) \Leftrightarrow$(iii).

Theorem 3.25 Let F a degenerate foliation in a Semi-Riemannian manifold (M,g). The next assertions are equivalent:

(i) The screen distribution S (F) is parallel with respect to ∇^F;

(ii) The second fundamental form of S (F) h^* identically vanishes;

(iii) The Weingarten operator A_N takes values in N .

Proof. From (3.82) we have that (i)$\Leftrightarrow$(ii) and from (3.48) that (i)$\Leftrightarrow$(iii).

In the final of this section it is interesting to see when the null distribution is integrable (from the point of view of the new geometrical objects).

Theorem 3.26 Let F a degenerate foliation in a Semi-Riemannian manifold (M,g). The next assertions are equivalent:

(i) N is integrable;
(ii)$h^L(\xi,P_2X)=0 \ \forall \xi \in N \ \ \forall X \in D_{F;}$
(iii) A^*_ξ identically vanishes on N .

Proof. From (3.84) we have $g(A^*_\xi \xi', P_2X)=g(\xi,h^L(\xi',P_2X)) \ \forall \xi,\xi' \in N \ \ \forall X \in D_F$. If (ii) holds then $g(A^*_\xi \xi', P_2X)=0 \ \forall \xi,\xi' \in N \ \ \forall X \in D_F$ therefore (iii) and reciprocally if (iii) is true then $g(\xi,h^L(\xi',P_2X)) \ \forall \xi,\xi' \in N \ \forall X \in D_F$ from where (ii). From (3.91) follows that if (i) is true that is N is integrable then $A^*_\xi \xi'=0 \ \forall \xi,\xi' \in N$ therefore (iii). If (iii) is true then $\forall \xi,\xi' \in N \ \ \forall X \in D_F$ follows:

$g([\xi,\xi'],P_2X)=g(\nabla_\xi\xi',P_2X)-g(\nabla_{\xi'}\xi,P_2X)=-g(A^*_{\xi'}\xi,P_2X)+g(A^*_\xi\xi',P_2X)=0$

therefore $[\xi,\xi']\in N \ \forall \ \xi,\xi'\in N$ which is the same thing with (i).

4. Totally geodesic degenerate foliations

Definition We call a degenerate foliation (F, g, S (F), S ($F^\perp$)) of codimension m of a (m+n)-dimensional Semi-Riemannian manifold (M,g) totally geodesic degenerate foliation if any geodesic of an arbitrary leaf of F is a geodesic of M.

Theorem 4.1 Let (F, g, S (F), S ($F^\perp$)) a degenerate foliation of a Semi-Riemannian manifold (M,g). F is totally geodesic if and only if one of the following statements is true:

(i) $h^L=h^S=0$;

(ii) ii_1) $A^*_\xi X=0 \ \forall \ \xi\in N \ \ \forall X\in D_F$;

ii_2) $A_W X\in N \ \ \forall W\in S(F^\perp)) \ \forall X\in D_F$;

ii_3) $D^L(X,P_3V)=0 \ \forall X\in D_F \ \forall V\in tr(F)$

Proof. The condition that F is totally geodesic is equivalent with $\nabla_X X\in D_F \ \forall \ X\in D_F$. From (3.37) we see that this is equivalent with $h^L(X,X)=h^S(X,X)=0$ and from the symmetry of h^L and h^S we have (i). Let prove now that (i)$\Rightarrow$(ii). If $h^L=h^S=0$ from (3.84) follows that $g(A^*_\xi X,P_2Y)=0 \ \forall\xi\in N \ \forall X,Y\in D_F$ therefore: $A^*_\xi X=0$ that is ii_1). From (3.83) and ii_1) follows that $\nabla^F_X\xi\in N$ and from (3.37): $\nabla_X\xi\in N$. We have now with (3.40):

$0=\nabla_X g(\xi,P_3V)=g(\nabla_X\xi,P_3V)+g(\xi,\nabla_X P_3V)=g(\xi,\nabla_X P_3V)=g(\xi,D^L(X,P_3V))$

$\forall\xi\in N \ \forall X\in D_F \ \forall V\in tr(F)$ therefore ii_3). Finally, from (3.44) and ii_3) we have: $g(A_W X,Y)=g(Y,D^L(X,W))=0 \ \forall X,Y\in D_F \ \ \forall W\in S(F^\perp)$ therefore ii_2).

If we shall suppose now that (ii) is true then from (3.84) and ii_1) follows: $g(\xi,h^L(X,P_2Y))=g(A^*_\xi X,P_2Y)=0$ therefore $h^L(X,P_2Y)=0 \ \ \forall X,Y\in D_F$. From the theorem 3.7 follows that $h^L(\xi,\xi')=0 \ \forall\xi,\xi'\in N$. We have finally that $h^L=0$. From (3.44), ii_2) and ii_3) we have now: $g(h^S(X,Y),W)=0 \ \ \forall X,Y\in D_F \ \forall W\in S(F^\perp)$. Because S ($F^\perp$) is nondegenerate follows that $h^S=0$ therefore finally (i).

Corrolary 4.1 Let (F, g, S (F)) a coisotropic foliation of a Semi-Riemannian manifold (M,g). The foliation F is totally geodesic if and only if one of the following statements is true:

(i) $h^L=0$;

(ii) $A^*_\xi X=0 \ \forall \ \xi \in N \ \ \forall X \in D_F$

Proof. In this case $h^S=0$, S $(F^{\perp})=\{0\}$, $P_3=0$ and the statement reduces to the theorem 4.1.

Corrolary 4.2 Let (F, g, S $(F^{\perp})$) an isotropic foliation of a Semi-Riemannian manifold (M,g). The foliation F is totally geodesic if and only if one of the following statements is true:

(i) $h^S=0$;

(ii) $D^L(X,P_3V)=0 \ \forall X \in D_F \ \ \forall V \in tr(F)$.

Proof. In the case of isotropic foliations, from the corrolary 3.1 follows $h^L=0$ and how S $(F)=\{0\}$ and $ii_2)$ is trivial follows the conclusions of the corrolary.

Corrolary 4.3 If (F, g) is a totally degenerate foliation of a Semi-Riemannian manifold (M,g) then the foliation F is totally geodesic.

Proof. From the corrolary 3.1 we have that $h^L=0$ and how $P_3=0$ we have $h^S=0$ therefore from the theorem 4.1 follows that F is totally geodesic.

Corrolary 4.4 If (F, g, S (F), S $(F^{\perp})$) is a totally geodesic degenerate foliation of a Semi-Riemannian manifold (M,g) then the null distribution N is integrable.

Proof. From the theorem 3.26 we see that N is integrable if and only if $h^L(\xi,P_2X)=0 \ \forall \ \xi \in N \ \ \forall X \in D_F$. From the theorem 4.1.i) the condition is satisfied by the totally geodesic degenerate foliation.

From the theorems 3.8, 3.9, 3.10 and 3.11 we have:

Theorem 4.2 In a totally geodesic degenerate foliation (F, g, S (F), S $(F^{\perp})$) of a Semi-Riemannian manifold (M,g) the induced connection ∇^F on F is independent of the screen distribution

Remark From (3.37) follows that ∇^F coincides with the restriction of ∇ on D_F.

From the theorem 3.13 we have the following:

Theorem 4.3 In a totally geodesic degenerate foliation (F, g, S (F), S ($F^{\perp}$)) of a Semi-Riemannian manifold (M,g) the induced connection ∇^F on F is metric.

5. Totally umbilical degenerate foliations

Definition We call a degenerate foliation (F, g, S (F), S ($F^{\perp}$)) of codimension m of a (m+n)-dimensional Semi-Riemannian manifold (M,g) totally umbilical degenerate foliation if $\exists H_L \in deg(F)$, $H_S \in S(F^{\perp})$ with the property that:

(5.1) $$h^L(X,Y)=g(X,Y)H_L$$

(5.2) $$h^S(X,Y)=g(X,Y)H_S$$

$\forall X,Y \in D_F$.

Remark From the theorem 4.1 follows that a totally umbilical degenerate foliation is totally geodesic if and only if $H_L=0$ and $H_S=0$.

Remark In the cases of coisotropic or totally degenerate foliations, because $P_3=0$, only the axiom (5.1) is necessary for totally umbilicality.

If we consider now the totally umbilical degenerate foliation F, the formula (3.37) becomes:

(5.3) $$\nabla_X Y=\nabla^F{}_X Y+g(X,Y)H_L+g(X,Y)H_S \quad \forall X,Y \in D_F$$

Also, the formula (3.44) becomes:

(5.4) $$g(H_S,W)g(X,Y)+g(Y,D^L(X,W))=g(A_W X,Y) \quad \forall X,Y \in D_F \quad \forall W \in S(F^{\perp})$$

From (3.45) we have:

(5.5) $$g(Y,\nabla^F{}_X \xi)=-g(X,Y)g(H_L,\xi) \quad \forall X,Y \in D_F \, \forall \xi \in N$$

If we shall note $(D\xi)(X)=\nabla^F_X\xi$ we have from (5.5):

(5.6) $$g(Y,(D\xi)(X))=g(X,(D\xi)(Y)) \ \forall \ X,Y\in D_F$$

From (5.6) follows therefore:

Theorem 5.1 On a totally umbilical degenerate foliation for any $\xi\in N$ the operator $D\xi$ is self-adjoint on D_F with respect to g.

From the definition, we have also

(5.7) $$h^L(X,\xi)=0$$

(5.8) $$h^S(X,\xi)=0$$

$\forall \ X\in D_F \ \forall\xi\in N$.

Theorem 5.2 If (F, g, S (F), S ($F^{\perp}$)) is a totally umbilical degenerate foliation of a Semi-Riemannian manifold (M,g) then the null distribution N is integrable.

Proof. From the theorem 3.26 we have that N is integrable if and only if $h^L(\xi,P_2X)=0 \ \forall\xi\in N \ \forall X\in D_F$. From (5.7) follows this type of foliations satisfies that this.

Theorem 5.3 A totally umbilical isotropic foliation is totally geodesic degenerate.

Proof. If F is isotropic then $N=D_F$. From (5.7) and (5.8) follows that $h^L=h^S=0$ and from the theorem 4.1.i) follows that the foliation is totally geodesic.

Because the totally degenerate foliations are totally geodesic and after the theorem 5.3 the isotropic are also totally geodesic from this moment we shall consider only the cases of r-degenerate with $r<\min\{m,n\}$ or coisotropic foliations.

From (5.7) and the theorem 3.26 we have therefore:

Theorem 5.4 On a totally umbilical r-degenerate with $r<\min\{m,n\}$ or coisotropic foliation we have that $\forall\xi\in N$ the operator A^*_ξ of S (F) vanishes identically on N .

From (3.67) we have now:

(5.9) $(\nabla^F_X g)(Y,Z)=g(H_L,Z)g(X,Y)+g(H_L,Y)g(X,Z)$

$\forall\ X,Y,Z\in D_F$.

From (5.9) follows:

(5.10) $\nabla^F_\xi g=0$

Theorem 5.5 On a totally umbilical degenerate foliation ∇^F is a linear connection metric on S (F).

Proof. From (5.9) for $Y\to P_2Y$ and $Z\to P_2Z$ we have:

(5.11) $(\nabla^F_X g)(P_2Y,P_2Z)=0$

$\forall X,Y,Z\in D_F\ \forall\xi\in N$.

Theorem 5.6 On a totally umbilical r-degenerate with $r<\min\{m,n\}$ or coisotropic foliation the induced connection ∇^F is metric if and only if $H_L=0$ (or $h^L=0$).

Proof. From (5.9) we have:

(5.12) $(\nabla^F_X g)(P_2Y,\xi)=g(H_L,\xi)g(X,P_2Y)+g(H_L,P_2Y)g(X,\xi)=g(H_L,\xi)g(X,P_2Y)$

$\forall X,Y\in D_F\ \forall\xi\in N$. If F is r-degenerate with $r<\min\{m,n\}$ or coisotropic then S (F) does not coincides with the null distribution, therefore we can choose a non-null vector field $X\in S(F)$). If ∇^F is a metric connection then from (5.12) follows: $0=(\nabla^F_X g)(P_2X,\xi)=g(H_L,\xi)g(P_2X,P_2X)$ therefore $g(H_L,\xi)=0\ \forall\xi\in N$ that is $H_L=0$. Reciprocally, from (5.12) follows that if $H_L=0$ then $(\nabla^F_X g)(PY,\xi)=0\ \forall X,Y\in D_F$ $\forall\xi\in N$. Also, from (5.9) follows: $(\nabla^F_X g)(\xi,\xi')= g(H_L,\xi')g(X,\xi)+g(H_L,\xi)g(X,\xi')=0$ $\forall\xi,\xi'\in N$. From (5.11) we have:

$(\nabla^F_X g)(P_2Y,P_2Z)=0\ \forall X,Y,Z\in D_F$ therefore ∇^F is a metric connection.

Corrolary 5.2 On a totally umbilical coisotropic foliation the induced connection ∇^F is metric if and only if it is totally geodesic.

Proof. On coisotropic foliations we have $P_3=0$ and therefore from the theorem 5.6 follows that ∇^F is a metric connection if and only if $h^L=0$. But this does not means else that the foliation is degenerate totally geodesic.

Theorem 5.7 On a totally umbilical foliation r-degenerate with r<min{m,n} or coisotropic any vector field of the screen distribution is proper for the Weingarten operator of S (F): A^*_ξ $\forall\xi\in N$.

Proof. From (3.84) we have:

(5.13) $g(A^*_\xi P_2X,P_2Y)=g(P_2X,P_2Y)g(H_L,\xi)$

$\forall X,Y\in D_F$ $\forall\xi\in N$.

Because $A^*_\xi P_2X\in S(F))$ $\forall\xi\in N$ $\forall X\in D_F$ and S (F) is nondegenerate, from (5.13) follows:

(5.14) $A^*_\xi P_2X=g(H_L,\xi)P_2X$ $\forall\xi\in N$ $\forall X\in D_F$

Theorem 5.8 On a totally umbilical foliation (F, g, S (F), S ($F^\perp$)) r-degenerate with r<min{m,n} the following statements are equivalents:

a) ∇^t is a linear metric connection relative to S (F);
b) $A_WP_2X=g(H_S,W)P_2X$ $\forall X\in D_F$ $\forall W\in S(F^\perp)$

Proof. a)⇒b) From (3.72)-(3.74) follows:

(5.15) $g(A_WP_2X,N)=0$

$\forall X\in D_F$ $\forall W\in S(F^\perp)$ $\forall N\in deg(F)$.

From (5.15) we have that $A_WP_2X\in S(F))$ $\forall X\in D_F$ $\forall W\in S(F^\perp)$.

From (5.4) we have also:

(5.16) $g(A_WP_2X,P_2Y)=g(H_S,W)g(P_2X,P_2Y)+g(P_2Y,D^L(P_2X,W))=g(H_S,W)g(P_2X,P_2Y)$

therefore:

(5.17) $$g(A_W P_2X - g(H_S,W)P_2X, P_2Y)=0$$

$\forall X,Y \in D_F \; \forall W \in S(F^{\perp})$. How $S(F)$ is nondegenerate follows from this:

(5.18) $$A_W P_2X = g(H_S,W)P_2X \; \forall X \in D_F \; \forall W \in S(F^{\perp})$$

b)$\Rightarrow$a) If (5.18) holds then $A_W P_2X \in S(F)) \; \forall X \in D_F \; \forall W \in S(F^{\perp})$. From (3.72)-(3.74) follows $(\nabla^{t}_{P_2X} g)(V,V')=0 \; \forall V,V' \in tr(F))$.

Theorem 5.9 Let $(F, g, S(F), S(F^{\perp}))$ a foliation r-degenerate with $r<\min\{m,n\}$ or coisotropic of (M,g). Then F is degenerate totally umbilical if and only if the following statements hold:

(i) $h^L(X,\xi)=h^S(X,\xi)=0 \; \forall X \in D_F \; \forall \xi \in N$;

(ii) $\exists \alpha \in \Lambda^1(S(F^{\perp}))$ such that $g(A_W P_2X, P_2Y)=\alpha(W)g(P_2X,P_2Y) \; \forall X,Y \in D_F \; \forall W \in S(F^{\perp})$;

(iii) $\exists \beta \in \Lambda^1(N)$ such that $A^{*}_{\xi} P_2X = \beta(\xi)P_2X \; \forall X \in D_F \; \forall \xi \in N$.

Proof. If F is totally umbilical then (i) follows from (5.7) and (5.8). From (5.16) defining $\alpha(W)=g(H_S,W) \; \forall W \in S(F^{\perp}))$ follows (ii). Finally, from (5.14) defining $\beta(\xi)=g(H_L,\xi) \; \forall \xi \in N$ follows (iii).

Reciprocally, let suppose that (i), (ii), (iii) are true. We define now $H_S \in S(F^{\perp})$ such that:

(5.19) $$g(H_S,W)=\alpha(W) \; \forall W \in S(F^{\perp})$$

and $H_L \in deg(F)$ such that

(5.20) $$g(H_L,\xi)=\beta(\xi) \; \forall \xi \in N$$

From (3.44) we have $g(h^S(P_2X,P_2Y),W)=g(A_W P_2X,P_2Y)=g(H_S,W)g(P_2X,P_2Y)$ and because $S(F^{\perp})$ is nondegenerate follows that $h^S(P_2X,P_2Y)=g(P_2X,P_2Y)H_S$. From (i) we have now (5.2). From (3.84) follows that $g(h^L(P_2X,P_2Y),\xi)= g(A^{*}_{\xi}P_2X,P_2Y)=g(H_L,\xi)g(P_2X,P_2Y)$ therefore $h^L(P_2X,P_2Y)=g(P_2X,P_2Y)H_L$ and with

(i) we have (5.1). From (5.1) and (5.2) follows that F is totally umbilical degenerate foliation.

Let see now some examples that illustrate the phenomenon of totally geodesibility or umbilicality.

5.1. If we go back to the example 2.1 we have:

$$\overline{\nabla}_X X = -\frac{\varphi'(x^1,x^2)}{\sqrt{\varphi^2(x^1,x^2)-1}^3} W - \frac{\varphi(x^1,x^2)\varphi'(x^1,x^2)\sin\alpha}{\cos^2\alpha\left(\varphi^2(x^1,x^2)-1\right)^3}\xi$$

$$\overline{\nabla}_X \xi = \frac{2\varphi(x^1,x^2)\varphi'(x^1,x^2)}{\sqrt{\varphi^2(x^1,x^2)-1}^3}\xi$$

$$\overline{\nabla}_\xi \xi = 0$$

therefore:

$$\nabla_X X = -\frac{\varphi(x^1,x^2)\varphi'(x^1,x^2)\sin\alpha}{\cos^2\alpha\left(\varphi^2(x^1,x^2)-1\right)^3}\xi$$

$$\nabla_X \xi = \frac{2\varphi(x^1,x^2)\varphi'(x^1,x^2)}{\sqrt{\varphi^2(x^1,x^2)-1}^3}\xi$$

$$\nabla_\xi \xi = 0$$

Finally we have:

$h^L(X,X)=0$, $h^L(X,\xi)=0$, $h^L(\xi,\xi)=0$, $h^S(X,X)=-\frac{\varphi'(x^1,x^2)}{\sqrt{\varphi^2(x^1,x^2)-1}^3}W$, $h^S(X,\xi)=0$, $h^S(\xi,\xi)=0$. If we consider now $H_S=-\frac{\varphi'(x^1,x^2)}{\sqrt{\varphi^2(x^1,x^2)-1}^3}W$ follows $h^S(X,X)=g(X,X)H_S$ therefore the foliation is totally umbilical 1-degenerate.

5.2. We shall present now an example from [2]. Let the 1-degenerate foliation in $\mathbf{R}^4_2(-,-,+,+)$ with a quasi-orthonormal basis given by:

$$\xi = \partial_1 + \partial_2 + \sqrt{2}\partial_3$$
$$X = \sqrt{2}\left[1+(x^1-x^2)^2\right]\partial_2 + \left[1+(x^1-x^2)^2\right]\partial_3 - \sqrt{2}(x^1-x^2)\partial_4$$
$$W = 2\left(x^2-x^1\right)\partial_2 + \sqrt{2}(x^2-x^1)\partial_3 + \left[1+(x^1-x^2)^2\right]\partial_4$$
$$N = -\frac{1}{2}\partial_1 + \frac{1}{2}\partial_2 + \frac{1}{\sqrt{2}}\partial_3$$

where N =Span(ξ),S (F)=Span(X),S (F $^{\perp}$)=Span(W) and deg(F)=Span(N).

The fact that it is a foliation follows from:

$$[\xi, X] = 2\sqrt{2}(x^1-x^2)\partial_2 + 2(x^1-x^2)\partial_3 - \sqrt{2}\partial_4 - 2\sqrt{2}(x^1-x^2)\partial_2 - 2(x^1-x^2)\partial_3 + \sqrt{2}\partial_4 = 0$$

We have now easy that $h^L=0$, $h^S(X,\xi)=h^S(\xi,\xi)=0$ and

$$h^S(X,X) = \frac{2\left(1-(x^1-x^2)^4\right)}{1+(x^1-x^2)^4}W$$

Because $g(X,X)=-(1+(x^1-x^2)^4)$ we have that $h^S(X,X)=g(X,X)H_S$ where $H_S=\frac{2\left((x^1-x^2)^4-1\right)}{\left(1+(x^1-x^2)^4\right)^2}W$. We have therefore that F is a totally umbilical 1-degenerate foliation in $\mathbf{R}^4_2$.

5.3. Let consider now the example 2.3. Because $h^L(\xi,X)=h(\xi,\xi)=0$ we have $h^L{}_1(X,X)=g(\nabla_X X,\xi)=-2y(\sin z+\cos z)(\sin z+\cos z-1)$. Let therefore:

$$H_L = \frac{-2(\sin z+\cos z)}{9y^2(\sin z-\cos z)^2(\sin z+\cos z-1)}\left[y(\sin z+\cos z-1)\partial_x + \right.$$
$$\left. y(3\sin z\cos z-2)\partial_y + 3(\sin z-\cos z)(\sin z+\cos z-1)\partial_z\right]$$

We have $H_L \in \deg(F)$ and $h^L(X,X)=g(X,X)H_L$. Because $0=h^L(X,\xi)=g(X,\xi)H_L$ and $0=h^L(\xi,\xi)=g(\xi,\xi)H_L$ follows that the foliation is coisotropic, totally umbilical.

5.4. From the corrolary 4.3 follows that the example 2.5 is a totally degenerate and totally geodesic foliation.

6. Examples of degenerate foliations

on manifolds provided with relativistic metrics

Let therefore the manifold M with the metric:

$$(6.1) \qquad ds^2 = V^2(r)dt^2 - \frac{1}{V^2(r)}dr^2 - r^2\left[d\theta^2 + \sin^2\theta d\varphi^2\right]$$

where $V \neq 0$.

Remark We have the following particular cases:

$1)V^2(r) = 1 - \frac{2m}{r}$ correspond to the exterior Schwarzschild metric;

$2)V^2(r) = 1 - \frac{2m}{r} + \frac{e^2}{r^2}$ correspond to the Reissner - Weil metric;

$3)V^2(r) = 1 - \frac{r^2}{R^2}$ correspond to the de Sitter metric;

$4)V^2(r) = 1$ correspond to the Minkowski metric;

$5)V(r)=Cr$, $C \in \mathbf{R}^*$

We shall note for the simplicity: $\partial_t = \frac{\partial}{\partial t}, \partial_r = \frac{\partial}{\partial r}, \partial_\theta = \frac{\partial}{\partial \theta}, \partial_\varphi = \frac{\partial}{\partial \varphi}$.

Theorem 6.1 Let the Semi-Riemannian manifold M endowed with the metric:

$$ds^2 = V^2(r)dt^2 - \frac{1}{V^2(r)}dr^2 - r^2\left[d\theta^2 + \sin^2\theta d\varphi^2\right],\ V \neq 0$$

If ∇ is the Levi-Civita connection on M then the following relations hold:

$$\nabla_{\partial_t}\partial_t = V^3V'_r\,\partial_r \quad \nabla_{\partial_t}\partial_r = \nabla_{\partial_r}\partial_t = \frac{V'_r}{V}\partial_t \quad \nabla_{\partial_r}\partial_r = -\frac{V'_r}{V}\partial_r \quad \nabla_{\partial_r}\partial_\theta = \nabla_{\partial_\theta}\partial_r = \frac{1}{r}\partial_\theta$$

$$\nabla_{\partial_r}\partial_\varphi = \nabla_{\partial_\varphi}\partial_r = \frac{1}{r}\partial_\varphi \quad \nabla_{\partial_\theta}\partial_\theta = -rV^2\partial_r \quad \nabla_{\partial_\theta}\partial_\varphi = \nabla_{\partial_\varphi}\partial_\theta = \frac{\cos\theta}{\sin\theta}\partial_\varphi$$

$$\nabla_{\partial_\varphi}\partial_\varphi = -rV^2\sin^2\theta\partial_r - \sin\theta\cos\theta\partial_\theta$$

restul componentelor fiind nule.

Proof. Through direct calculus.

6.1. Let the foliation F generated by the vector fields: $\xi=\frac{1}{V}\partial_t+\frac{1}{r}\partial_\theta$ and $X=e^{tV}\partial_\varphi$. We have $[\xi,X]=[\frac{1}{V}\partial_t+\frac{1}{r}\partial_\theta, e^{tV}\partial_\varphi]=e^{tV}\partial_\varphi=X$, $[\xi,\xi]=[X,X]=0$ therefore F is a foliation. Because $g(\xi,\xi)=g(\frac{1}{V}\partial_t+\frac{1}{r}\partial_\theta, \frac{1}{V}\partial_t+\frac{1}{r}\partial_\theta)=0$, $g(\xi,X)=0$, $g(X,X)=-r^2e^{2tV}\sin^2\theta\neq 0$ follows that the foliation is degenerate. We have therefore $N=Span(\xi)$ and $S(F)=Span(X)$. Considering $D_F=\{\alpha\xi+\beta X \mid \alpha,\beta\in F(M)\}$ we have: $D_F{}^{\perp}=\{ar\partial_t+b\partial_r+aV\partial_\theta \mid a,b\in F(M)\}=Span(r\partial_t+V\partial_\theta,\partial_r)=Span(\xi,\partial_r)$. We obtain therefore $S(F^{\perp})=Span(W)$ where $W=\partial_r$. If consider now $N=\frac{1}{V}\partial_t+V\partial_r$ we have $deg(F)=Span(N)$ and therefore the foliation is 1-degenerate with a local quasi-orthonormal basis given by $\{X,W,\xi,N\}$.

If we compute the principal geometrical objects we have:

$h^L(\xi,\xi)=h^L(X,\xi)=0$, $h^L(X,X)=re^{2tV}\sin\theta\cos\theta N$

$h^S(\xi,\xi)=(VV'_r-\frac{V^2}{r})W$, $h^S(\xi,X)=0$, $h^S(X,X)=-rVe^{2tV}\sin\theta\cos\theta W$

$$\nabla^F_\xi\xi=0,\nabla^F_\xi X=\left(1+\frac{\cos\theta}{r\sin\theta}\right)X,\nabla^F_X X=-re^{2tV}\sin\theta\cos\theta\xi,\nabla^F_X\xi=\frac{\cos\theta}{r\sin\theta}X$$
$$\nabla^*_\xi X=\left(1+\frac{\cos\theta}{r\sin\theta}\right)X,\nabla^*_X X=0$$
$$\nabla^{*t}_\xi\xi=\nabla^{*t}_X\xi=0$$
$$h^*(\xi,X)=0,h^*(X,X)=-re^{2tV}\sin\theta\cos\theta\xi$$
$$A^*_\xi\xi=0,A^*_\xi X=-\frac{\cos\theta}{r\sin\theta}X$$
$$A_W\xi=-\frac{1}{r}\xi,A_W X=-\frac{1}{r}X,A_N\xi=-\frac{V}{r}\xi,A_N X=-\frac{V}{r}X$$
$$D^L(\xi,W)=\left(\frac{V'_r}{V}-\frac{1}{r}\right)N,D^L(X,W)=0,D^S(\xi,N)=\frac{V^2}{r}W,D^S(X,N)=0$$

From these and the theorem 4.1 follows that F is not totally geodesic. In order that F be totally umbilical it must that: $h^S(\xi,\xi)=0 \Leftrightarrow VV'_r-\frac{V^2}{r}=0$ herefore $V(r)=Cr$, $C\in\mathbf{R}^*$. Reciprocally, if $V(r)=Cr$, $C\in\mathbf{R}^*$ then $h^S(\xi,\xi)=0$. If we define:

$$H_L=-\frac{\cos\theta}{r\sin\theta}N \text{ and } H_S=\frac{C\cos\theta}{\sin\theta}W$$

we have $h^L(X,Y)=g(X,Y)H_L$ and $h^S(X,Y)=g(X,Y)H_S$ $\forall X,Y\in D_F$ therefore the foliation is totally umbilical.

6.2. Let the foliation F generated by the vector fields $\xi=\frac{1}{V}\partial_t+V\partial_r$]i $X=e^{\alpha\int\frac{1}{V(r)}dr}\partial_\theta$, $\alpha\in\mathbf{R}$. We have $[\xi,X]=\alpha X$,$[\xi,\xi]=[X,X]=0$ therefore F is really a foliation. Because $g(\xi,\xi)=0$, $g(\xi,X)=0$, $g(X,X)=-r^2e^{2\alpha\int\frac{1}{V(r)}dr}\neq 0$ follows that the foliation is degenerate. We have therefore: N = Span(ξ) and S (F)=Span(X). Considering now $D_F=\{\alpha\xi+\beta X \mid \alpha,\beta\in F(M)\}$ we have: $D_F{}^{\perp}=\{a\partial_t+aV^2\partial_r+b\partial_\varphi \mid a,b\in F(M)\}$=Span($\xi,\partial_\varphi$). We obtain therefore S ($F^{\perp}$)=Span(W) where $W=\partial_\varphi$. Considering $N=\frac{1}{2V}\partial_t-\frac{V}{2}\partial_r$ we have deg(F)=Span(N) and therefore the foliation is 1-degenerate with a local quasi-orthonormal basis given by {X,W,ξ,N}.

If we compute the principal geometrical objects we have:

$$h^L(\xi,\xi)=0,\ h^L(\xi,X)=0,\ h^L(X,X)=rVe^{2\alpha\int\frac{1}{V(r)}dr}N$$
$$h^S(\xi,\xi)=0,\ h^S(\xi,X)=0,\ h^S(X,X)=0$$

$$\nabla^F{}_\xi\xi=V'(r)\xi,\nabla^F{}_\xi X=\left(\alpha+\frac{V}{r}\right)X,\nabla^F{}_X X=-\frac{rV}{2}e^{2\alpha\int\frac{1}{V(r)}dr}\xi,\nabla^F{}_X\xi=\frac{V}{r}X$$
$$\nabla^*{}_\xi X=\left(\alpha+\frac{V}{r}\right)X,\nabla^*{}_X X=0$$

$$\nabla^{*t}{}_\xi\xi=V'(r)\xi,\nabla^{*t}{}_X\xi=0$$
$$h^*(\xi,X)=0,h^*(X,X)=-\frac{rV}{2}e^{2\alpha\int\frac{1}{V(r)}dr}\xi$$

$$A^*_\xi \xi = 0, A^*_\xi X = \frac{V}{r} X$$
$$A_W \xi = 0, A_W X = 0, A_N \xi = 0, A_N X = -\frac{V}{r^2} X$$
$$D^L(\xi, W) = 0, D^L(X, W) = 0, D^S(\xi, N) = 0, D^S(X, N) = 0$$

From these and the theorem 4.1 follows that the foliation F is not totally geodesic because $h^L(X,X)\neq 0$. If we define now $H_L=-\frac{V}{r}N$ and $H_S=0$ we have $h^L(X,Y)=H_L g(X,Y)$ and $h^S(X,Y)=H_S g(X,Y)$ $\forall X,Y\in D_F$ therefore the foliation is totally umbilical.

6.3. Let the foliation F generated by the vector fields $\xi=f(r)\partial_t+V^2f(r)\partial_r$, $X_1=\frac{1}{r}\partial_\theta$, $X_2=\frac{1}{r\sin\theta}\partial_\varphi$ where $f:\mathbf{R}\to\mathbf{R}$ is a smooth map non-null everywhere.

We have $[\xi,X_1]=-\frac{V^2f(r)}{r}X_1$, $[\xi,X_2]=-\frac{V^2f(r)}{r^2\sin\theta}X_2$, $[X_1,X_2]=-\frac{\cos\theta}{r\sin\theta}X_2$ therefore F is really a foliation. Because $g(\xi,\xi)=g(\xi,X_1)=g(\xi,X_2)=0$, $g(X_1,X_1)=-1$, $g(X_2,X_2)=-1$ follows that F is a degenerate foliation.

We have therefore: N = Span(ξ) and S (F)=Span(X_1,X_2). Like upper we have: $N=\frac{1}{2V^2f(r)}\partial_t-\frac{1}{2f(r)}\partial_r$ therefore deg(F)=Span(N). The foliation is therefore coisotropic 1-codimensional with a local quasi-orthonormal basis given by $\{X_1,X_2,\xi,N\}$.

If we compute now the degenerate second fundamental form of F , we have:

$$h^L(\xi,\xi)=0, h^L(\xi,X_1)=0, h^L(\xi,X_2)=0, h^L(X_1,X_1)=h^L(X_2,X_2)=\frac{V^2f(r)}{r}N$$

Defining now: $H_L=\frac{V^2f(r)}{r}N$ follows that $h^L(X,Y)=H_L g(X,Y)$ $\forall X,Y\in D_F$ therefore the foliation F is coisotropic totally umbilical.

6.4. Let the foliation F generated by the vector field $\xi=f(r)\partial_t+V^2f(r)\partial_r$ where $f:\mathbf{R}\to\mathbf{R}$ is a smooth map non-null everywhere. Because $g(\xi,\xi)=0$ and $[\xi,\xi]=0$

follows that the foliation F is degenerate. If we shall proceed like in the first example, we have:

$$W_1 = \frac{1}{r}\partial_t + \frac{V^2}{r}\partial_r + \frac{1}{r}\partial_\theta,\ W_2 = \frac{1}{r\sin\theta}\partial_t + \frac{V^2}{r\sin\theta}\partial_r + \frac{1}{r\sin\theta}\partial_\varphi$$

where $g(W_1,W_1)=g(W_2,W_2)=-1$.

Also:

$$N = \frac{(V^2+r^2)\sin^2\theta + V^2}{2f(r)r^2\sin^2\theta V^2}\partial_t + \frac{(V^2-r^2)\sin^2\theta + V^2}{2f(r)r^2\sin^2\theta}\partial_r - \frac{1}{f(r)r^2}\partial_\theta - \frac{1}{f(r)r^2\sin^2\theta}\partial_\varphi$$

where $g(N,N)=1$.

The foliation is therefore isotropic 1-codimensional with a local quasi-orthonormal basis given by $\{\xi,W_1,W_2,N\}$.

If we compute the degenerate second fundamental form of F and the screen second fundamental form we have: $h^L=0$, $h^S=0$ therefore the foliation F is isotropic degenerate totally geodesic.

6.5. Because dim N =min {1,3}=1 we have therefore that on manifolds endowed with relativistic metrics does not exists totally degenerate foliations.

References

1 Bejancu, A., Duggal, K. L., *Lightlike Hypersurfaces of Semi-Riemannian Manifolds*, Bull.Inst. Polytechnic Iasi, Romania;

2 Bejancu, A., Duggal, K. L., *Lightlike Submanifolds of Semi-Riemannian Manifolds and Applications*, Kluwer Acad.Publishers, Dordrecht, 1996;

3 Bejancu, A., Duggal, K. L., *Lightlike Submanifolds of Semi-Riemannian Manifolds*, Acta Applicandae Mathematicae 38, 1995, 197-215;

4 Bonnor, W. B., *Null Hypersurfaces in Minkowski Space Time*, Tensor N.S., 24, 1972, 329-345;

5 Brito G. B., Walczak, P. G., *Totally geodesic foliations with integrable normal bundles*, Bol. Soc. Bras. Mat., Vol. 17, N°1 (1986), 41-46;

6 Cagnac, F., *Géométrie de Hypersurfaces Isotropes*, C.R.Acad.Sci.Paris, 201, 1965, 3045-3048;

7 Cairns, G., Ghys, E., *Totally geodesic foliations on 4-manifolds*, J. Diff. Geom., 23, 3 (1986), 241-254;

8 Cairns, G., *Totally Umbilic Riemannian Foliations*, Michigan Math.J., 37 (1990), 145-159;
9 Carriere, Y., Ghys, E., *Feuilletages Totalement Géodésiques*, An. Acad. Brasil, 53 (1981), Nº3, 427-432;
10 Chen, B. Y., *Geometry of Submanifolds*, Marcel Dekker, New York, 1973;
11 Eisenhart, L. P., *Riemannian Geometry*, Princeton University Press, 1926;
12 Ferus, D., *Totally Geodesic Foliations*, Math. Ann., 188 (1970), 313-316;
13 Haefliger, A.,*Varietes feuilletees*, Ann. Scuola Norm. Sup. Pisa Sci. Fis. Mat., 3, 16, (1962), 367-397;
14 Haefliger, A., *Some remarks on foliations with minimal leaves*, J.Diff. Geom., 2, (1980), 269-284
15 Hermann, R., *On the Differential Geometry of Foliations*, Ann. of Math., 72, 2 (1960), 445-457
16 Ioan, C. A., *Totally Umbilical Lightlike Submanifolds*, Bull. Math. Soc. Sc. Math. Roumanie, Tome 37 (87), No. 1-4, 1996, 151-172;
17 Ioan, C. A., *Degenerate Submanifolds of Semi-Riemannian Manifolds*, Tensor, N.S., Vol. 58, No.1 (1997), 1-7;
18 Ioan, C. A., *Totally Geodesic Foliations and Minimal Foliations on the Semi-Riemannian Manifolds*, Rev. Roum. Math. Pures et Appl., XLII, 3-4 (1997), 281-299
19 Ioan, C. A., *Totally Geodesic Foliations on Semi-Riemannian Manifolds*, Tensor, N. S., Vol. 58, No. 1 (1997), 31-34;
20 Johnson, D. L., Whitt, L. B., *Totally Geodesic Foliations on 3-Manifolds*, Proc. Amer. Math. Soc., 76, 2 (1979), 355-357;
21 Johnson, D. L.,Whitt, L. B.,*Totally Geodesic Foliations*, J. Diff. Geom., 15 (1980), 225-235;
22 Katsuno, K., *Null hypersurfaces in Minkowski space-time*, Tensor N.S., 40, 1 (1983), 7-12;
23 Kobayashi, S., Nomizu, K., *Foundations of Differential Geometry*, Interscience Publishers, New York, vol. I, 1963, vol. II, 1969;
24 Lang, S., *Differential Manifolds*, Addison Wesley, 1972;
25 Lawson, H. B., *Foliations*, Bull. Amer. Math. Soc., (80), 3, 1974, 369-418;
26 Molino, P., *Riemannian foliations*, Boston, Basel, Birkhauser, 1988;
27 O'Neill, B.,*Semi-Riemannian Geometry with Applications to Relativity*, Academic Press, 1983;
28 Reinhart, B. L., *Differential Geometry of Foliations*, Springer-Verlag, 1983;
29 Reinhart, B.L., *Foliated manifolds with bundle-like metrics*, Ann. of Math., (2), 69, 1959, 119-132;
30 Reinhart, B.L., *The second fundamental form of a plane field*, J. Diff. Geom., 12 (1977), 619-627;
31 Rosca, R., *On Null Hypersurfaces of a Lorentzian Manifold*, Tensor N.S., 23, 1972, 66-74;

32 Rosca, R., *Sur les Hypersurfaces Isotropes de Défaut 1 Incluses dans une Variété Lorentzienne*, C. R. Acad. Sci. Paris, 272, 1979, 393-396;
33 Solomon, B., *On foliations of R^{n+1} by minimal hypersurfaces*, Comment. Math. Helvetici, 61 (1986), 67-83;
34 Steenrod, N., *The topology of fibre bundles*, Princeton University Press, 1951;
35 Vaisman, I., *Varietes Riemannienes Feuilletees*, Czech. Math. J., 21 (96) (1971), 46-75;
36 Wolak, R., *Normal bundles of foliations of order r*, Demonstratio Mathematica, 18, 4 (1985), 977-994.

DEMOGRAPHIC CHANGE – NEW CHALLENGE FOR EUROPEAN POLICIES ON SOCIAL COHESION PROGRAMS

Associate professor, Doinita Ariton
Dunarea de Jos University from Galati

Abstract: *Demographic changes create a new society, and these changes are set to speed up from 2010 onwards: ever fewer young people and young adults, ever older workers, pensioners and very elderly people. Our societies will have to invent new ways of liberating the potential of young people and older citizens. Dealing with these changes will require the contribution of all those involved: new forms of solidarity must be developed between the generations, based on mutual support and the transfer of skills and experience.*

Keywords: *human resources policy, demographic change, pension reforms*
Jel Classification: *N10 - General, International, or Comparative*

In November 1998, Christian Marchal[1] presented a project pertaining to the ages' pyramid in the fifteen countries of the European Union and explained: ''since 1965 the annual number of births is almost constantly decreasing, it has already fallen by more than one third. Because of the increase of life expectancy the total population is still very slowly increasing but while the number of old people is rapidly increasing the number of young people is rapidly decreasing".

The problem of change of structure of population concerning the "age of pyramid" of was signalled as a contemporary reality, but "it will pose a major challenge for global prosperity and stability during the first half of the twenty-first century"[2].

In 2003, the European Employment Guidelines and the Employment taskforce report suggested adoption of "comprehensive active ageing policy centered on the

[1] World Demographic Braking, Christian Marchal.htm, Christian Marchal, Chairman of the polytechnic group X-Demography, Economy, Population,
[2] Richard Jackson and Neil Howe, The 2003 – Aging Vulnerability Index. An assessment of the Capacity of Twelve Developed Countries to Meet the Aging Challenge, the Center for Strategic and International Studies, March 2003, www.csis.org, www.watsonwyatt.com

appropriate financial incentives to longer working lives, lifelong learning strategies and improved quality of work. Extended lifelong learning opportunities should be created for supporting the ageing part of the population in an independent and healthy lifestyle, as long as possible and for extending their social network, reinforcing their active citizenship rights in all areas of every day life and avoiding social exclusion."[1]

The situation of demographical terms of Europe between 2005 and 2025, is characterized by the growth at a very slow rate — 10 million individuals. In same period, the working age population will already be decreasing, since it will lose 12 million people. "It is hard to imagine how our continent will maintain economic growth without a growing population — a 'growth without cradles'. The recent publication of a Green Paper by the European Commission calling for a new solidarity between generations, is trying to fill that gap".[2]

The idea of social partnership must be extended to demographic issues too. The ageing populations will increasingly span four generations, so, the Commission is calling for intergenerational partnership – one that recognizes the diverse needs of different generations, while placing particular emphasis on opportunities for young people. "*This intergenerational approach has been further explored in a recent Green Paper, which analyses the demographic changes of European populations and their consequences".*[3]

"*If people are living longer, there have got to be opportunities – perhaps a necessity – for them to work longer. But we've got to look at how we handle these things*".[4]

The role of the European and national policies is to contribute in achieving an average employment rate for the European Union (EU) of 70 % overall, of at least 60 % for women and of 50 % for older workers (55 to 64) by 2010, in order to

1 The social situation in the European Union 2004 - DG Employment and Social Affairs and Eurostat the European Foundation on Living and Working Conditions, Professor Géry Coomans of ISMEA; Professor Dr M. Harvey Brenner of the Berlin University of Technology; Professor Yannis Yfantopoulos of the University of Athens and Professor Christian, Haerpfer and Dr. Claire Wallace of the Institute for Advanced Studies Vienna.

2 Confronting demographic change: a new solidarity between the generations, Green Paper, European Commission, Directorate-General for Employment, Social Affairs and Equal Opportunities, Manuscript completed in March 2005, http://europa.eu.int/yourvoice/consultations/index_en.htm

3 http://europa.eu.int/comm/employment_social/social_policy_agenda/social_pol_ag_en.html

4 John Monks, General Secretary of the European Trade Union Confederation, spoke to Social Agenda Magazine, The European Commission's magazine on employment and social affairs ISSUE N° 11 May 2005

reduce unemployment and inactivity. Member States should consider setting national employment rate targets.[1]

An ageing employer perspective presupposes:[2]

- pressure on public finances leading to increased taxation and rising labor costs: Financial sustainability of pension and health systems will be under pressure;
- decreased labor market participation: If the current pattern of low levels of employment among people aged over 50, it does not change the overall employment;
- rate will further drop due to a purely mechanical effect;
- rising of the discrepancy between the evolution of productivity and labor costs: after the age of 50-55 the worker productivity curve tends to be below the labor costs, because wages have an important element of seniority;
- aggravation of the skills gap. The process of skills renewal through the entry of young people into the labor market will decrease, while the pace of technological change will increase. The figures offered by statistics[3] today, 80% of the workforce will have acquired their education and training more than 10 years earlier, while 80% of the technology used at work is less than 10 years old. This has important implications for the ability of companies to compete in the global economy which increasingly depends on workers' skills.

This inter-generational approach can improve the overall functioning of labor markets for the entire workforce as opposed to policies by categories of workers in which it measures for one group and it could be detrimental to other groups. But the problem in this approach is as follows: young people or older workers are not homogenous categories and policy responses targeted at one or other do not allow to sufficiently differentiating according to individual needs. And, "the key components of an inter-generational approach are:

1 Guideline No. 17: Implement employment policies aiming at achieving full employment, improving quality and productivity at work, and strengthening social and territorial cohesion, European Commission

2 Intervention by Thérèse de Liedekerke, Director, Social Affairs, Conference "Confronting demographic change: a new solidarity between the generations", 11-12 July 2005, Brussels, Union of Industrial and Employers' Confederations of Europe – Union des Confédérations de l'Industrie et des Employeurs d'Europe AISBL, Website:/www.unice.org

3 "The transition from education to working life: the key data on vocational training in the European Union", 2001, DG Education and Culture, Eurostat and Cedefop (European Centre for the development of Vocational Training).

- avoiding shifting the burden of present policies on the next generations by pursuing sound public finances with progressive removal of public deficits, hence the urgent need to reform pension and health care systems and to cut the public debt ratio
- improving the efficiency of education and training systems in order to give real access to lifelong learning opportunities to workers of all age groups;
- improving the functioning of labor markets to integrate more people on the labour market throughout the population spectrum (young and older workers)
- offering a favorable context to both young and older entrepreneurs willing to take risks in order to create wealth and jobs
- modernizing social protection systems also to remove unemployment and poverty traps, in particular youth unemployment traps
- increasing the employment rates of older workers in particular by discouraging and progressively removing incentives for early exit from the labor market, providing incentives for people to stay longer on the labor market and for companies to employ and keep older workers."[1]

A new place for the elderly[2]**:**

Service and product needs of this old generation provide a potentially significant market for economic development. However, the conception that all older people have reaped the rewards of economic growth ignores the reality of the situation of so many older people.

National experiences and initiatives

Most Member States received a specific recommendation. There are efforts to define the national active ageing strategies (CY, CZ, EE, FI, FR, LV, NL, PT, UK), but the action often relies on piecemeal measures in the area of tax-benefit and pension reforms discouraging early retirement rather than on increasing employability and participation throughout the lifecycle, including young people. There seems to be a high degree of reliance on the impact of pension reforms (FR,

[1] Intervention by Thérèse de Liedekerke, Director, Social Affairs, Conference "Confronting demographic change: a new solidarity between the generations", 11-12 July 2005, Brussels

[2] Main points raised by Anne Sophie PARENT (President of European Social Platform) at final panel on July 12th, Conference "Confronting demographic change: a new solidarity between the generations", 11-12 July 2005, Brussels

IT). Moreover, the commitment of some Member States was essentially limited to proposals for tripartite social dialogue (BE, LU).[1]

Finland experience:[2]

The government has implemented extensive programmes (Programme for Ageing Workers, Work attraction 'Veto' programme). Demography is one of the key points of the new European Social Model along with handling restructuring and delocalization much better than we do now, and running our economies at much higher levels of growth and employment.

Romanian experience:[3]

The main active measures are recruitment incentives to employers, training and retraining programmes, support to job creation in SMEs and to business start-ups, community work programmes and mobility grants.

Anglo-Saxon countries experience:[4]

This experience portrays a fairly flexible labor market, small public sector, low-wage service sector, welfare-to-work. Social enterprises were asked to identify the most significant challenge to the voluntary base of the social enterprise. Reluctance on the behalf of young people to volunteers, an ageing volunteer base and reluctance to become involved in the board due to legal responsibilities were the most significant challenges (22%, 20% and 16.4% respectively of all social enterprises). These challenges were more significant to rural rather than urban social enterprises. Lack of strategic skills among the Board of Directors of the enterprise, failure to develop and implement policies regarding volunteers and tensions between paid

1 Council (Employment, Social Policy, Health and Consumer Affaires), attached the text of the Joint Employment Report 2004/2005, as adopted by the Council and the Commission on 3 March 2005, to be forwarded to the European Council in view of its meeting on 23-24 March 2005, st07010.en05.pdf

2 Intervention by Sinikka Mönkäre Minister of Social Affairs and Health, Conference "Confronting demographic change: a new solidarity between the generations", Brussels, 11-12 July 2005

3 Joint Assessment of Romania's short-term employment and labor market policy priorities. This document presents an agreed set of employment and labor market objectives necessary to advance the country's labor market transformation, to make progress in adapting the employment system so as to be able to implement the Employment Strategy and to prepare it for accession to the European Union, 28 October 2002

4 Intervention by Linda HANTRAIS Director of the European Research Centre, Loughborough University (UK) , Conference "Confronting demographic change: a new solidarity between the generations", Brussels, 11 July 2005

staff and volunteers were the more common challenges suggested to currently face urban social enterprises.[1]

Research project

According to the Green Paper "Confronting demographic change: a new solidarity between the generations" (COMMUNICATION FROM THE COMMISSION):

> "*The structure of society is also changing radically. Family structures are changing: there are more "older workers" (55-64), elderly people (65-79) and very elderly people (80+), fewer children, young people and adults of working age …*"

Hence:

> "*…new forms of solidarity must be developed between the generations, based on mutual support and the transfer of skills and experience*."

Initiatives in this respect are welcomed and here there are few attempts around Europe. Two of well focused projects started in the Fifth Framework Programme are presented below.

ACTIVAGE – Overcoming the barriers and seizing the opportunities for active ageing policies in Europe.

Start date: 2002-11-01	End date: 2005-10-31
Duration: 36 months	Project Reference: HPSE-CT-2002-00102
Project cost: 1891563 EURO	Project Funding: 1139996 EURO
Programme Acronym: HUMAN POTENTIAL	Programme type: Fifth Framework Programme
Subprogramme Area: Key action Socio-Economic Knowledge Base	Contract type: Cost-sharing contracts

[1] Research on Social Enterprise in the UK: Trends and models, practices and challenges. Co-ops Research Unit and Public Interest Non-Profit management Research Unit Open University, Milton Keynes "Exploring the differences – A Comparative analysis of Irish urban and rural social enterprises" Mary O' Shaughnessy, Margaret Fenton & Patricia O' Hara Centre for Co-operative Studies, National University of Ireland Cork, Ireland

The overall aim of the ActivAge project is to identify and analyse the socio-institutional, economic, political realities facing the implementation of active ageing policies in Europe.

This implies the following:

- Chart and analyse the existing active ageing policy landscape in Europe;
- Identify and outline the barriers to and opportunities for implementing active ageing policies in Europe;
- Highlight and explore means of overcoming barriers and seizing opportunities for active ageing policies in Europe.

The ActivAge project assumed to provide:

- A map of European active ageing policies and their socio-institutional contexts;
- A catalogue of socio-institutional, economic and political barriers to and opportunities for implementing active ageing policies;
- A set of recommendation on how to overcome these barriers and seize the opportunities for active ageing policies.

The project reports discuss:

- ageing matters on the whole population of Europe, considering that the number of older people will relatively grow fast in the future decades;
- legal and institutional aspects on active ageing for the 10 countries involved in the project;
- aspects on the labor legislation and on companies' human resources policy regarding young and old people employment;
- problems on health of elder people that prevent them to continue the active life;
- recommendations on all aspects above that may help aged people to continue the active life.

ActivAge project offers a synthesis of the policies meant for active aging across Europe (member and non-member states), along with a summary of basic principles known in the field, e.g. Walker's seven key principles of Active Ageing Policy:

Walker's seven key principles of Active Ageing Policy[1]

The nature of Activity: 'activity' should consist of all meaningful pursuits which contribute to the well-being of the individual concerned, his or her family, local community or society at large and should not be concerned only with paid employment or production" .

Scope: "active ageing must encompass all older people, even those who are, to some extent, frail and dependent".

Active ageing is/should be a preventative concept. This implies adopting a 'life course' approach to understanding the ageing issue: active ageing policy, if it is to be effective, cannot solely concentrate on the current old but also needs to include policies for the future old. Centrality of intergenerational solidarity for active ageing policy: "This means fairness between generations as well as the opportunity to develop activities that span the generations"

Active ageing encompasses rights and obligations: "*Thus the right to social protection, life-long learning and training and so on may be accompanied by obligations to take advantage of education and training opportunities and to remain active in other ways*".

Active ageing should be "participative and empowering": bottom-up initiatives must qualify and complement top-down measures. Active ageing has to respect national and cultural diversity in Europe: what counts as a 'meaningful activity' is likely ton differ across Europe. APPLE Ageing populations - policy lessons from the east, which tries to find out policy challenges and solutions sought in economically dynamic Asian countries.

Start date: 2002-07-01	End date: 2003-08-31
Duration: 14 months	Project Reference: QLK6-CT-2002-30201
Project cost: 64209 EURO	Project Funding: 64209 EURO
Programme Acronym: LIFE QUALITY	Programme type: Fifth Framework Programme
Subprogramme Area: Key action The Ageing Population and Disabilities	Contract type: Preparatory, accompanying and support measures

[1] Walker, Alan and Gerhard Naegele (eds.) (1999), The Politics of Old Age in Europe, Buckingham, Open University Press.

The project mainly organized workshops that offered the opportunity for debates on the theme.

Conclusions

A new solidarity between the generations

Demographic changes create a new society, and these changes are set to speed up from 2010 onwards: ever fewer young people and young adults, ever older workers, pensioners and very elderly people. Our societies will have to invent new ways of liberating the potential of young people and older citizens. Dealing with these changes will require the contribution of all those involved: new forms of solidarity must be developed between the generations, based on mutual support and the transfer of skills and experience.

What broad policy goals should a European active ageing agenda pursue?[1]

First, in order to overcome ageism in the labor market and attain the Lisbon targets, Europeans need active employment strategies (Amann, 1999). Policy-makers, he argues, should take into account that older people in the future will rely on four sources of income: income from public pension systems, from occupational pensions, from private savings and from earned income (c.f. Giarini and Liedtke, 1998). However, this implies that any policy limited to removing financial incentives to early retirement alone is likely to fail. Without simultaneously combating age discrimination, retrenching pension systems alone would set older people in marginal employment to poverty and destitution. Effective anti-discrimination policy, in turn, requires hands-on age management at the organizational level.

Secondly, advocates of the senior citizenship acknowledge that pension systems are in need of reform. However, unlike the general thrust of pension reform in Europe (with the notable exception of Norway), pension reform should aim at providing older people with the material means to fully participate in social life (Amann, 1999; Ney, 2003). For many marginal groups in the labor market, such as women or people with disabilities, this means that pension systems provide some form of basic or guaranteed minimal income regardless of prior contribution. What is more, pension systems should not erect barriers to labor market participation of older people. This implies abandoning mandatory retirement ages, reducing the punitive nature of taxation on earned income during retirement, as well promoting pension arrangements that permit part-time employment (Walker, 2003).

[1] Walker, Alan (2003), "A Strategy for Active Ageing", unpublished paper.

Thirdly, since good health is the pivotal precondition for activity in old age, any active ageing policy must weave health care and social services into the overall policy fabric. Rather than expending scarce policy-making resources on structural and financial reforms to health care systems, advocates of the senior citizenship discourse suggest reforming the underlying approach to providing health care. Avoiding skyrocketing health care costs in the future involves breaking the link between poor health and employment (Walker, 2003): European health care systems need to prevent ill-health rather than curing disease at great costs. Moreover, activity and societal participation for the very old and frail, a group set to increase considerably in the coming decades, call for effective long-term care. Here, the active ageing policy imperative is to develop innovative concepts of long-term care and social service provision.

And finally, but by no means least, any active ageing agenda must be about democracy, rights and citizenship. In a very real sense, combating discrimination in the labour market, providing adequate old-age income and ensuring good health in old age empowers older people. However, while societal reforms represent one side of what David Held (1995) calls "double democratization", real change in political systems is the indispensable other side. Active ageing policy, argue the advocates, has to provide older people (and, by extension, everyone) with a real say in decision-making. In this way, European citizens can take active control and responsibility for their ageing. This also includes promoting the civic engagement of older Europeans.

References:

1. Aiken, M. and Spear, R, *Gateways into employment: third sector organizations working with people disadvantaged in the labor market*, NCVO Research Conference, University of Central England, Birmingham, 2-3 September 2003;
2. Bode, I., and Aiken, M., *Non-profit organizations tackling unemployment in Germany and the UK: Vigorous independent enterprises meeting social needs or the emergence of 'entrepreneurial not-for profits with limited social liability'?* VSSN Conference Paper, 12 May 2004, Manchester University;
3. Calvó-Armengol, A., Jackson, M., *Social Networks in Determining Employment: Patterns, Dynamics, and Inequality*, forthcoming American Economic Review, 2003;
4. Community Fund (2001) *Annual Report 2000/2001*, Community Fund: London;
5. Blaga, I, *Populatia activă în România*, Bucureşti, Editura politică, 1979;
6. Constantinescu, N. N., *Problemele tranziţiei la economia de piaţă în România*. Studii, Bucureşti, Editura Fundaţiei "România de Mâine", 1997;
7. Defourney, J., and Spear, R., *Economics of Co-operation* in Spear, R., and Voets, H., *Success and Enterprise: the significance of employee ownership and participation*, Aldershot: Averbury, 1995;

8. Deloitte and Touche, *Department of Enterprise, Trade & Employment, Final Report, Review of Community Employment Programme*, Dublin, Stationery Office, 1988;
9. Denny, K., Harmon, C., O'Connell, P. J., *Investing in People : The Labor Market Impact of Human Resource Interventions Funded under the 1994-1999 Community Support Framework in Ireland*, ESRI Policy Research Series No.38. Dublin, ESRI., 2000;
10. Done, I., *Salariul şi motivaţia muncii*, Bucureşti, Editura Expert, 2000;
11. FAS, *Social Economy Enterprise Guidelines*, 2000;
12. FAS, *The Job Initiative Operational Guidelines*, 1998;
13. Grigorescu, C., *Îmbătrânirea demografică în România. Trecut, prezent şi perspective*, volume *Probleme ale tranziţiei la economia de piaţa, Studii* vol. I, Bucureşti, Editura Fundaţiei "România de Mâine", 1997;
14. Hill, M., *Understanding Social Policy*, 7th Oxford: Blackwell, 2003;
15. Lazarsfeld, P., Merton, R., *Friendship as a social process: A substantive and methodological analysis*", in Berger, M., Abel, T., and Page, C. (eds), Freedom and Control in Modern Society, Princeton NJ: Van Nostrand, 1954;
16. ONS, *Labor Market Trends*, (July, August and September 2002, vol. 110:6, 110:7, 110:8); 2002;
17. O'Hara, P., *Ireland: Social Enterprises and Local Development in Borzaga*, C. & Defourny, J., (eds), *The Emergence of Social Enterprise*, Routledge, 2001;
18. O' Hara, P., & O' Shaughnessy, M., *Work Integration Social Enterprises in Ireland*, EMES European Research Network Liege: Belgium, 2004;
19. Roman, M., Mihăescu, C., Hurduzeu, M, Danciu, A., Grădinaru, G., *Culegere de probleme de demografie*, Bucureşti, Editura ASE, 2002.

ROMANIA'S ECONOMIC CONTRIBUTION TO MAKING A LONG WORLD WAR II SHORTER

Senior Lecturer Ştefan GHEORGHE, PhD
„Danubius" University from Galati

Abstract: *Romania's participation in World War II was brought about by political reasons and strategic needs that resulted from the international political situation at the middle of the twentieth century. One can hardly say that Romania did not do its best to avoid becoming involved in the war. From September 1939 to June 1941, the foreign policy laid focus on non-belligerency and neutrality. But eventually Romania was drawn in, too, right after the series of unfortunate events in the summer of 1940*

Keyword: *economic contribution, Statistic Service, Romanian currency*
Jel classification: N10 - General, International, or Comparative

Romania's participation in World War II was brought about by political reasons and strategic needs that resulted from the international political situation at the middle of the twentieth century. The loss of approximately one third of the national territory and of six million inhabitants to the three neighbouring countries (that is the Soviet Union, Hungary and Bulgaria) basically accounted for Romania's taking part in military operations on both the East and the West front during World War II[1].

One can hardly say that Romania did not do its best to avoid becoming involved in the war. From September 1939 to June 1941, the foreign policy laid focus on non-belligerency and neutrality[2]. But eventually Romania was drawn in, too, right after the series of unfortunate events in the summer of 1940[3], so that on the 22nd of June

[1] Aurică Simion, *Dictatul de la Viena(the Vienna Dictate),* Bucureşti, Editura Albatros, 1996
[2] Ion Constantin: *România, Marile Puteri şi problema Basarabiei (Romania, the Great Powers and the Issue of Basarabia*, Bucureşti, Ed. Enciclopedică, 1995, pp. 27-41
[3] Mihai Bărbulescu, Dennis Deletant, Keith Hitchins, Serban Papacostea, Pompiliu Teodor, *Istoria României (The History of Romania),* Ed. Enciclopedică Bucureşti 1999, pp 449-451

1941, Romanian troops were crossing the Prut as ordered by the head of the state at the time, General Ion Antonescu[1].

As far as the requirements of waging a modern war are concerned, Romania was taken aback by the outbreak of the war. Romanian effective forces, though in large number[2], lacked in modern equipment and war techniques that such a conflagration asks for[3].

The wall of fire and steel which the leaders of the time spoke of in order to reassure people and public opinion turned out to be a bluff made up by the for the sake of propaganda by the authoritarian monarchic regime. Within few weeks, Romanian frontiers would collapse under the pressure of the most important political and military forces of the time that is Hitlerite Germany and the Soviet Union, dragging along Carol II's regime.

The new head of the state, Ion Antonescu, would start at once reconstructing the army, as he was well aware that ongoing international political events could provide Romania with an opportunity to make its borders stand again and to reintegrate all territories that had been temporarily under foreign rule. Antonescu was to join forces with the Axis and wage war against the Soviet Union, as strongly believed that the Axis stood a firm chance of winning the war[4].

The military campaign led by the Romanian army on the East front had as an objective to set free Basarabia and the North of Bucovina, territories which had been torn apart by the Soviet Union as a result of the ultimatum delivered in June 1940[5]. It is also true that Ion Antonescu was hoping that an estimation of Romania's siding with Germany could bring along the retrocession of Ardeal; that was the very same reason for which Hungary also sent troops to the East front and declared war to the Soviet Union[6].

---

[1] Florin Constantiniu, Ilie Schipor, *Trecerea Nistrului 1941*, Bucureşti, Editura Albatros, 1995, pp 163-170

[2] Alessandru Duţu, Florica Dobre, Leonida Loghin*, Armata română în cel de-al doilea război mondial 1941-1945 (The Romanian Army during World War II 1941-1945)*, Bucureşti, Ed Enciclopedică, 1999, pp 213-214

[3] G-ral Tiberiu, Urdăreanu, *Factorul ethnic in razboaielr Romaniei Moderne (Modern Romania's Techniques of Waging War)* , Editura Militară Bucureşti 1994, pp. 153-173, 183-258

[4] Constantin, Kiriţescu, *Romania during World War II,* Bucureşti, Ed. Univers Enciclopedic, 1995, pp. 187-192

[5] Valeriu-Florin, Dobrinescu, Ion Constantin, *Basarabia during World War II,* Institutul European, Iaşi 1995 pp 216-221

[6] Florin, Constantiniu, *The Romanian Communist Party, Pătrăşcanu and Transylvania (1945-1946)*, Bucureşti, Ed Enciclopedică 2001, p. 51: *the only reason for which we sent troops against the Russians were the Romanians*, stated the Hungarian Prime-Minister Miklos Kallay, around the years 1942-1944

Owing to the alliance with Germany, the two provinces were set free in July, not without major human sacrifice[1], and political controversy whether or not war against the Soviets should be waged beyond the Nistru soon broke out between political opposition, represented by the *historical parties*, and marshal Antonescu's government. It stands to reason that the head of the state had the last say in this matter and that Romanian soldiers would go and fight in the wilderness of Russia. They fought courageously besides German allies who would often show gratitude for their support and sacrifice.

Three times were the Romanian and German troops defeated, in Stalingrad, in Crimea and on the Moldavian front Iaşi - Chişinău, and the defeats meant military disaster as they put an end to the Romanian campaign on the East front. The campaign had taken three years and two months (the 22nd of June 1941 – the 23rd of August 1944). The defeat of the Romanian army on the front in Moldova during the Soviet offensive Iași – Chișinău, together with the effects of the blow at the palace and the arresting of the marshal and his people would eventually result in Romania's withdrawal from its alliance with Germany[2] against a country which had taken away a big part of our national territory[3].

In the new political context, the Romanian authorities in Bucharest would manage to stand alone and face Germany's anger; Romanian troops disarmed and set free most part of the national territory, including the capital, and paved the way for the Soviet Army. As for the significance and the importance of what happened on the 23rd of August, it has become clear by now that the act will always be interpreted according to the evolution of political events[4].

Nevertheless, the Romanian troops' volte-face would have unpleasant consequences, especially on the front in Moldova. As soon as the Romanian army units received an order to cease fire against the Soviet army and they were loose from the German troops to withdraw at once to the south of the fortified frontline Focşani-Nămoloasa-Brăila[5], the commanding officers would obey the Department of Military Structures and cease hostilities against the Red Army, trying to save what could still be saved of the Romanian army after the front had been broken by the Soviets[6].

[1] Constantin, Kirițescu, op cit p. 269
[2] Alessandru, Duțu, op cit pp. 214-216
[3] C-tin, Kirițescu, op cit p 188
[4] Lily Marcou, *Under Stalin and Dej, the Memoirs of a left-wing politician*, Ed Antet, pp 68-87. The writer looks back on the events, making allowances for a growing communist society
[5] Al Duțu, op cit p 170
[6] Ibidem . further reference , the talks between marshal Antonescu and the representatives of the Romanian Army IV, on the necessity of withdrawing along this fortified frontline in the South

The Soviet commanding structures would make no allowances for this new situation, ordering Fronts II and III in Ukraine to continue the offensive against the enemy. Thus, a large number of Romanian soldiers who had ceased fire were taken prisoners and, strangely enough, they were held prisoners until the 12th of September and even afterwards[1] . The condition of the officers and crew in the Danube Delta and on the Black Sea turned out to be just as disastrous. They were made to cease fire on receiving orders from the Romanian authorities and then forced to land and confined to Soviet camps. This is how a large part of the Romanian trading ships and, of course, the whole of the war fleet was lost to the Soviets[2].

Romanian soldiers would still be captured by Soviet troops even after the truce was concluded on the 12th of September; strangely enough, most of them were captured in areas where no military operations between Romanian and Soviet forces had been carried out, such as Muntenia / Walachia. All these were happening while Romanian soldiers had to face German attack, too, for the Germans had received the Fuhrer's command to restore the situation in Romania, which threatened to cause the collapse of the entire German military device in the Balkans.

On the 23rd of August 1944, the Romanian military campaign in the war against the Soviets came to an end, not without big casualties as far as the number of missing people was concerned, not to mention the material losses in which bombardment, the seizure of territories and the military techniques resulted. Since 1942, Romania was the most important ally of Germany on the East front, as it disposed of some 26 Romanian divisions as compared to the eight Italian ones, the twelve Hungarian divisions and the varying number of Finnish military units. Obviously, among the 46 allied military units that the Germans could dispose of on the East front, more than half the number was represented by the Romanian units; as a consequence, it was only normal that Romania suffered significant human losses, directly proportional to the number of soldiers that had been sent to the front.

In October 1945, an informative note of the Statistic Service of the Romanian army was issued to estimate the total human losses during the campaign in the East to 309,503 military men. Mention should be made of the fact that the figures referred both to the ones taken prisoners and to those who had died on the battlefield, since the Romanian Department of Military Structures did not have the means to count the dead and the prisoners separately[3].

[1] Archives of the Defense Department, fund 948, file 1799, pp 23-24. The number of Romanian military men captured on the front in Moldova is somewhere between 60,000 and 160,000 prisoners. The most likely figure is provided by the Department of Military Structures, i.e. 130,000

[2] Idem, fund 379, file 10, p. 45

[3] Idem, fund 948, file 1585, pp 14-22

The economic cost of waging war alongside Germany turned out to be of no lesser importance; on the 23rd of August, Romanian economy was under the control of German capital, as Germany's debt to the Romanian state went up to some 1.5 billion deutsche marks. Calling up a significant military contingent was to diminish Romania's capacity for industrial production, whereas the country had to go to great lengths to keep the same production rate as before. Moreover, as Romania moved towards an economy war, many financial and economic resources which were normally meant for various other enterprises were now taken up to keep Romanian troops on the front[1]. Allied bombardments, particularly on the major economic centers, will also result in significant losses for Romanian economy; oil industry was mainly aimed at, and human losses did not lack in significance. Among indirect damage and losses induced upon Romanian economy, mention should be made of exchanges which did not amount to Romanian currency as the deutsche mark was made to go up artificially as compared to the Romanian *leu*; thus the price of the products imported from Germany went up without any regard for the price of Romanian goods for export[2]; at the same time, the purchasing power of the German military increased and they would take certain products off the Romanian market only to cause their price to go up artificially.

The support given to German troops on Romanian territory, the payment system, the deductions that resulted from transport and communication were further issues to be tackled during collaboration with the Third Reich in the war against the Soviets. Whereas German military units and import-export companies purchased essential goods at very low prices, the products imported from Germany were of little necessity to the Romanian market and they came in at extremely high prices. Romania's complex international position made the country extremely vulnerable when confronted with the terms of the German ally which was well aware of the fact that Romania could not object under the circumstances. These were some of the reasons which cast a shadow on the co-operation between Romania and Germany and they were mainly accounted for by the fact that, on accepting to become involved in the military campaign on the East front, Romanian authorities had overlooked some details, although, legally, Romania's position was, to a certain extent, that of an independent, autonomous state fighting against mutual enemy. Obviously, the two armies had not joined forces in response to some military or political convention; Romania joined in the war of its own accord with a view to setting Basarabia and Bucovina free.

The events on the 23rd of August were to bring about deep changes, as far as Romania's foreign policy was concerned; all of a sudden, Romania was made to

[1] Constantinescu, N. N. *Istoria Economiei Naţionale (The History of National Economy)*, Bucureşti, Editura Economică, 2000, vol. II p. 60
[2] Ibidem p66

fight - with the same objectives in mind - by the side of its former enemies and against its former allies. This sudden turn was sure to serve the cause of the United Nations, but Romania would have to pay dearly for it; for a while, Romanian authorities and military forces *had to resist two enemies* and part of the price it had to pay was the large number of Romanian soldiers who had been taken prisoners by Soviet troops[1].

Despite the significant losses of human lives, war materials and techniques, due to cautious course of action taken by the Romanian Department of Military Structures, Romania was able to present, on the 23rd of August, five operational divisions fully equipped for war and some other 29 divisions consisting mainly of recruits and other units which were billeted with the non-mobile troops on the battlefield. Romanian military units around the front area were not very well organized as they had been destroyed by Soviet offensive, that is why the military campaign against Nazi Germany was mainly carried out by these divisions of recruits provided inland by the Department of Military Structures.

The concluding of the Truce Convention by Romania and the United Nations on the 12th of September imposed the most significant terms as to how Romania should carry out the military campaign against Germany and its allies. The provisions were to be brought into operation under the control of an Allied (Soviet) Control Commission which, as the three great powers had established, would basically leave it to the Soviet Union. The rough interference of the representatives of the commission for control was to change the course of home policy and to affect the balance of the state budget, as goods were drawn in keeping with the Truce Convention.

Romanian authorities were determined to improve collaboration with the Soviet Union as soon as possible and to set Transylvania free, that is why they would send to the front more units than the 12 military divisions that the truce asked for, thus moving towards applying the economic provisions of the convention. As for the number of Romanian military men involved in military operations against German troops, it amounted to some 538,536 soldiers out of a total of 1,100,000 who had been called up to serve the cause of the United Nations. Between the 23rd of August 1944 and the 12th of May 1945, Romanian troops had traveled for about 1,700 kilometers at around 6 kilometers a day, they moved across 12 waterways and 20 groups of mountains and they took 3,831 villages and towns, among which there were 53 big cities. All these were accomplished through the sacrifice made by 169,822 military men who were killed, injured or declared missing. The strategic, material and human support Romania gave to the United Nations was significant enough and it helped make the war shorter by some 200 days.

[1] *The Archives of the Defense Department*, fund 514, file 8, p. 96

Although its support amounted to 1,200,000,000 dollars (1938 currency) and it came fourth in the hierarchy of the United Nations that fought against Germany, Romania was not granted the status of co-belligerent country which it was entitled to. The refusal was based on some political reasons that only the Great Powers knew of. Among all the states that had a similar situation, Romania had taken the greatest efforts to destroy the Nazi war machine[1].

Despite the fact that it may have varied to some extent in keeping with the provisions of the Romanian-Soviet protocol on the 26th of November, the number of Romanian troops involved in military operations was always higher than the one imposed by the Truce Convention. Moreover, Romania's contribution hardly knew any limits[2]: both the authorities and the civil population went to great lengths to support the military campaign of Romanian troops. No military unit or organization carried on the fight against the United Nations after the 23rd of August. For example, Italy contributed some 100,000 partisans to the cause of the United Nations, whereas 5 infantry divisions and 2 air-force squadrons would carry on fighting in the north of Italy on Germany's side. As for Hungary, they came up with 10 divisions, Bulgaria – one pro-Nazi division, Yugoslavia - 3 divisions, 9 brigades, a division of guards and a regiment of cavalry. In France the number of those who carried on fighting soon amounted to the effective force of a quick division, whereas in Poland it amounted to some 20-25,000military men[3]. The statement Romania made at the Peace Conference in Paris dwelled on the country's military and economic direct involvement and on the extent to which they had discharged their duties until the 1st of July 1945 in keeping with article 3 of the Convention; expenses came to 77 billion lei (1938 currency.

The defeat of Nazi Germany saluted by most countries in Eastern and South-Eastern Europe did not bring about utter joy since the alternative was now the communist regime imposed on by the Soviet model. The alternative was rejected by most Eastern European societies[4] which were still vexed by the war and by the release of the Red Army. Hundreds of thousands of people were to fully experience the *advantages of the Soviet release* and the consequences of their countries' political regimes turning into *popular regimes* under Soviet influence.

The fact that these *popular democratic regimes* were forced upon the states in Eastern and South-Eastern Europe would prevent further economic progress of these societies; we all had to pay tremendous costs to return to an efficient state, and we are still paying them today, more than 50 years after the conflagration. But little

[1] A.N.I.C.B, *Royal House* fund, file 21/1945, pp. 1-12
[2] *Archives of the Defense Department*, fund 948, file 177, passim
[3] Idem, file 151, pp 105-110
[4] Stephen, Fischer, Galaţi, *Eastern Europe and the Cold War*

mention is made of the moral trouble, the personal failures or the fear in which most inhabitants of these countries lived, particularly the war prisoners. They have come a long way; they were first regarded as heroes on the battlefield, subsequently they were considered traitors to their country, as they were accused of having fought against the Soviet Union and of having upset the actual "big brother" that meant to control all East-European states and under the influence of which these states would embrace communism in their social and political structures.

Bibliography:

1. Bărbulescu, Mihai, Deletant, Dennis, Keith, Hitchins, Papacostea, Şerban, Pompiliu, Teodor, *Istoria României (The History of Romania)*, Ed. Enciclopedică Bucureşti 1999, pp. 449-451;
2. Constantiniu, Florin, Schipor, Ilie, *Trecerea Nistrului 1941*, Bucureşti, Editura Albatros, 1995, pp. 163-170;
3. Constantiniu, Florin, *The Romanian Communist Party, Pătrăşcanu and Transylvania (1945-1946)*, Bucureşti, Editura Enciclopedică 2001, p. 51;
4. Constantin, Ion: *România, Marile Puteri şi problema Basarabiei (Romania, the Great Powers and the Issue of Basarabia*, Bucureşti, Ed. Enciclopedică, 1995, pp. 27-41;
5. Constantinescu, N. N., *Istoria Economiei Naţionale*, Bucureşti, Editura Economică, 2000, vol. II p. 60;
6. Dobrinescu, Valeriu-Florin, Constantin, Ion, *Basarabia during World War II*, Institutul European, Iaşi, 1995 pp. 216-221;
7. Duţu, Alessandru Dobre, Florica, Loghin,Leonida, *Armata română în cel de-al doilea război mondial 1941-1945 (The Romanian Army during World War II 1941-1945)*, Bucureşti, Ed Enciclopedică, 1999, pp. 213-214;
8. Fischer, Stephen, *Eastern Europe and the Cold War, Galati;*
9. Kirițescu, Constantin, *Romania during World War II*, Bucureşti, Ed. Univers Enciclopedic, 1995, pp. 187-192;
10. Marcou, Lily, *Under Stalin and Dej, the Memoirs of a left-wing politician,* Ed Antet, pp 68-87;
11. Simion, Aurică, *Dictatul de la Viena (the Vienna Dictate),* Bucureşti, Editura Albatros, 1996;
12. Urdăreanu, Tiberiu, *Factorul tehnic în războaiele României Moderne (Modern Romania's Techniques of Waging War)* , Bucureşti, Editura Militară, 1994, pp. 153-173, 183-258;
13. ****The Archives of the Defense Department*, fund 514, file 8, p. 96;

14. ***A.N.I.C.B, *Royal House* fund, file 21/1945, pp. 1-12;
15. ****Archives of the Defense Department*, fund 948, file 177, passim.

ASPECTS CONCERNING THE OPPORTUNITIES OF CAREER DEVELOPMENT IN THE HUMAN RESOURCES DOMAIN

Senior Lecturer Maria – Madela ABRUDAN, PhD
Senior Lecturer Horia DEMIAN PhD
University from Oradea

Abstract: *The function of human resources is an unusual matter concerning the evolution of the other functions of the organization. The enriching and changing process of its status determined, at the same time, a movement of its instruments and activities as far as all the departments and activities are concerned, so that it can ensure a true partnership with the personnel and be closer to reality. Our step can't situate itself outside the new context of the evolution of the human resources function .One of the important tasks of human resources department refers to predicting and supervising the employees' career of the business organization[1]. In order to achieve the adequate objectives, in the best conditions, it must be taken into account, simultaneously, the business organization's requirements, the employees' determination to develop their career, but also their individual potential. The department of personnel/human resources, as an essential function in the business organization, is divided into two domains, Human Resources Management (RUM) and Human Resource Development (HRD). Even though many companies do not distinguish as separate domains, human resources management and human resources development, it is important to understand and to remember that they are two main human resources domains– they are actually, the two major options in human resources career.*

Keywords*: Human Resource Management (RUM) and Human Resource Development (HRD,)*

Jel classification: *O15 - Human Resources; Human Development;*

[1] Cerdin J-L., *Gérer les carriers, Vade Mecum,* Editions EMS Management & Société, Collection Pratiques d'Entreprises, Caen, 2000.

Human Resource Management (RUM)

Human Resource Management is the traditional direction of human resources activities that leads and supports the business organization staff, and it assumes that every business organization has at least one person responsible for this domain.

Human resources management domain as the main activities include:

- The selection and the recruit;
- The rewarding system;
- Relations with the employees;
- Health and safety.

The role of human resources management is to maintain human resources competent and also to be able to fructify them. Let's imagine what would happen if your business organization would stop paying salaries. The Human Resources Management has the responsibility to make the business organization to work well and the department of Human Resources Management has to intervene in order to discover what employees expect from their workplace. The Department of Human Resources Management is also responsible for the organization to function as a whole. This process operation may cost a lot of money, and human resources management has the qualification to take decisions helping the company to save money and to ensure that the staff is well attended. In every important domain of human resources management process, there are prepared continuous assessment and also implement programs and new systems for better serving the business organization.

The following examples are relevant:

• The staff recruitment and strategy: recruitment management systems (RMS) or the applicants' identification system (AIS) are the one used in electronic management of managing the CVs flow during the recruitment busy times. The business organizations save money by accelerating the process of recruiting and involving a smaller number of personnel to manage employees' records.

• Working relationship with the employees: training the managers on topics like sexual harassment and rules at workplace which become increasingly common. These are necessary in order to proactively reduce conflicts and pending lawsuits connected with the behavior in the workplace.

• Work safety: while accidents are common in factories and on construction sites, specialists in human resources faced also a growing number of work accidents in the office. Many specialists in work safety proposed, for example, one approach

to office furniture ergonomically. Although these sophisticated chairs and computer monitors for reducing the brightness are expensive, such investment could prevent future accidents and related costs.

• Rewards and Benefits: the outside benefits are a popular way to reduce the costs and the responsibilities for the business organization. Some of the departments that are responsible for the rewards and the benefits, contract outside companies that lead benefits programs, such as: savings plans for employees. As soon as these specialized companies have the expertise and the necessary systems to run these programs, the companies are often saved from additional expenses.

Since advertising based on jobs online has become expensive, finding new ways of recruiting is an important solution to save money. According to the studies on human resources management, an important part of the work consists in improving the way the employees are "attended", so that in the past five years, one of the concerns was directed towards attend better "the customers"; the companies have achieved Intranet networks and the process of options and benefits, as all the personnel procedures and policies have become on-line procedures without bureaucracy.

Human resource development (HRD)

Human resource development represents the second domain (by some specialists, much more limited) of the human resources department. The department of human resources development from a business organization focuses those activities that help the employees to progress. Many organizations refer, in a simplistic way, to develop human resources in terms of training or learning, but in reality it is much more than that. The department of human resources development includes:

• Training and learning

• The Development Program of the Organization, which includes:

- planning the succession/development of the organizational career;
- coaching;
- performance management.

Human resource development is the domain of the human resources that is increasingly developing in the recent years, as organizations recognize the need to do much more than simply lead the workforce. While smaller organizations often have a generalist in human resources who assumes responsibility for training, among

other tasks, and the big companies have a component dedicated to fully develop human resources in the organization.

The department of human resources development may be responsible for certain activities - for example, training the staff in the sales department - or it can also offer as an internal consultancy service for some projects - for example, relating to the restructuring of the departments or the restructuring succession plan for the entire division. Other responsibilities of this department may also include: assessing the employee's performance, training the new employees, and helping the company regarding the changes resulting from the new programs, technologies, mergers or acquisitions.

The Department of career development is expanding every year. Training and development are areas that in the years to come they will know a broad development. This is due not only that jobs become increasingly complex, but also because of the numerous changes of technology, which require much more training and processing development programs of the personnel. What does this means for human resources?

Although the human resources development is not the only one which develops, it should be taken into account the fact that it is differently structured from one business organization to another. Human resource development should be seen as viable part of career, and it is very important to integrate it in the company's strategy. Organizations that have focused only on the training activity may not consider human resources development that important, as those who have a specific function of organization development.

The responsibilities of a DRU include:

- Internal Trainer;
- Consultant Executive;
- Specialist on development organization;
- Training designer (who designs the training);
- Training coordinator.

Leadership's development and training have become increasingly important for a company to attract and keep talented people. Many companies turn in for external consultants to help the executive to solve the issues related to the performance assessment that can prevent the promotion of objective criteria. Currently, many consulting firms have created divisions on the human resources development, on the basis that shortly, the human resources development will gain a great importance in most business organizations. The external consultant may work directly with the

head of the compartment or the vice-president, helping them to achieve an effective work.

In many organizations, managers, executive directors employ young professional trainers to assist in problems regarding the achievement performance or other specific areas of development. There are Professional Training Programs at the consultancy firm "Capital One", that is known in the human resources domain for its innovation of bringing closer leadership to coaching. Capital One has centralized the process, saving money and insuring that the coaching is used for practical reasons

Tabel 1

	Specific steps in the chain of coaching values	Specific activities
Who identifies the need for coaching and how?	The identification of executive directors training needs	- Training needs for an executive director are often identified by the manager, by the executive or by the Human Resources consultant in discussions relating to management performance. - The executive Managers and the Training Program Department assesses each training application to ensure that it is closely linked to the objective or business needs. - Professional training is addressed to the executive at the director's level or at a higher level.
Who selects a trainer for specific needs? How?	Identifying the trainer with the executive director	- Use specialist trainers that collaborate with the organization depending on the level of applications. - The Department for the Executive Training Program identifies the specialist consultants, with the Executive Director, based on two criteria: the needs of executive development and consultants' preoccupation. - The leading council interviews two or three candidates as trainers, recommended by the Department of Executive Training Program, before making the final selection.
What are the parameters of the specialized coaching commitment?	Management commitment to coaching	One typical cycle of counseling takes six months, during which the director may benefit up to 30 hours of counseling. Each counseling commitment is justified by a contract, between the trainer, manager and the counseled, that specifies the measurable goals of development.

		Every two or three months, the counseled, the trainer and the direct manager will have a separate meeting to analyze the progress.
How to analyze the impact of coaching?	Results analysis and the effectiveness of coaching.	- After 6 months, the counseled, the trainer and the manager revise the progress and the effectiveness of consultants by comparing them with the goals specified in the contract. - The manager provides feedback on behavior changes that took place (or not) on the basis of their own comments and feedback on the team - The manager, trainer and the counseled determine the future extension of the contract, and in some other cases it may mean to renew the commitment to yet another 6 months.

An example of the Professional Training Program. The source[1]Coaching is a phenomenon of modern management, which finds its place also on the Romanian market, especially on the multinational companies. Although the attempts to define its domain are numerous, the easiest way of seeing coaching is as a process that answers a need, namely, the need for performance management. The coaching relation often leads to unexpected good results for its beneficiaries. While many systems of personal and professional development have "legitimate parents", coaching does not have such a history. It appeared in the U.S. as a result of evolution that took a lot of techniques from different systems.

Management has as an essential dimension the leading idea: a person with authority that leads another person or a group in order to achieve a result. In coaching, it is emphasized the relation between two people as "equals". On the other hand, the specific coaching skills are absolutely necessary to managers. A good manager must have the ability to treat their subordinates as equals at least at some moment of the activity in order to create team cohesion.

Bibliography:

1. Cerdin, J-L., *Gerer les carriers, Vade Mecum*, Editions EMS Management & Société, Collection Pratiques d'Entreprises, Caen, 2000;
2. David, A., De Cenzo, Stephen, Robbins, P., *Human Resources Management: Concepts & Practices*, Printed by Englewood Cliffs, N. J.: Prentice Hall, 1994;
3. Mathis, R., Nica, P., Rusu, C., coord., *Managementul resurselor umane*, Bucureşti, Ed. Economica, 1997;
4. *** The collection of *Ziarul financiar;*
5. *** www.vault.com;
6. *** www.homedepot.com.

[1] www. homedepot.com

Business Administration

THE CONTRIBUTION OF RURAL TOURISM TO THE SUSTAINABLE DEVELOPMENT OF THE RURAL AREAS

Associate Professor Anca Turtureanu, PhD
"Danubius" University from Galati

Abstract: *There are some elements related to the concept of rural tourism which has nowadays become very important around the world. The rural tourism can revitalize the conventional concepts and views on tourism, and bring in a new dimension to the sustainable development of tourism. It has been realized that tourism can play a major role in many countries economies, especially in developing ones, where it can substantially contribute to the increase of the national income. In this respect, mention should be made that Romania has a lot of resources to develop this tourism branch: villages with well conserved traditions, folklore, wildlife, natural heritage. All these natural elements put in value together with investments in infrastructure have determined an increasing demand for the Romanian rural destinations.*

Keywords*: sustainable development, rural tourism, sustainability*
JEL Classification*: Q56 - Environment and Development; Environment and Trade; Sustainability; Environmental*

1 Introduction

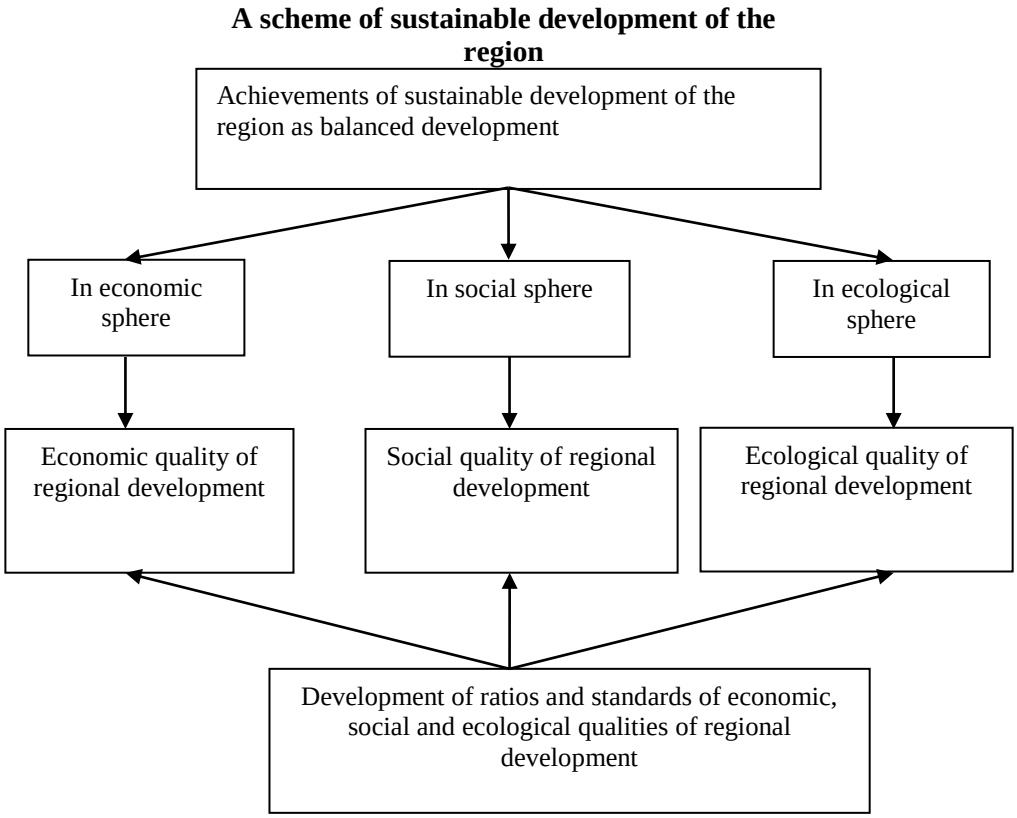
The World Tourism Organization (WTO) defined sustainable tourism development as "that which meets the needs of present tourist and host regions while protecting and enhancing opportunities for the future. It is envisaged as leading to the management of all resources in such a way that economic, social and aesthetic needs can be fulfilled while maintaining cultural integrity, essential for ecological process, biological diversity and life supporting system".

The economic quality of a regional development is ability of the region due to its own resources to produce such a gross income, which can provide high levels of consumption and accumulation in the region for a long time. The social quality of a regional development is ability of the region due its own demographic potential and social infrastructure to provide stable population in the region and to maintain high standards of life quality in the region during a long period of time. The ecological quality of a regional development is ability of the region to maintain its natural-resource potential and high qualities of environment during a long period of time. A sustainable development of the region is its development with high qualities of a regional development maintained for a long period of time (tens years). Selection of the main parameters - indices and calculation of criteria and limitations of sustainable development is the major phase in creation of the model of sustainable development of the region (Fig.1).

Thus tourism has to be human and adapted to the needs of the tourist, respond to the needs of the local communities, be socio-economic and culturally well planned and environmentally sound. The tourism must offer products that are operated in harmony with the local environment, community attitudes and culture so that they become permanent beneficiaries and not the victims of tourism. The basic cultural identity of these local people should not be adversely affected. Sustainability also ensures economically sustainable-development process in the efficient management of resources and such management to ensure that the resource supports the future as well as the present generation.

Thus sustainable tourism aims to:

- Improve the quality of life of people.
- Provide good experience to the tourists
- Maintain the quality of environment that is essential for both tourists and the local community.



Sursa: [P. Y. Baklanov, 2007, "Model of SD"]

Figure 1. A scheme of sustainable development of the region

2. Emerging dimensions

Tourism will expand greatly in future mainly due to the revolution that is taking place on both the demand and supply side. The changing population structure, improvement in living standard, more disposable income, fewer working hours and long leisure time, better educated people, ageing population and more curious youth in developing the countries, all will fuel the tourism industry growth.

To tap the immense opportunities, coordinated actives of all agencies involved in the development are required. A carefully planned and properly implemented development will definitely benefit the community economically and improve the quality of life in the villages. The success of such development depends upon the people's participation at grass root level for the development of tourist

facilities and for creating a tourist friendly atmosphere. Development of rural tourism is fast and trade in hotels and restaurants is growing rapidly. Increase in the share of earnings through rural tourism will no doubt; provide an attractive means of livelihood to the poor rural community. It increases the purchasing power at all levels of community and strengthens the rural economy. Development of infrastructure facilities such as rail, electricity, water, health and sanitation will definitely improve the quality of life.

As it is said above, rural areas represent the home of a quarter of the European population and more than 80 % of the European territory (Cork Declaration, 1996).

Rural tourism takes an important place at the European level and the European Community is doing a lot to develop it. Many publications were done about the issues of tourism in Europe. Even the WTO commission for Europe organized various conferences about rural tourism in Europe9 to present the trends and evolutions of rural tourism in western and Eastern Europe, and to establish a document on the European way to organize and develop rural tourism.

The WTO commission for Europe points out that rural tourism could be used as a tool for rural development since it could "deliver additional sources of income to rural communities"10, such as transport, services or health. It also could break the isolation of some communities and give cultural exchange opportunities. Rural tourism could also contribute to the preservation and improvement of environment since it depends on its attractiveness. This commission drew the economic and social costs that could create the development of rural tourism: increase demand for public services, increase in the prices of land and traditional buildings and even of goods; what would prevent local people from buying it. These statements show that, at the European level, authorities want to use tourism for developing rural areas but are also conscious of the negative effects to which this development could bring.

As rural development is a priority at the European level, EU (European Union) has adopted measures that support it. Rural tourism being considered as part of rural development, it can be funded through its instruments in the areas of information, training, marketing, integration of the tourist activity in the area. The European funds granted for rural development are:

- European Regional Development Fund to assist infrastructure development, promotional activities and training;
- European Social Fund to promote training;

- European Agriculture Guarantee and Guidance Fund to promote agricultural adjustment and diversification.

As explains the European Commission on its website 11, LEADER is one of the four initiatives financed by EU structural funds. Its role is to make rural actors aware of the long-term potential of their region. It encourages integrated strategies and innovative projects with the purpose of a high - quality and sustainable development. It gives a large place to partnerships and networks to exchange experiences. LEADER + is the third generation of LEADER programme. LEADER I in 1991 was a new approach of rural development policy territorially based, integrated and participative. LEADER II in 1994 put the emphasis on the innovative aspects of projects. LEADER + aims also, for the period 2000-2006, at the implementation of integrated development programmes for local rural areas, drawn and implemented by LAGs (local action groups) that represent the partnership between the private and public sectors and ensure the local adaptability of the LEADER + programme.

The priority themes of LEADER + are:

- making the best use of natural and cultural resources,
- improving quality of life,
- adding value to local products,
- the use of new know-how and new technologies.

Member States have also an important role to play through the principle of subsidiary, since they are responsible for the implementation of the LEADER initiative. They have to settle national and regional programmes and plans. They also have to evaluate each project funded by LEADER + making a mid-term evaluation by the end of 2003 and another one in 2006.

LEADER is considered by Jenkins et al. (1997) as "one of the most significant strategies for rural development". It is regarded as a 'bottom-up', an endogenous approach to problems of rural development.

The LEADER programme has been of significant use for rural tourism projects. Indeed, even if Kearney remarked in 1992 that "European tourism has long suffered from the benign neglect of governments which have still to recognize its economic importance in modern economies", an article from Info LEADER magazine explains that "tourism seems to offer a natural development path for rural areas in Europe, particularly in the most disadvantaged of them" since "in 71 of the 217 LEADER 1 areas, it is the dominant activity in the business plans of the local action groups".

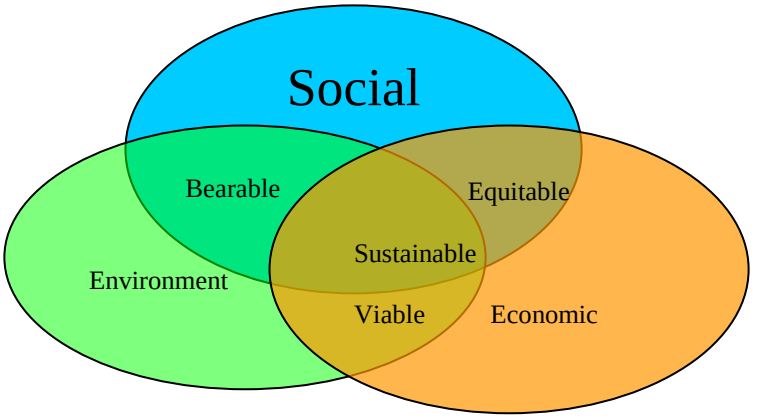
Indeed, about one third of LEADER I initiative was already turned towards tourism. This observation is reinforced by Airey (1983) that noticed that regional development was the most frequent justification for tourism policies. However, this statement shows how slow tourism development has been, especially in the rural areas of southern Europe.

The concept of sustainability means that mankind must live within the capacity of the environment that supports. Sustainable development has been defined briefly as "that which meets the needs of the present without compromising the ability of future generations to meet their own needs".

The definition brings out certain important aspects of sustainable tourism:

- Conservation and enhancement of resources for the future generation;
- Protection of biological diversity;
- Equity within and between generations;
- Integration of environmental, social and economic considerations.

The concept of sustainable development is all about conservation and stewardship of resources for the future. The support for ecologically sustainable development emerging strongly in the tourism sector, as it is the logical way of balancing environmental concerns with growth and development of the industry. Environmental problems facing the world today are of such magnitude that urgent actions have to be taken at the highest levels to counter this fatal degradation. But it is not just enough for government organizations to work towards sustainable development. Every individual, every neighborhood, and every community has to contribute in every away possible to get close to the goal.



Sursa: [UNESCO, 2001]

Figure 2: Scheme of sustainable development: at the confluence of three preoccupations

Many tourism professionals have already identified Romania as the country with the highest potential in Europe regarding the development of rural tourism as a major source of income, both for investors, as for the budget of the country. Nevertheless, and in spite of the fact that more than 18 years have passed from the December Revolution, rural tourism in Romania is still at its beginnings. And it doesn't seem to have an easy way, as Romanian authorities don't yet fully acknowledge the importance and the potential of this type of tourism. They still support major investments in classic mass tourism. But more and more antrepreneurial-spirited people begin to see the benefits and the potential of practicing rural tourism. First steps in this field have started several years ago, shyly, in some traditional places of the country (Prahova Valley, Braşov neighbourhood), and later in other areas. The present-day offer extends from types of accommodation to location.

But certain things able to add value to Romanian rural tourism are missing. First of all, the infrastructure (especially roads) to facilitate the access to the places most wanted by those tourists who want something different, not just sun and a patch of sand near other thousands of persons. An isolated mountain area guesthouse owner will never be able to access by himself a professional event in Vienna or in any other city of the world. And last but not least, the know-how, the science to offer tourists an experience that goes beyond good accommodation and rich meals.

Tourism represents a major factor of general economic progress; it contributes to increasing the Gross Domestic Product, to equilibrating the external balance of payments and to improving life quality.

The main reason for which visitors choose rural destinations is the desire of enjoying the quality of rural, but also human landscape. Maintaining the environmental quality represents an essential condition for that area to be attractive.

The offer of transportation means and other services should take into account the needs of the visitors and of local people, but also environmental policies. It is necessary to endorse the increase in the number of visitors who use public transportation means to get to their destination by establishing attractive itineraries and offering quality services and promotion.

An essential role is played by some technical town endowments that increase the degree of comfort and hygiene, without leading to aggressive urbanization of rural areas. Satisfying these requirements becomes a priority condition for increasing and maintaining a constant tourist circuit that can insure repayment and profit accumulation. Another important factor is transport

development and modernization, especially through the qualitative and quantitative increase in transportation means and telecommunication.

Lack of information:

Rural tourism-an activity which in the last years enjoyed strong development- is confronted with problems due to the fact that pension owners are from different activity sectors and don't have the necessary training to face the exigencies in current tourism. On the world tourist market and especially on the European one there is a phenomenon of strong segmentation of consumer demand. Perfecting promotion methods is essential even if now there are more diffuse and smaller tourist activities in the rural area. Capitalization of all present tourist resources creates the conditions for real broad products with a special way of presenting and selling which will attract the tourist.

Human resources:

Due to the recent changes in the Romanian economy, the role of human resources became extremely important. The current economic conditions and the unprecedented dynamic of change especially in the services sector led to the acute need of training and retraining force labor in this field.

- Lack of rural tourism laws: It is necessary to elaborate new norms of classification for the tourist pensions so that we can be aligned to the European standards;
- Stimulating small entrepreneurs in creating and maintaining the facilities (transportation means, pensions, gardens, forests, alleys, parks and others) through programs and governmental facilitation is another aspect that must be considered in drafting a development strategy of this kind of tourism and implicitly, of national export strategy.

A decisive factor in developing rural tourism is the lack of facilities such as: no taxation or reducing to half the taxes for pensions with a capacity of maximum 10 places for 5 years after they began their activity for pensions owners who have the possibility and want to develop ECO agricultural farms, in accord with the European standards and to sign them in the program

Limited access to the European funds due to the lack of information in some regions of the country and no organisms that offer services for potential investors who may create and implement projects (such as: mountain bike routes, nature trips, riding, fishing) that will develop tourism in that area and will offer more amusement to the tourist, stops the development of tourist packages that could be successfully sold on foreign market.

The existent tourist potential must be capitalized at maximum and the support to any innovative projects in the field can be successfully applied in Romania.

Promotion-an essential factor in rural tourism development

The creation of a tourist information office in Bucharest, especially meant for the foreign tourists is a necessary measure. The promotion methods must be designed according to the demands of the foreign tourist who is attracted by nature, by rural and eco tourism. Currently, an increasing demand for active holydays, for novelty, is being recorded. These sorts of tourist packages must be promoted on the foreign market.

Low quality of services and tourist information in many of the rural tourist areas is a factor which decisively affects the development of this type of tourism. The low professional standard of tourism personnel due to the absence of information in practicing and promoting rural tourism is the main cause of bad services from some areas, and this is because the pension owners from the rural area are from other activity sectors and don't have the necessary training to face the exigencies in current tourism.

ANTREC carries out in partnership with THR training courses for "Pension Administrator" and "Worker in rural tourism". Qualification in "Pension Administrator" trade is a necessary condition for obtaining the homologation and classification certificate. This conditioning was adopted by National Authority in Tourism at the request of ANTREC. The reason for this request was the lack of good services in the agro-tourism pensions where the owners were not acting in a professional manner. ANTREC also contributed to increasing the professional qualifications of those who practice rural tourism by practical training courses for its members in order to make them aware of European standards.

Qualification of human resources in this sector is a priority in the conditions in which the number of foreign tourists is increasing. To meet requests and exigencies of quality tourism, to make tourists come back, to attract new market segments, new sources of financing must be identified for regular such courses, and this because most of those who work in this sector are not professionally qualified for working in it.

The exigencies of good quality modern tourism reject unqualified personnel and the obsolete or arbitrary norms of the service provider. Because Romanian tourism must be aligned to European legislation and the quality of the services must

increase, it is normal to consider the human factor, which is decisive in insuring the quality of the services in tourism field.

Tourism development will open new perspectives for unemployed people and for reducing the seasonal character of jobs, through the job offers from tourism industry and through the opportunities given to the services performers by the increasing number of visitors. Tourism can become one of the key factors in the process of re-launching the economy, taking into account the fact that Romania has a huge tourist potential, adequate to the varied types of tourism at national and regional level. The only way to face competition in the tourist market is the continuous improvement of quality.

In 2004 the following trends manifested in the Romanian tourism development:

Tourism with foreigners had a positive evolution, 6,600 thousand entering in Romania, 18% more compared to 2003. Most of the visitors are coming from Europe (95.5%), EU total share being of 58.3%. Hungary, with 2,603 thousand persons was on the first place as no. of visitors (39.4%) followed by Moldavia with 1,212 thousand (18.45%), Bulgaria with 375 thousand, Germany with 296 thousand, Italy with 230 thousand, Serbia and Montenegro with 220 thousand, Turkey with 195 thousand, Poland with 132 thousand, France with 93 thousand, Austria with 90 thousand. 1,354 thousand foreign tourists were accommodated in hotels, 23% more than last year. The number of foreign tourists who benefited from hotel services represent only 20.5% of the total people who entered in Romania, the rest of approximately 80% didn't use these services or used services that are not statistically tracked, such as relatives, friends.

Tourism with Romanian tourists also increased, and through the tourist agencies were attracted in touring activities 1,649 thousand persons, 15.9% more than last year. Of them 1,429.2 thousand persons participated in domestic tourism and 215.7 thousand persons in foreign tourism activities, which means that from the total of 6,972 thousand Romanians who traveled abroad in 2004, only 3.1% used the services of tourist agencies.

The e number of Romanians who benefited from hotel services increased by 8.3% while spending the night increased with only 0.6% for this category of tourists. This shows the fact that Romanian tourists are oriented towards shorter vacations in the country and have an increased interest in the vacations abroad, with negative impact on the payment balance.

As a consequence of the increase of the tourist circulating, especially with foreigners, the usage of hotel capacities was superior to 2003, an increase of 3,7% for spending the night. From the total number of 18,500 thousand times just spending the night registered in 2004, 5,554 (30%) were in balneal places, 4,917 thousand (25.2%) in the capital and regional capitals, 4,337 thousand (23.4%) at the seaside, 2,060 thousand (11.1%) in the mountains 1,502 thousand (8.1%) in other places and tourist circuits, 128 thousand (0.6%) in the Danube Delta.

The economic and social context of Romania corroborated with the trends from the international tourism markets allow us to anticipate for 2005 an increasing dynamic of tourism in our country. An increase of 15% of foreigners visits are estimated, with favorable consequences over the increase with approximately 5% in the usage of hotels capacities.

Today's rural communities provide a stunning image by their contradictory dimensions and very mobile structure. They describe a system in which private property predominates and they often show "parallel economies".

Romanian rural communities represent a significant asset which proves itself useful from the perspective of touristic development.

In this context, research on effective use of rural tourism potential has been begun, at the level of Romanian rural communities, underlining the possibilities of integrated urban development in Dorna District, Suceava County, a region with a remarkable potential, but still underexploited.

It is very likely that in the near future, rural tourism and especially agricultural tourism will contribute to increasing the touristic offer, redistributing touristic rush, a superior revaluation of the resources and to attaching young people to their native areas. These activities can largely contribute to globally developing the Romanian villages, thus human communities will unite in associative forms, action groups, to solve problems like roads and communications networks, water supplying and sewerage, or environment protection.

Tourist hosting in villages has been practiced for a long time in most countries, in a more or less organized manner. But the present-day extension of this phenomenon is something new and it can be explained, on the one hand, by the relaunch of rural areas development, and on the other, by the variety of forms recently taken by mass tourism.

Inside local economy, rural tourism can be defined as a form of revaluation of rural spaces through exploiting natural assets, cultural and historic values and

traditions, agricultural products and by consecrated brand products of regional, ethnographic and cultural identity that could meet consumers' needs of welcoming, food, leisure activities, entertainment and various services.

Largely speaking, rural tourism includes a large variety of guest housing ways, activities, events, festivities, sports and entertainment, and all happen in a typically rural environment. It is a concept which covers touristic activity organized and led by rural local people and which generates from a tight contact with the natural and human environment.

Agricultural tourism can be defined as an additional activity, based on the excess of housing space available in rural homes, especially prepared for guests; it is made of a whole of goods and services provided by rural homes for the benefit of those persons who search rural environment for temporary relaxing, rest and leisure, or therapeutic cures, or business, or hobbies, to initiate in traditional peasants' crafts, for studies and research, as well as many other specific activities.

The farm is still a powerful symbol for urban dwellers. It is the farmer's home – that person who knows the nature's secrets, the best fishing or mushroom-gathering places around. It is the place where domestic animals are bred, with whom urban people lost contact; it is the place where fresh fruit can be eaten, the place where generations succeed, representing at the same time a specific place and lifestyle.

The village means something special for urban people: human dimension, local village life, town hall, local pub, school, the church, places that have been marking people's lives for centuries. Here live craftsmen, marketers, small investors, local actors who make village life easier. It also represents the cradle of the most beautiful feasts, wedding and christening customs, or those specific to wintertime.

The farm, the rural village and space, taken together or separately, represent the charm of rural tourism through attractiveness. Rural tourism must be understood as a form of activity that provides urban dwellers the most adequate conditions of therapy against stress, created by the uproar of everyday life. This form of tourism is strongly influenced by psychological factors and mainly addresses nature lovers, those who know how to use it for the benefit of their own health and mental comfort, without destroying it.

Globally, agricultural tourism has been imposing itself more and more. Considering the numerous problems in rural space and agriculture, European rural tourism has acquired major importance. European rural spaces need new

perspectives and viable alternatives, lest the social position of rural population degraded.

Rural tourism and agricultural tourism have developed differently in various countries. There are big qualitative differences particularly about equipment and services. Each country has its own strong points and potential that allows the development of rural spaces.

In Western and Central Europe, the most attractive and best-developed area, the Alps zone is representative from the point of view of rural tourism and agricultural tourism. Austria is considered the country for rural tourism. International experiences must be carefully selected and adapted to Romanian conditions. Western specialized rural tourism service providers are going to show their influence, and in a united Europe, Romania is about to become a more and more attractive and looked-for touristical destination. Ethnographic assets have a distinct role among rural resources, as each community owns a spiritual and material patrimony, as a result of its evolution and the geographical spreading of its people. That's why foreign people will assimilate this patrimony, as it appears as unique and original.

Usually, ethnographic assets are characterized by a permanent combination of attractive objectives and specific events. There is nowadays a paradox concerning ethnographic patrimony, as its lowest value can be found in most urbanized and industrialized countries, and its highest value in developing countries. Because genuine pop culture can be found but inside rural life, where traditions are preserved and enriched by the experience of its own creators.

Several major attractive rural events: trades and crafts; peasant clothes, dances and songs; traditional feasts; peasant architecture and technical equipment; human communities. Trades and crafts show a great regional diversity. The way rural people make their lives differ from a climatic type to another. They are so attractive because the way they are used is different, as well as the tools that are used, or the final result of human activities. Such trades and crafts are: plants' cultures, farm animals breeding, wood working, hunting and fishing, bee-breeding, gold and iron working, pottery, furrier's trade, spinning, weaving, etc.

Traditions are creative forms of rural spirituality, which consider various events in the community life as symbol status and oracle's practice. They are associated to the natural cycles of seasons, family or individual events.

Peasant clothes, songs and dances are very different from an area to another and from a nation to another. In Romania there are genuine treasures as far as these

are concerned, represented by clothes, peasant songs and dances. We wish to underline the absolute originality of Romanian folklore, its great variety and its exceptional preservation up to our times. Peasant clothes from Nasaud, Oas, Bucovina, Oltenia, Muntenia or Banat are unique landmarks for Romanian peasants' spirituality. Peasant musical instruments – the Panpipe, the shepherd's flute, the dulcimer, etc. have also their own specificity.

Architecture and traditional equipment confirm the genius of the anonymous artist, whose love for beauty and practical insight materialized in special buildings and production means. Peasant architecture has certain regional features: gates sculpted with solar or floral motives, or spiral-shaped in Maramures, Maramures, Salaj or Apuseni Mountains wood churches. The ways they are built, as well as the materials that were used, confer uniqueness. Traditional equipment: water mills, whirlpools, are in their turn very complex and varied.

Human communities are the essence of the above-mentioned elements, a communion of buildings and spirituality. The rural village is a self-sufficient whole, defined by its dwellers' creativity. Human habitats are attractive due to their values: age, structure, space placing etc.

The revaluation of the originality of the Romanian village and its geographical personality impacts the transformation of local activity, an objective reached on the following conditions:

- crafts are reactivated and services are developed - they should ensure a stable work market;
- alternative activities are stimulated or those that bring additional income (rural tourism, agricultural tourism);
- local economy is promoted and stimulated (processing food and non-food products);
- the organization of production structures and their processing through pattern guides;
- the organization of association forms: family micro-production farms, family associations, professional associations, etc;
- a local institutional structure is created and of "public - private" partnership type;
- legislation comprises the real problems of the rural space, including social protection.

The main targets that must be aimed to fully revaluate the rural potential are:

- the reconstruction of rural homes and of the whole agricultural policy;

- the stimulation of complementary non agricultural activities, especially through the creation of new economic units and through the turning to good account of the touristic and agricultural potential;
- the selection of certain towns and villages specific to Romanian ethnographic areas in order to gradually finance some projects of complex planning for rural tourism;
- research for local fitting up of rural space, especially as a project for practicing sports, cultural entertainment, touristic programs, in order to offer tourists as many attractions and activities as possible.

The revaluation of the assets of the Romanian villages can be done through various fairs, festivals, contests, meant to ameliorate the positive image of the village. Nevertheless, the most important way to turn to good account these assets is rural tourism - as it contributes to introducing certain natural conditions into the domestic and world touristic circuit and to revaluating traditional or modern culture of the rural space.

This large activity is based on three interdependent elements:

- the attraction towards natural beauty, ethnography, events happening in rural life;
- meals and accommodation that, even if they don't meet hotels' standards, must be qualitative and offered with hospitality;
- transports to rural environment are vital to provide a constant rush of tourists.

A cradle of old civilization, where history blends with legend, Bucovina is known for its famous painted monasteries - UNESCO patrimony: Moldovita, Sucevita, Voronet, Humor. As to the architectonic and historical aspects, the monasteries of Putna and Dragomirna are not of less importance.

This entire architectonic jewel is completed by an extraordinary natural surrounding. The Rarau Mountain, the valley of Sucevita and of Moldova with its tributary stream, Moldovita, the pass of Golden Bistrita river, the centenary old forests from Slatioara are only a few highlights of this area.

The popular architecture is really unique. The verandas, doors and window - frames, wells and gates are real wooden embroideries. In addition to this we can mention outside decorations as well, with geometrical or floral motives, stylised and coloured, found mostly in the villages along the Golden Bistrita valley, like Ciocanesti. The homespun, the embroideries, the decoration of Easter eggs with geometrical miniatures, the manufacturing of traditional costumes, are also

important part of Bucovine's unquestionable treasury. Guests are welcomed heartily in Bucovina, with a glass of wine or of traditional home-made "tzuica" (plum brandy), and the local gastronomy is a real adventure with temptations: the traditional "mamaliga" (polenta) with cheese, the smoked trout and cheese, and the "sarmale" (meat rolls in cabbage leaves) are all local specialties waiting to be tasted by visitors. The hosts are welcoming and eager to show you their day to day life and their traditions.

A good example for Romania is The Dorna District, known as "The Bucovina Gem", well-known for the natural assets it has been providing, for its quality products, ("La Dorna"milk and processed cheese, the "Dorna" mineral water), for its generous pastures which made possible the secularly tradition of animal-breeding, for its rich sources of mineral water. A better revaluation of the touristic potential of this area and a greater involvement of the local villages are necessary.

The Dorna District has a surface of 222,194 square km, that is 0,63% of the surface of the Oriental Carpathians and 0,093% of the surface of the country. Within this area, there are 10 counties with 49 villages and two towns: Brosteni and Vatra Dornei (Suceava county).

The Dorna District provides numerous assets which facilitate the development of local tourism: the mountains around with a lot of touristic itineraries, the rivers that, while crossing the mountains, form beautiful gorges (the Zugreni gorge on the Bistrita river), the resinous forests that spread fresh air, a lot of game as well as many monuments, historic objectives, and last but not least genuine traditions.

Such a potential, as well as the technical and material basis, have allowed several types of tourism types in the area, such as: mountain tourism, hunting, sportive tourism (ski, river-rafting), therapeutic tourism and rural tourism.

Touristic activities such as accommodation, serving, special food, medical cures, leisure and entertainment, etc. are a key to the social and economic development of the Dorna District. Thus, tourism, along with its cultural and recreative impact on tourists, creates new jobs, limiting people's migration to different zones. But economic impact is the most powerful.

In The Dorna District one can notice the growth of the living standard of those dwellers who practice rural tourism. As they had to meet certain rules imposed by the Worldwide Tourism Organization, people in the area had to ameliorate infrastructure and accommodation; moreover, they had to extend the

accommodation space in order to be able to practice tourism in the long term. Together with greater incomes and a higher living standard, tourism encourages the development of traditional peasant activities.

Although the Dorna District enjoys high potential, touristic population is mainly domestic, foreign tourists counting only about. 2%. This can be explained by the lack of an adequate material basis.

For a bigger number of foreign tourists, who would bring greater income to local people, the material basis needs to be ameliorated and intensely promoted, something that recently showed up, through various methods: flyers, illustrated touristic guides, web pages, TV etc.

Vatra Dornei spa is an area enjoying many investment opportunities that would lead to a faster economic growth. Thus, we recommend:

- the consolidation and the arrangement of the Spa Casino and its rendering to the touristic circuit;
- the building of the market and the planning of the neighbourhoods;
- a parking system;
- the improvement of the heating system of the spa;
- the arrangement and the ratifying of the ski and sledge tracks;
- the turning to good account of the Lunca Dornelor and Runc leisure areas;
- transforming the Dorna river into a touristic objective by building a modern water storage dam and equipping the lake with boats and water bikes;
- building a berry-processing plant;
- building a wood plant.

If these investments are made, the Dorna District will have more tourists and maybe it will get worldwide recognition. For the moment, it just follows its slow but sure evolution.

The Dorna District doesn’t allow cereal and technical crops, forage crops etc., in exchange here one can get the basis for the development of the zootechnic field; it is the best area to breed bovines (especially the ones for milk) and sheep. Among agricultural crops, potatoes are representative.

Secondary mountain lawns are composed of natural hayfields and pastures, occupying extended surfaces and being at the same time good food for cattle. It has

made possible ancestral pastoral economy, that represented a major factor of unification between local people and those in Transylvania and Moldavia.

The landed stock of the Dorna District sums 222194 ha; the major part is natural pastures (28894 ha), natural hayfields (22681 ha), and arable land represents only 2349 ha. The population of the area sums 50.730, of whom 34,8% live in the urban environment and 65,2% in the rural environment. In comparison to the national average (54,7% urban and 45,3% rural), this confirms the generally low economic level development.

Nowadays, the active population of the Dorna District sums 22.453 persons, (44,26% of the total population), of which 10.594 persons are employed (47,18% of the active population). 11.859 persons work in agriculture (52,18% of the active population). A survey of the denseness of the animal species of the Dorna District, that takes part to the creation of this marker by summing (50,90 UVM/100 ha arable land) shows that animal loading /ha. is sub-optimum (the optimum is, on the average, 1UVM/ha).

In the rural space one cannot speak about a regional development pattern but only about local patterns, where the territorial image of the investigated area is marked by the specificity of the agricultural structures which generate their own manners to deal with space, and with natural and social capital. The multiplication of the economic structures and especially the emergence and the encouragement of the specific structures of rural tourism lead to a bigger employment rate and reinserted the rural universe into different terms in the economic paradigm. This has cultural and communicational effects, as tourism is a dynamic element that can bring tradition closer to modernity.

Research shows that inside the rural space of the Dorna District, the factors favoring development are: various natural assets (mainly mineral resources of the subsoil, forest vegetation, productive agricultural land allowing the development of zootechnics, valuable elements of the natural environment); the human potential (the generous and cheap workforce, the youth, partially instructed in various agricultural and non-agricultural activities); the forest potential; natural parks; special landscape and cultural values (historic, cultural, architectural and etnographic); local experience in animal breeding, crafts, trades and rural tourism. The unfavourable factors to the development of the rural space are: the higher emigration; the stagnation of economic activities; the low-result agriculture; people's low incomes; the low-quality roads – most of the communal roads are not modernized and over 61% of the rural population has no direct access to the main roads and the railway network; the water supply is insufficient and inadequate; the education network is not diversified, schools run into inadequate buildings and are hardly equipped with

specific instruments; the degradation of the forests, mainly due to an uncontrolled deforestation.

Conclusions

Tourism industry may become one of the major segments of the Romanian economy. It generates employment and helps infrastructure development. Romania has a lot of resources to develop this economic branch, and especially the rural tourism: villages with well preserved traditions, folklore, wildlife, natural heritage. The rural tourism has specific resources of Vatra Dornei area from Bucovina county, namely:

- the village, as natural entity, playing an important role in Bucovina's image as touristic destination
- the natural attractions as mountains, forests, mineral waters, large low polluted areas
- the traditional workmanship, customs, folklore and the ecological products

All these elements may constitute a solid support for developing the rural tourism in this county, obviously accompanied by economic measures to encourage improvement of the road infrastructure, renovation of the old houses, churches and monasteries, building of new accommodation capacities with modern utilities, implementing of facilities for time-spending and various out-door activities like skiing, trekking, cycling, rafting, swimming.

Adding a good promotion, both within and outside the country, of this region by the help of the local and national organizations, as a safe, healthy and beautiful Romanian destination, the authors consider that the rural tourism may substantially contribute to the economic development of this Romanian region, and Bucovina and Vatra Dornei area may therefore become one of the a pearls of the European rural tourism destination, internationally recognized.

BIBLIOGRAPHY

1. Arnold, J., *Why rural tourism is no picnic*, BBC News, 2004;
2. Baldock, B., Beaufoy, G., 1993, *Nature Conservation and New Directions in the EC Common Agricultural Policy*, Institute for European Environmental Policy. 1993;
3. Fleischer, A., Tchetchik, A., *Does rural tourism benefit from agriculture?* Tourism Management, 2005;
4. Goosens, C., *Tourist information and pleasure motivation*, *Annals of Tourism Research*, vol. 27, no. 3, 2000;

5. Tchetchik, A., *Rural Tourism in Israel: Structure Analysis and Policy Implications*, Ph.D. Dissertation, The Hebrew University, Jerusalem., 2006;
6. Tribe, J., E*conomics of Leisure and Tourism*, Butterworth-Heinemann, London, 1999;
7. Tureac, C.E., *Management*,"Danubius" Academic Foundation Publishing-House, Galati, 2007
8. Turtureanu, A. G., *The Actual Management- from theory at mathematic tooling*, Galati Synthesis Publishing house, , 2005;
9. WTO, *Agenda 21 for the Travel and Tourism Industry: Towards Environmentally Suitainable Development*, 1995.

RESEARCH IN PUBLIC RELATIONS

Senior Lecturer Coriolan PĂUNESCU, PhD
"Danubius" University from Galati

Abstract: *Research has an important role in public relations (PR) being necessary in developing strategies in this area. Therefore, we can speak of two types of research, the applied research and theoretical research, both being successfully used in the work of public relations. Applied research, can be strategic (used in programs in order to identify attitudes and opinions of the target public, to develop strategies for formulating and transmitting messages, to establish the criteria for evaluating the work) and evaluative by which it determines the communication efficiency, in carrying out the program of public relations (in fact it is the comparison between the established and achieved objectives).*

Keywords: *public relations (PR), applied research, theoretical research*

Jel Classification: *I22 - Educational Finance*

Introduction

Research has an important role in **public relations (PR)** being necessary in developing strategies in this area. Therefore, we can speak of two types of research, the **applied research** and **theoretical research**, both being successfully used in the work of public relations.

Applied research can be: **strategic** (used in programs in order to identify attitudes and opinions of the target public, to develop strategies for formulating and transmitting messages, to establish the criteria for evaluating the work) and **evaluative** by which it determines the communication efficiency, in carrying out the program of public relations (in fact it is the comparison between the established and achieved objectives).

Theoretical research "*is abstract and conceptual. It comes in support of applied research results and it identifies new directions of investigation or*

summarizing new theories" (MILO, Katie and colaborators, *Introducere în relaţii publice*, Bucureşti, Editura NIM, 1998).

Research Applications in PR

The specialists identified five areas in which research can be used, namely: **monitoring** the environment in which an organization develops its activity in order to identify the changes in public opinion and skills (in this case, the research is reduced to recording some data that allow the anticipation of social current); the **general periodic checking** of the public relations activity (which determines how the internal public or the external organization perceives its image, the conclusions being used in making decisions about the company's policy and the organization of the public relations campaigns) the **communication checking**, namely the analysis of channels, of methods, materials and messages used in the communication process of an organization (basically, there are determined the differences between what is intended and what succeeds to transmit and to receive, in order to take action, in the cases of shrinking information flows of unequal charging of some technical components of some people in the organisation's communication system, detection of antagonistic activities, identification of contradictions or gaps between the concepts that are meant to describe **what is** and **what does** the organization **do**). After an analysis of the environment where the organization operates, the **social checking** provides clues on the level of support and what can managers expect from a community; **evaluative correcting** used both in planning and implementing the public relations programs (in other words, it measures the effectiveness of taken actions, that is testing for the public for which a program is designed before and after its development, and it compares the results).

Obtaining the information for projects

For the achievement of projects, that is some strategies for public relations, there is a need for information; this information can be obtained through **formal** and **informal** methodologies of research.

The first category, **formal research**, regards the history, the context of an issue and other general data that you can familiarize the agent with the case specific to which he works.

The sources of **informal investigation** may be specialized libraries (including personal library of the public relations agent or the company to which is engaged), personal files or archive of the client (the organization that helps an agent

to overcome the existing difficulties of communication) annual reports, publications, internal or external drive in question, recording the decisions taken and their operation.

Studying the management decisions, the impact that they have had on the public, both very necessary because they can obtain interesting conclusions for how could the crisis occur and the committed mistakes at the highest level. In other words, at this stage it is confirmed the origin of the appeared difficulties, or the idea that the deadlock occurred at a different level of organization, that is outside the leading department, which means that the reasons of the crisis are not of managerial nature which consequently must be sought in other areas of information and communication flow.

In order to obtain information, the **informal investigation** may appeal on **public libraries** (government documents, press excerpts about the nature of the occurred crisis, academic papers) so as the **virtual library**, accessible via global network of computers, known as **Database,** registered a big development because of the Internet. In this context, the electronic publishing, the new concepts of interactivity (**Wold Wild Web, e-mail, Home page**), entered into the vocabulary and practice of millions of people that use the computer in order to obtain all kinds of information, is a great and effective support to informal research, in designing and in many other areas.

Thus, the informal research (which is more expensive), the agent can obtain important data and conclusions specific to the studied case. Among the most used methods include: **surveys, interviews and analysis of content.**

The first surveys are used in social, political, economic, cultural sporting issues that aim for a wide audience, as spectrum and number. In fact, **surveys** determine the state of the public, its trends and its psychological motivation. The results of the **opinion polls** are mainly **descriptive** (in the sense that the status of fact, of the moment and of perspective) or **explicative** because it shows why a certain phenomenon took place (for example, the wide-spread union events at Bucharest and other social protests). It is understood that only the opinion polls fairly and professionally conducted, can provide conclusive results; therefore, the public relations need to master well the methodology survey. In this regard, note that in addition to formulating a correct questionnaire, selecting a representative pattern is very important in order to obtain the most accurate results. The pattern is chosen by two methods, which should be well mastered; they are **probabilistic method** and **non-probabilistic**.

In **probabilistic** method included four procedures:

- **Simple random choice** (which gives to all members of a public community equal chances to be selected);
- **Systemic Random choice** (in which the population is considered a randomly composed range), it is chosen randomly a prime topic and then, until the completion of the number of subjects; each person on the line is selected (which is suitable to some polls street);
- **Stratified choice** according to which the population is divided by age, gender, financial position, education, etc., after which it can be chosen a pattern from these categories;
- **Choice of subgroups** (by dividing the population into subgroups), and selecting a pattern.

Non-probabilistic method may have two proceedings: **choosing the unstructured, non systemic conjuncture**, in which the subjects are intentionally chosen to emphasise ideas, opinions, points of view, as it happens when journalists get interviews on street or in the large markets of towns, and **the intentional choice** that aims at selecting a category of subjects that have in common, a certain feature (a party affiliation, religion, age, etc.).

According to some specialists polls can be made directly, that is face to face, by telephone, by mail (by which there are questionnaires sent with the request to respond), only in limited groups. Some institutions may organize the polls with prizes in order to know the reflection of their work in the eyes of the public (tours, goods, money, etc.). Such practices distort the results; each subject is feeling indebted to accommodate the one who pays.

Interviews are methods that often offer to the public relations' agent the possibility of knowing from the source the depth of opinions and information, regarding the researched issue. The most effective interviews are conducted **face to face**, but also the interviews by phone can provide significant and efficient data, which, unfortunately, is more difficult due to suspicion of some (in particular, managers and politicians) that, at the on the other end of the phone, it might not be a journalist, but another person interested in a diversion or in an unofficial process of research.

Content analysis is used to examine a message or a group of messages, for example checking the functioning of a communication system within an organization. For example, a **newspaper article** can be analyzed, according to specialists in the field, by criteria such as: how many official statements were

necessary in order to release the article, the position and the pagination of the article, in what spreading area was the publication included, why were the announcements used (only for the informational content, or for the organization's position and purposes), how many changes were made before publication and what effect did the changes have on the message, what kind of references occurred in the organization (positive, neutral, negative).

The subject to a content analysis (which can be done after other criteria) may be an article from a newspaper, a speech, a television program, only if each (or all combined) have been included in a **public relations** strategy; if a problem occurs such as stopping communication, deterioration of the image, it must be established accordingly the causes of the phenomenon in question.

Finance and Banking

PUBLIC BUDGETARY POLICY ASSOCIATED WITH THE REQUIREMENTS OF THE EUROPEAN UNION INTEGRATION

Professor Dragomir Georgeta, PhD
„Danubius" University from Galati

Abstract: *In the complex process of accession to the European Union and the entry into the Euro Zone, Romania is bound to focus its efforts within finalizing the necessary reforms for fulfilling its commitments. Economic boost, low inflation, budget deficit remained within sustainable and stable exchange rates, all represent priorities and benchmarks of the European construction. In each state, budgetary policy is a result of the elaboration project of several categories of related budgets that make up a system. The budget system is variable depending on the organizational structure of each state: unitary type (France, England, Sweden etc.) and federal type (U.S., Canada, Switzerland, etc.). In Romania the need of resources at the level of society and their possibilities are reflected in the general consolidated budget. The law on Public Finances indicates that the management of public financial resources is carried out by a unified budget system.*

- **Keywords***: GPD, local budget, state budget, monetary policy, value added tax*

Jel Classification: *F - International Economics, F2 - International Factor Movements and International Business, F20 - General*

1. The budget in the context of the European Union enlargement

In the complex process of accession to the European Union and the entry into the Euro Zone, Romania is bound to focus its efforts within finalizing the necessary reforms for fulfilling its commitments. The economic boost, low inflation, budget deficit remained within sustainable and stable exchange rates, all represent

priorities and benchmarks of the European construction. In each state, budgetary policy is a result of the elaboration project of several categories of related budgets that make up a system.

The budget system is variable depending on the organizational structure of each state: unitary type (France, England, Sweden etc.) and federal type (U.S., Canada, Switzerland, etc.). In Romania the need of resources at the level of society and their possibilities are reflected in the general consolidated budget. The law on Public Finances[1] indicates that the management of public financial resources is carried out by a unified budget system that includes:

- The state budget
- The social security budget
- The local budgets
- The budgets of special funds
- The state treasury
- The budgets of other institutions

The general consolidated budget reflects the financial public flows of forming the fiscal and non-fiscal tax revenues and their distribution to destinations in accordance to the social need. The role of the budget is considered an instrument by which it is achieved the granting and the redistribution of resources and it also presents utility in the regulation of economic and social activity.

The state budget in the finance science can be addressed legally and economically. Under its legal appearance the budget is an act that is stipulated and approved the annual revenues and expenditures of the state. State budget provides for and authorizes, from the legislative point of view, the expenditures and state resources, being compulsory and submitted to the Parliament approval. It represents an act of foreseeing the public resources and its use; it is elaborated for a period of one year. The law of the state budget is influenced by political economic and social concepts, specific to each period.

Economic approach concerns the macroeconomic correlation and in particular the connexion between the level and the evolution of the gross domestic product. A certain level of GDP and the extent to which it is assigned to the gross capital formation and consumption, it determines the level of budget indicators and it represents the basis for favourable development of resources in the future.

[1] The Public Finance Law no. 500 of 11 July 2002 with subsequent amendments, published in the "Monitor Oficial no. 597 of 13 August 2002"

State budget expresses economic relations in cash which rises in the process of the distribution of the gross domestic product, according to the objectives of social, economic and financial policy of each period. These relationships have double meaning: on the one hand, the relationship that mobilizes resources and money, and on the other hand the relationships of distributing these resources.

The relations of resources mobilization are distribution relations of GDP in favour of the state through taxes, fees and contributions, the levies from incomes and transfers attracting the temporary free availability by the intern state loans. It also creates the relations state with foreign countries materialized in external financing.

The budgetary construction on medium term is projected in close connection with the combination of overall macroeconomic policy, that is the monetary policy and the revenue for:

- continuation of the disinflation process and its supplementary reducing;
- maintaining the current account deficit to sustainable limits;
- strengthening the budgetary revenue to cope with expenditures' commitments, that Romania will face in the coming years;
- preparing to insure the necessary conditions to the absorption capacity of structural instruments (structural and cohesion funds) since 2007.

The public policies integrated into a coherent and predictable multiannual scope will take into account the effectiveness of budgetary allocations, which will focus on:

- financing converged policies with those promoted by the European Union - infrastructure, research - development, environment and rural development;
- financing of some public policies of structural adjustment - transport, agriculture industry – for increasing the competitiveness of Romanian economy performance;
- continuing and consolidating the human capital policies - education, health - which will provide premises for achieving sustainable savings in labour domain.

In the domain of public investment it will be stressed upon:

- the development and modernization of transport infrastructure;
- environmental protection by developing the infrastructure in this sector, the investments focus on works of waste water treatment, organic waste storage of waste;

- developing drinking water programs and achieving programs against flooding;
- the investment in agriculture consisting of continuing the works on irrigation systems and improving the soil;
- programs of rehabilitation and consolidation schools and hospitals.

As an official document, the state budget outlines the approved level of expenditures that will make in the near future also the size and revenues that can be to the state.

2. **The evolution of budget's revenues and expenditures in Romania**

The revenues of the state budget for 2004 are of 288279.8 billion lei, while those in 2005 are of 357366.6 billion as follows:

Billion lei ROL

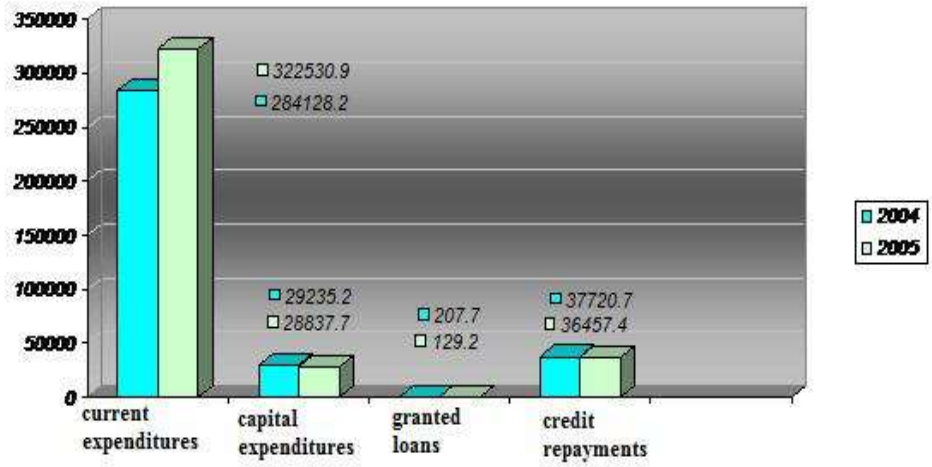
	2004	**2005**
Income – TOTAL	**288.279,8**	**357.366,6**
Out of which:		356.158,6
Current income	287.856,8	
Of which		
a) fiscal income s	269.306,8	336.119,6
- direct taxes	58.417,0	75.238,1
Of which:		
Tax on profit	49.210,0	60.710,0
Taxes on wages and income	64.921,0	76.887,0
Quota and deducted amounts on income tax for local budgets (they are subtracted)	- 62.424,0	- 69.718,9
other direct taxes	5.810,0	6.540,0
Contributions	900,0	820,0
Indirect taxes	210.889,8	260.881,5
Of which:		
Value added tax	110.150,8	126.692,5
Levied Value added tax	156.189,0	186.094,0
Deducted amounts from value added tax for local budgets (they are subtracted)	- 46.038,2	- 59.401,5
Excise	68.915,0	97.733,0
Custom duties	13.824,0	16.752,0
Other indirect taxes	18.000,0	19.704,0
b) Non-fiscal revenue	18.550,0	20.039,0
Income from capital	350,0	1.133,0
Recovers from repayment of granted	73,0	75,0

loans

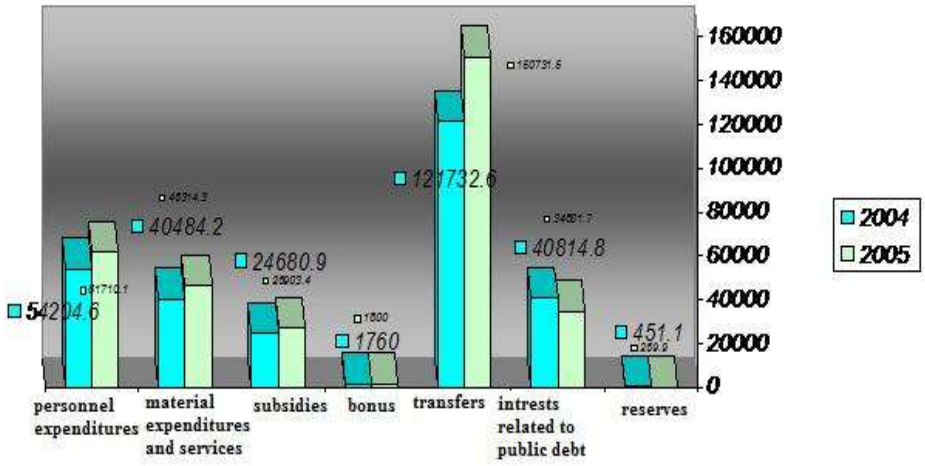
The expenditure of the state budget for 2004 is 351291.8 billion in economic structure and in 2005 is 387955.2 billion, as follows:

		Billion lei ROL
	2004	**2005**
Expenditures – TOTAL	351.291,8	387.955,2
Out of which:		
Current expenditures	284.128,2	322.530,9
Of which		
a) personnel expenditures	54.204,6	61.710,1
b) material expenditures and services	40.484,2	46.314,3
c) subsidies	24.680,9	26.903,4
d) bonus	1.760,0	1.800,0
e) transfers	121.732,6	150.731,5
f) interest related to public debt and other expenditures	40.814,8	34.801,7
g) reserves	451,1	269,9
2. Capital expenditures	29.235,2	28.837,7
3. Granted loans	207,7	129,2
4. Credit repayment, interest payment and credit commission	37.720,7	36.457,4
Of which		
a) Repayments of external credits and interest payments and commissions to external credits	37.400,2	36.135,5
- repayments of external credits	26.092,6	24.066,6
- payments of interest and commission	11.307,6	12.068,9
b) Repayments of internal credits and interest payments and commissions	320,5	321,9
- repayments of internal credits	101,7	101,7
- payments of interest and commission		

Total expenditures – Billions ROL



Current Expenditures- Billions ROL



3. The analysis of budgetary expenditures

A. The value of total expenditures at each of the two years [1]

Billions Lei ROL

	2004	**2005**
Current expenditures	284.128,2	322.530,9
Capital expenditures	29.235,2	28.837,7
Granted loans	207,7	129,2
Credit repayments, interest payments and commissions on credit	37.720,7	36.457,4
Total value on expenditures	**351.291,8**	**387.955,2**

A1. *The value of current expenditures at each of the two years:*

Billions Lei ROL –

	2004	**2005**
Personnel expenditures	= 54.204,6	61.710,1
Material expenditures and services	= 40.484,2	46.314,3
Subsidies	= 24.680,9	26.903,4
Bonus	= 1.760,0	1.800,0
Transfers	= 121.732,6	150.731,5
Interest related to public debt and other expenditures	= 40.814,8	34,801,7
Reserves	= 451,1	269,9
The value of current expenditures	**284.128,2**	**322.530,9**

B. *The share of each item of expenditure under the structure of the section A in total expenditures in each of the 2 years* [2].

The analysis of expenditure can be made on the basis of their structure on various categories of expenditure, being established the specific share and the

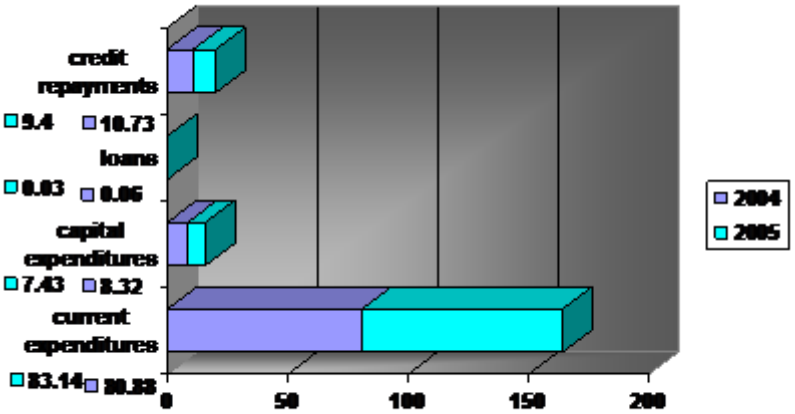
[1] It is determined by summing the current expenditures, granted loans and loan repayments.

[2] The share of each item of expenditure under the structure of the section A in total expenditures at each of the 2 years are calculated on the basis: share = (current expenditures / total expenditures) * 100

weight of each category of expenditure in total. With the help of shares it will be established the proportion in which the state resources were targeted at specific objectives, changing in dynamics the budgetary options, comparisons between countries with different levels of development.

Billion lei ROL

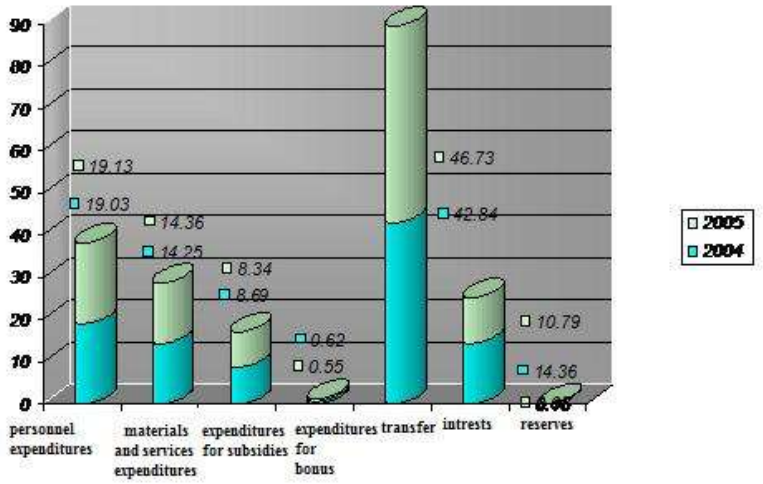
	2004	**2005**
The share of current expenditure in total expenditures	= 80.88%	83.14%
The share of granted loans in total expenditures	= 8.32%	7.43%
The share of capital expenditure in total expenditures	= 0.06%	0.03%
The share of credit repayments, interest payments and commissions on credit to total expenditures	= 10.73%	9.40%



B1- The share of each item of expenditures under the structure of section A1 in current expenditures at each of the 2 year.

Billion lei ROL

	2004	**2005**
The share of personnel expenditures in current expenditures	=19,08 %	19,13 %
The share of material expenditures and services	=14,25 %	14,36 %
The share of expenditures for subsidies in current expenditures	=8,69 %	8,34 %
The share of expenditures for bonus in current expenditures	=0,62 %	0,56 %
The share of transfers in current expenditures	=42,84 %	46,73 %
The share of interest related to public debt in current expenditures	=14,36 %	10,79 %
The share of reserves in current expenditures	=0,16 %	0,08 %



*C. Nominal increase in absolute **a** values*[1]:

Total expenditures	= 387.955,2 – 351.291,8	= 36.663,4
Current expenditures	= 322.530,9 – 284.128,2	= 38.402,7
Capital expenditures	= 28.837,7 – 29.235,2	= - 397,5
Granted loans	=129,2 – 207,7	= - 78,5
Credit repayments, interest payments and commissions on credit in total expenditures	= 36.457,4 – 37.720,7	**= - 1.263,3**

*C1- Nominal increase (year 2005 values minus year 2004 values) in **a** absolute values:*

Personnel expenditures	=61.710,1 – 54.204,6	=7.505,5
Material expenditures and services	=46.314,3 – 40.484,2	=5.830,1
Subsidies	=26.903,4 – 24.680,9	=222,5
Bonus	= 1.800,0 – 1.760,1	= 39,9
Transfers	=150.731,5 – 121.732,6	= 28.998,9
Interest related to public debt and other expenditures	=34,801,7 – 40.814,8	=-6.013,1
Reserves	=269,9 – 451,1	=- 181,2

*D. Nominal increase in **a** relative values*[2]*:*

Total expenditures	= (387.955,2 – 351.291,8)/351.291,8	= 10,44%
Current expenditures	= (322.530,9 – 284.128,2)/284.128,2	= 13,51%
Capital expenditures	= (28.837,7 – 29.235,2)/29.235,2	= - 1,35%

[1] The Nominal increase in absolute value represents current expenditure base expenditure, expressed in current prices and it is calculated based on the relationship between : expenditures in 2005 (current year) minus the expenditures of 2004 (base year).

[2] The nominal increase in relative value is calculated: (2005 values minus values 2004) / values 2004)*100

Granted loans	= (129,2 – 207,7)/207,7	= -37,79%
Credit repayments, interest payments and commissions on credit in total expenditures	= (36.457,4 – 37.720,7)/37.720,7	= -3,35%

D1 Nominal increase in relative value (2005 values minus values 2004) / values 2004):

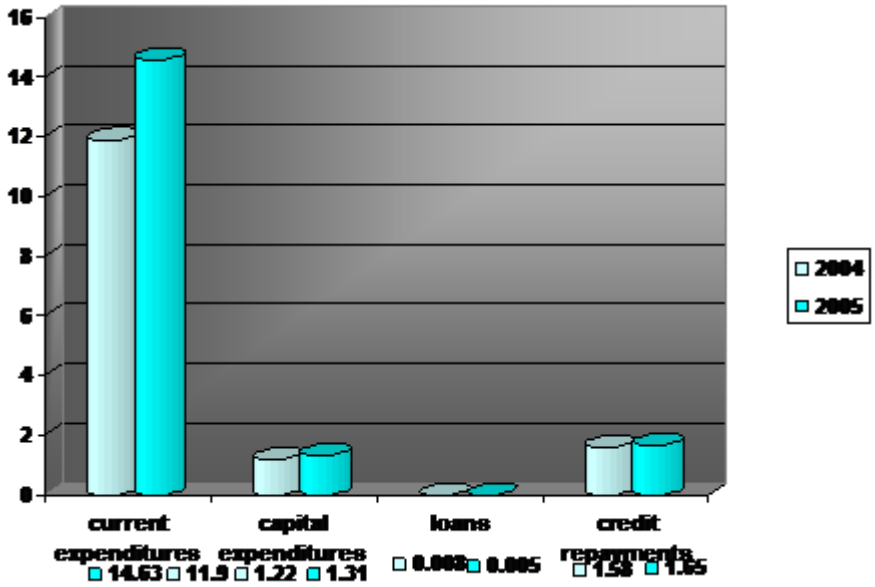
Personnel expenditures	=(61.710,1 – 54.204,6)/54.204,6	=13,85%
Material expenditures and services	=(46.314,3 – 40.484,2)/40.484,2	=14,40 %
Subsidies	=(26.903,4 – 24.680,9)/24.680,9	= 0,90 %
Bonus	=(1.800,0 – 1.760,1)/1.760,1	= 2,27 %
Transfers	=(150.731,5 – 121.732,6)/121.732,6	=23,82%
Interest related to public debt and other expenditures	=(34,801,7 – 40.814,8)/40.814,8	=14,73%
Reserves	=(269,9 – 451,10)/451,1	=40,17%

E. *The share of each item of expenditure under the structure of section Ain internal gross product*[1]ressed a characterization of the expenditures and their dynamics in a certain period of time. Gross domestic product for 2004 is 2,387,914 billion lei ROL, and for 2005 is 2,494,670 GBP billion Lei.

Billion lei ROL

	2004	**2005**
The share of current expenditures in GDP	= 11,90 %	12,93 %
The share of capital expenditures in GDP	= 1,22 %	1.16%
The share of granted loans in GDP	= 0008%	0005%
The share of credit repayments interest payments and credits commissions in GDP	= 1,58%	1,46%

[1] The relation of calculating the share of each expenditure item under the section A in GDP is: (expenditures/GDP)*100



E1. The share of each item of expenditure under the section

Billion lei ROL

	2004	**2005**
The share of personnel expenditures in GDP	2,27%	2,47%
The share of material expenditures and services in GDP	1,70%	1,86%
The share of expenditures for subsidies in current expenditures in GDP	1,03%	1,08%
The share of expenditures for bonus in current expenditures in GDP	0,07%	0,07%
The share transfers in current expenditures in GDP	5,09%	6,04%
The share of interests related to public debt in GDP	1,71%	1,40%
The share of reserves in GDP	0,02%	0,01%

F. The elasticity of expenditures compared to GDP

F.1. The elasticity of total expenditures compared to GDP

a) Total expenditures 2005 – total expenditures 2004 = 387.955,2 – 351.291,8 = 0.095
Total expenditures 2005 387.955,2

b) GDP 2005 – GDP 2004 = 2.494.670 – 2.387.914 = 0,043
GDP 2005 2.494.670

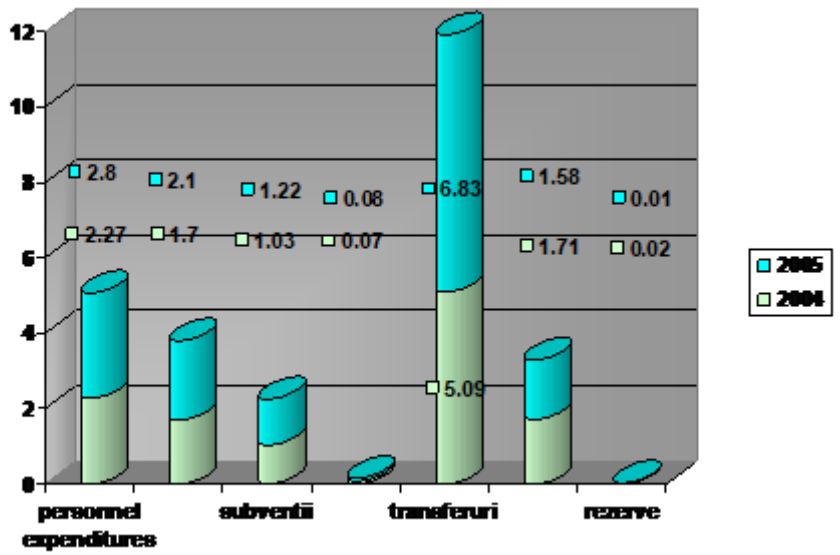
c) 0,095 / 0,043 = 2,209 ⟶ there is the tendency of using in a great extent the GDP for financing the total expenditures

F.2 The variability of the current expenditure compared to GDP

a) Current expenditures 2005 – Current expenditures 2004 = 322.530,9 – 284.128,2 = 0,119
Current expenditure 2005 322.530,9

b) GDP 2005 – GDP 2004 = 2.494.670 – 2.387.914 = 0,043
GDP 2005 2.494.670

d) 0,119 / 0,043 = 2,767 ⟶ there is the tendency of using in a great extent the GDP for financing the current expenditures



F.3 The variability of the capital expenditure compared to GDP

a) Capital expenditure 2005 – Capital expenditure 2004 = 28.837,7 – 29.235,2 =-0,014
Capital expenditure 2005 28.837,2

b) GDP 2005 – GDP 2004 = 2.494.670 – 2.387.914 = 0,043
GDP 2005 2.494.670

c) – 0,014 / 0,043 = - 0,326 ⟶ it expresses the tendency of restraining the proportion of capital expenditures in GDP

F.4. The variability of granted loans compared to GDP

a) Granted loans 2005 – granted loans 2004 = 129,2 – 207,7 = - 0,607
Granted loans 2005 129,2

b) $\frac{\text{GDP 2005} - \text{GDP 2004}}{\text{GDP 2005}} = \frac{2.494.670 - 2.387.914}{2.494.670} = 0,043$

c) – 0,607 / 0,043 = - 14,116 ⟶ it expresses the tendency of restraining the proportion of granted loans in GDP

F.5. The variability of credit repayments, interest payments, commissions to credits compared to GDP

a) $\frac{\text{Credit repayments 2005 - Credit repayments 2004}}{\text{Credit repayments 2005}} = \frac{36.457,4 - 37.400,2}{36.457,4} = -0,026$

b) $\frac{\text{GDP 2005} - \text{GDP 2004}}{\text{GDP 2005}} = \frac{2.494.670 - 2.387.914}{2.494.670} = 0,043$

c) – 0,026 / 0,043 = - 0,604 ⟶ it expresses the tendency of restraining the proportion of credit repayments, interest payments, commissions to credits in GDP

The correspondence between the growth of public spending and growth of gross domestic product [1]

(387.955,2/351.291,8) / (2.494.670 / 2.387.914) = 1,104 / 1,045 = 1,056 ⟶

it shows that gross domestic product has increased more slowly compared to expenditure, but at the same time, increased expenditure leads to an increase in GDP, depending on the value of the coefficient of the multiplication[2]. Obtaining such a result implies that fiscal pressure should not increase, because it diminishes the disposable income of private individuals who carry out the allocation between consumption and investment. So, expenditures exercise an effect maximum multiplier effect, when they are financed the budgetary deficit.

[1] Relationship of calculation is: (expenditure in 2005 / 2004 expenditure year) / (GDP year 2005 / GDP year 2004).

[2] The concept of a multiplier designates the effect of a exogenous variable (tax, unemployment, expenditure) on an endogenous variable (national income, GDP, disposable income, savings, investments)

4. The conclusions regarding the characteristics of expenditures for the analyzed period

a) Personnel expenditures are increasing in 2005 with 7505.5 billion ROL; this growth is determined mainly by:
   - Full ensuring funding necessary for the payment of salaries of budgetary staff as a result of salary increases for all categories of budgetary personnel starting with 1 January 2005;
   - Court resolution that grants the wage differences to the authorities;
   - Elimination of reduction by two percentage points of the social security contributions share.

b) Expenditures for materials and services are increasing comparing with 2005. The growth of these expenditures is influenced mostly by the:
   - Providing medicines and medical services (5075.7 billion lei);
   - To increase tariffs for the use of broadcasting stations and video and sound circuit to the Romanian Society for Broadcasting and Romanian Television Society (450.0 billion).

c) Interest payments on public debt in 2005 with a share of 1.58% of gross domestic product going less by 0.13 percentage points to the year 2004 .The decreasing tendency of the expenditures with its interest related to public debt will continue; there are factors that influence favourably:
   - The decline in interest rates on the interbank market with the consolidation of the disinflation process;
   - Using since 2000 the revenues from privatization and the recovery of non-performant banking assets for the redemption of state titles;
   - Extension of the maturity curve of financing and refinancing instruments of government internal debt which will reduce the monthly volume of refinanced titles;
   - Reducing granted loans.

d) The expenditures with loans diminish; this reduction represents, mainly, influences in reducing the exchange rate leu/euro comparing with the one planed for 2005 budget, and also a resizing them depending on the progress of external loan agreements. The most significant reductions are located: ensuring the compliance of the commitments made by Romania in the process of accession to the European Union, the implementation of new fiscal measures adopted recently, the correlation of the general consolidated budget deficit with other macroeconomic parameters

e) The subventions, the bonus and the transfers continue to hold the largest share of the gross domestic product, and 8.13% in 2005, going up by 1.94 percentage points comparing with 2004, due to social transfers. The increasing transfers are determined on the one hand, as **supplementing** for providing necessary funds:
   - recalculation of pensions (5426.3 billion lei);
   - full payments of farmers and military pensions, allowances and other entitlements for revolutionaries (5300.5 billion lei);
   - indexing allowances for children (514.2 billion lei);

On the other hand, as **reducing** by resizing the nature of these expenditures as a result of estimating the reduction of exchange rate compared to that used in elaborating the 2005 budget;

f) Capital expenditures for 2005 are 28837.7 billion lei diminished compared to 2004, with 397.5 billion lei, in circumstances where the capital expenditures is part of other expenditures of the general consolidated budget (transfers, loans, own revenues, external non-refundable funds).

5. Budget policy - debates[1]

The public expenditures are instruments used in the budgetary policy. They are targeted for implementing the decisions of the members of society. The used budgetary policy instruments are diverse. They serve to purchases of goods and services by institutions which implement state policy or subventions granting, perform social services, etc.; all have the mission of orienting, at the microeconomic level, those that benefit from budget appropriations adopted by budget laws, in order to provide the necessary and efficient public services, in accordance with the general interest of society. The efficiency and effectiveness of budget expenditures encompasses the modern methods of sizing expenditures as project and choosing the optimal variants.

The stress on quantity, quality and efficiency of public expenditures may be found also in the budgetary strategy, finalized in budgets – the multiannual program. They design and pursue the implementation (including the stipulations allocated

[1] The source: The press conference in November – December 2004 and 2005 presented by Calin Popescu Tăriceanu, Vladescu, Ionut Popescu.

annually in the budgetary law), the observing the sustainable development implementation of the vision in the short term, with the medium and long term.

Budgetary policy for 2004 was based on the addressability of the obligatory social expenditures, protecting the priority expenditures and reducing the non-essential expenditure, which will allow **improving the stability policy of expenditures and, consequently, the effectiveness of budgetary allocations.**

The objectives of budgetary policy aim the following:

- **maintaining the budget deficit at a reduced level**, funded with non-inflationist estimated at 3.0% of gross domestic product;
- **improving the prioritization of public expenditures and a reconsideration of social policy** by providing social protection based on measures focused on the level of the population categories which are most disadvantaged;
- **the continuous adjustment of budgetary expenditure to real possibilities of accumulating the budget revenues** and achieving a budgetary stability;
- continuation of the budgetary programming by substantiating budget and the allocation of public funds based on programs; the programs will be targeted in priority sectors of the economy and they will contain performance goals clearly defined, with a clear positive impact on economic growth and improving the living standards of the population.

Romania has registered a strong economic growth, a deepening of the current account deficit and a slowing of the process of disinflation. The fiscal pro-cyclical policy, combined with the development of policy wages of public sector has widened the existing macroeconomic imbalances. Real GDP growth was 8.3% in 2004, reducing to 4.9% in the first half of 2005 as a result of the negative impact of flooding and a slowing rate of growth of exports.

The inflation at the end of the year fell to 9.3% in 2004 on the rapid appreciation of the national currency, but stalled in 2005 due to the increase of administered prices, international prices of energy and real wage growth by more than 13%.

Wage increases recorded during the pre-election in 2004 and they have continued in 2005, while wage increases in the private sector were generally correlated with productivity.

The degree of employment with the legal age of employment has increased by 0.4% in 2004 and the rate of employment remained at 57.7%. Unemployment rate rose from 6.8% in 2003 to 7.1% in 2004, but during 2004 it was registered a gradual decrease, reaching in the first quarter of 2005 to 5.6%.

The mix the policy has become less prudent, especially in terms of reducing taxes and strong wage growth.

Total expenditures exceeded the original budget for 2004, particularly due to salary expenditures and other current expenditures, which were higher than those set by the budget. The pre-election expenditures were in the form of important increases in pensions and wages in the public sectors, which together with a considerable increase in the minimum wage have been added the wage costs, which were already very high. Real wages have increased by more than 10% in 2004 and 13.4% in first 8 months of 2005.

A major fiscal reform was implemented in January 2005, consisting of a significant reduction of profit tax from 25% to 16% and the introduction of unique quota of income tax of 16%. Improving the tax collection, broadening the base of taxation and restrictive expenditure policy aimed at offsetting the reduction in budgetary revenue as a result of these tax reductions. The reform can contribute to increasing private investment and employment in the formal sector in the medium term. However, it generates concerns by the tendency to be pro-cyclical, aggravating the existing imbalances, and the risks that cause a decline at the already reduced level of the revenue / GDP. The measures to increase revenue and reduce expenditures decided in May and June 2005 were aimed at limiting the adverse effects, but the prudence regarding the wage policy from the public sector and the expenditure policy that has not been applied as it had been announced. There was done a budgetary consolidation although limited in the context of strong economic growth, and the public finances are fragile in Romania in reducing growth. With the possible adverse effect of fiscal reform on the revenue budget, this represents a concern regarding the fiscal position of Romania's accession to the EU. The need for budgetary financing for supporting the process of real convergence and preparation for the accession to the EU stresses, also the need to increase the budgetary revenues, in particular by improving the collection, broadening the tax base or increase the indirect taxes, and the better prioritization of expenditures in order to ensure that Romania will be ready to become a EU member.

In its periodic report it was noted that **fiscal sustainability** should be strengthened by advancing the expenditure reform and further improving income tax compliance.

There has been recorded progress regarding the compliance in paying taxes, but the accumulation of new arrears was not stopped. You can not notice any progress in the reform of expenditure. As part of efforts to strengthen the budgetary framework, the public finance law has been applied to elaborate the multi-annual budgets, establishing limits on budget spending for budgetary agencies and the use of budget reserve funds. These efforts were completed by continuing the process of fiscal decentralization, which increased the fiscal autonomy of local authorities regarding the allocation of revenues and expenditures, but without the simultaneous strengthening of mechanisms for control of local public finances. Romania has not set up so far medium-term expenditure in order to support the reallocation of public spending to human capital, infrastructure and administrative capacity. The public investment increased to 3.5% of GDP in 2004, but the expenses investments were limited in 2005 in an attempt to counterbalance the negative impact on the budget, exercised by the increased wages in the public sector.

For the reduction of transfers from the EU, efforts have been made in improving the growth of EU funds absorption, including through the use of incentive pay for the retention and attraction of qualified staff. The education was a priority area for spending, additional resources being provided. Caused by a fragile public pension system from the financing point of view, which is affected also by the considerable increase of pensions in 2004 for a large number of pensioners, Romania has decided to continue with the implementation of the second pillar, while limited measures have been taken to counter the pressure of increasing costs of health system. To ensure both the consistency over time and that the public finances will contribute increasingly more to increase of growth potential of the economy; there was necessary a stronger link between the budgetary implementation and a global strategy for reallocating of budgetary expenditure, including a more active role for the multi-annual budgetary framework.

The pro-cyclical effect of the reform of taxation and of wage policy in the public sector is responsible for this and they are not in line with the call in 2004 for a mix of prudent policy. Priority should be given to restoring a prudent fiscal policy, in particular implementing additional measures to permanently strengthen the

collection basis and through a wage policy of the public sector in a prudent expenditure strategy, aiming at strengthening the potential of economic growth and preparations for accession to the European Union.

Selective Bibliography

1. Dragomir, Georgeta, *Finanţele – politici, tehnici, strategii*, Galaţi, Editura Fundaţiei Academice *Danubius*, 2001;
2. Dragomir, Georgeta, *Sistemul resurselor financiare publice*, Galaţi, Editura *Academica*, 2004;
3. Dragomir, Georgeta, *Finanţe publice,* Galaţi, Editura Fundaţiei Academice *„Danubius"* 2005;
4. Moşteanu, T, şi colectiv, *Buget şi Trezorerie publică*, Editura Universitară, 2003;
5. Ţâţu, L., Panaite, A., Dragotă, V., Stoian, A., Vasilescu, D., *Finanţe locale şi instituţii publice*, Bucureşti, Editura ANER, 2002;
6. Văcărel, I, şi colectiv*, Finanţe publice*, Bucureşti, Editura Didactică şi Pedagogcă, 2003;
7. ****Legea bugetului de stat pe anul 2004 nr. 507 din 28 noiembrie 2003*, published in Monitorul Oficial nr. 853 from 2 December 2003;
8. ****Legea bugetului de stat pe anul 2005 nr. 511 din 22 noiembrie 2004*, published in Monitorul Oficial nr. 1121 from 29 November 2004.

LE RISQUE DE TAUX D´ INTÉRÊT

Senior lecturer Rodica PRIPOAIE, PhD
„Danubius" University from Galaţi

Abstract: *La mondialisation des marchés de capitaux et les fluctuations des taux d´intérêt nécessitent la mise en place dans les entreprises et les banques d´une politique de gestion des taux, ainsi ont été mis au point des instruments financiers pour se couvrir contre le risque de taux d´intérêt. Les fluctuations des taux d´intérêt font courir des risques des entreprises; le risque de taux d´intérêt est le risque de perte réelle par une entreprise ou une banque sur des placements ou emprunts actuels ou futurs. Les instruments de couverture contre ces risques sont les contrats futurs et les options de taux. Les instruments de couverture contre ces risques sont les contrats futurs et les options de taux. Un marché à terme sur taux d´intérêt permet de couvrir le risque de taux d´intérêt, soit par opération d´arbitrage, soit par spéculation. Un contrat future ou à terme de taux d´intérêt représente un engagement de livrer par vendeur ou de prendre livraison par acheteur à une date future bien définie, des titres financières spécifiés pour une montant déterminé.*

Mots clé: *taux d´intérêt, le risque de taux d´ intérêt, les contrats futures, les option sur taux*

Jel - G32 - ***Financing Policy; Financial Risk and Risk Management; Capital and Ownership Structure***

Les fluctuations des taux d´intérêt font courir des risques des entreprises; le risque de taux d´intérêt est le risque de perte réelle par une entreprise ou une banque sur des placements ou emprunts actuels ou futurs. Les instruments de couverture contre ces risques sont les contrats futurs et les options de taux.

Le risque de taux se peut manifester au niveau du bilan ou du compte de résultats. *Le risque de bilan* est la conséquences d´une variation des taux qui dévalorise la valeur de certains actifs ou augmente la valeur des dettes. Le risque de taux en ce qui concerne le compte de résultats c´est *le risque d´exploitation* qui perturbe l´équilibre financière soit en augmentant les charges financière, soit en diminuant les produits financières.

Le risque de taux d´intérêt concerne : les créances et dettes actuelles, les créances et dettes futures, ou les créances et dettes conditionnelles. Une entreprise a une position longue de taux d´intérêt lorsque le risque de la position est celui de la hausse des taux d´intérêt. Une entreprise a une position courte de taux d´intérêt lorsque le risque de la position est celui de la baisse des taux d´intérêt.

Les instruments de couverture contre ces risques sont les contrats futurs et les options de taux. Un marché à terme sur taux d´intérêt permet de couvrir le risque de taux d´intérêt, soit par opération d´arbitrage, soit par spéculation. Un contrat future ou à terme de taux d´intérêt représente un engagement de livrer par vendeur ou de prendre livraison par acheteur à une date future bien définie, des titres financières spécifiés pour une montant déterminé.

Les contrats *futures* de taux d´intérêt sont :

- les contrats à court terme permettent aux entreprises de se couvrir contre des risques à court terme;
- les contrats à long terme permettent aux entreprises de se couvrir contre le risque de taux long.

Les prix des contrats *futures* de taux d´intérêt reflètent les anticipations des acheteurs et des vendeurs en ce qui concerne le taux d´intérêt. La base est la différence entre le taux à terme et le taux comptant. La base peut être positive ou négative pendant la durée du contrat; si la durée d´un placement ou d´un emprunt n´est pas égale à la durée du contrat, l´entrepris s´exposent à un risque de base.

Les opérations de couverture peut annuler le risque de taux d´intérêt; ainsi, le profit ou la perte réalisé en dénouant un future de taux par une opération en sens inverse permet de compenser la perte ou le profit résultant de la hausse ou de la baisse des taux d´intérêt de l´actif réel sur lequel porte le contrat.

L´entreprise qui veut se couvrir contre une hausse des taux vend des contrats pour un montant et une durée équivalente à la position qu´elle veut couvrir; ainsi, si la hausse des taux se produits, le gain qu´elle réalise en rachetant moins cher les contrats qu´elle avait vendus sert à compenser les pertes au comptant qui découlent de la hausse des taux. Lorsque le taux d´intérêt baisse, le cours d´un future de taux augmente et l´achat de contrats permet de couvrir le risque de baisse des taux.

Le risque de hausse de taux d´intérêt est celui auquel est confrontée l´ entreprise qui a contracté un emprunt à taux révisable ou l´entreprise qui doit emprunter des fonds dans le futur.

L´entreprise qui veut se couvrir contre une baisse des taux achète des contrats pour un montant et une durée équivalente à la position qu´elle veut couvrir; ainsi, si la baisse des taux se produits, le gain qu´elle réalise en vendant plus cher les contrats qu´elle avait achetés sert à compenser les pertes au comptant qui découlent de la baisse des taux.

Le risque de baisse de taux d´intérêt est celui auquel est confrontée l´ entreprise qui a déjà emprunté à taux fixe ou l´entreprise qui envisage de placer ses fonds dans le futur.

Les contrats *futures* pour le taux d´intérêt représentes des instruments efficaces pour diminuer les risques de taux, ainsi ces sont utilisés pour :

- arbitrer entre la marche au comptant et le marche à terme;
- diminuer les effets des variations de taux sur le bilan;
- diminuer les effets des fluctuations de taux sur les positions anticipées;
- spéculer en fonction de ses anticipations de taux.

Les options sur taux permettent de :

- se couvrir contre des mouvements adverses de taux, couvrir un portefeuille d´obligations, sans perdre le bénéfice d´une évolution favorable de taux d´intérêt;
- couvrir des opérations incertaines;
- améliorer la performance d´une portefeuille;

La mondialisation des marchés de capitaux et les fluctuations des taux d´intérêt nécessitent la mise en place dans les entreprises et les banques d´une politique de gestion des taux, ainsi ont été mis au point des instruments financiers pour se couvrir contre le risque de taux d´intérêt.

GENERAL CONSIDERATIONS ON THE INFLUENCE OF PRICES ON BUSINESS

Senior Assistant Cornelia Elena Tureac, PhD
"Danubius" University from Galati

Abstract: *In most companies, there is an ongoing conflict between managers in charge of covering costs (finance and accounting) and managers in charge of satisfying customers (marketing and sales). Accounting journals warn on prices that fail to cover full costs, while marketing journals argue that customer willingness to pay must be the sole driver of prices. The conflict between these views wastes company resources and leads to pricing decisions that are imperfect compromises. Profitable pricing involves an integration of costs and customer value. To achieve that integration, however, both need to let go of misleading ideas and to form a common vision of what profitability means.*

Keywords: *decision, pricing, cost.*

JEL Classification: *E3 - Prices, Business Fluctuations, and Cycles*

1. INTRODUCTION

The concept of economic value assumes not only that customers are aware of alternatives but that they can accurately evaluate what the alternative suppliers have to offer. In fact, it is often quite difficult to determine the true attributes of a product or service prior to purchase. For example, consumers suffering from a headache may be aware of many alternative pain relievers that are cheaper than their usual brand and that claim to be equally effective, but if they are unsure that a cheaper brand is as effective or as free of unwanted side effects as the one they usually buy, they will consider it an inferior substitute even though it could be chemically identical. Most customers will continue paying a higher price for the assurance that their regular brand offers what the substitutes do not: the confidence accumulated from past experience that their brand can do what the others only promise to do.

Even price itself can be difficult to compare across brands, thus reducing price sensitivity. Catalog and Internet retailers often divide their prices into two parts: one part for the items plus a fixed or variable charge for "shipping and handling". Research shows a wide variance among customers in their ability to make accurate comparisons with the single prices offered by traditional stores. Similarly, branded grocery products are often packaged in odd shapes and sizes, making price comparisons with cheaper brands difficult. When, however, stores offer unit pricing, grocery shoppers can identify the cheaper brands. In one study of unit pricing, the market shares of cheaper brands increased substantially after stores ranked brands by their unit prices.

Companies with new products for which they are trying to build cash flow often make the mistake of building the start-up cost of acquiring and servicing a new customer into a large, up-front fee. Because high uncertainty undermines perceived value, such companies lose potential sales and win sales only at lower prices than they otherwise could. By absorbing the up-front cost in higher monthly fees, the seller communicates confidence that customers will be satisfied and enables customers to pay as they enjoy a known value from product usage. Consequently, the seller should close more sales and, assuming that the product or service delivers the promised value so that the customer continues to buy it, the seller can ultimately expect a greater cash flow and a higher net present value (NPV) per customer acquired.

2. Switching Cost Effect

The greater the added cost (both monetary and non-monetary) switching suppliers, the less sensitive buyers are to the price of a product. The reason for this effect is that many products require that the buyer make product-specific investments to use them. If those investments do not need to be repeated when buying from the current supplier, but do when buying from a new supplier, that difference is a switching cost that limits interbrand price sensitivity.

This is the *switching cost effect: The* greater the product-specific investment that a buyer must make to switch suppliers, the less price sensitive that buyer is when choosing between alternatives. Since this effect is often attributed simply to consumer "inertia", it is easy to underestimate its predictability and manageability.

Aspiring suppliers often absorb part of the switching cost in order to eliminate this effect. They should not do this simply by offering a lower price; however, since then they must give the discount even to previous customers who are not incurring a switching cost. The key is to target the discount selectively to new customers without lowering the price expectation. New suppliers do this by

providing free training, by giving generous "trade-in allowances" to customers who replace competitive equipment, or by giving a discount on the first order placed under a long-term contract.

3. Price-Quality Effect

Generally, price represents nothing more than the money a buyer must give to a seller as part of a purchase agreement. For a few products, however, price means much more. Such products fall into three categories: image products, exclusive products, and products without any other cues to their relative quality. In these cases, price is more than just a burden; it is also a signal of the value a buyer can expect to receive. In such cases, price sensitivity is influenced by the *price-quality effect,* which states that buyers are less sensitive to a product's price to the extent that a higher price signals better quality.

Often, the perception of higher quality at higher prices reduces price sensitivity *even when* consumers seek *neither* prestige nor exclusivity. This *occurs* when potential buyers cannot ascertain the objective quality of a product before purchase *and* lack other cues, such as a known brand name, a country of origin, or a trusted endorsement to guide their decision for example, the name of a restaurant in a strange location, a folk artist at a fair, or a totally new brand with which the buyer has no prior experience. In such cases, consumers will rely somewhat on relative price as a cue to a product's relative quality, apparently assuming that the higher price is probably justified by corresponding higher value.

As an illustration of how strong this effect can be, researchers have reported cases where a new synthetic car wax faced strong consumer resistance until its price was raised. Similarly, sales of new creamy-style cheesecake were poor until the company raised the price to equal that of its heavy (and more costly to produce) regular-style cheesecake. Buyers could not judge the quality of either product before purchase. Consequently, buyers played it safe by avoiding cheap products that they believed were more likely to be inferior.

Extreme cases such as these, where sales respond positively to a higher price, are admittedly rare. They lead one to expect, however, that in other cases sales simply respond less negatively to a higher price than they would if buyers did not associate a higher price with higher expected quality. Numerous studies have shown that, even when the objective quality of a brand is unaffected by its price, consumers use price as a quality cue to the degree that:

1. they believe qualities differ among brands within the product class.
2. they perceive that low quality imposes the risk of a large loss.

3. they lack other information (such as a known brand name) enabling them to evaluate quality before purchase.

The more consumers must rely on price to judge quality, the fewer prices sensitive they will be. For most purchase decisions, consumers can either examine a product before purchase or infer its quality from past experience with the brand (the difficult comparison effect). Studies indicate that under these conditions, price is not used as a quality cue. Nevertheless, the conditions for using price as a quality cue occur in one very important case: when new products are first offered to a market.

4. Expenditure Effect

A buyer's willingness to evaluate alternatives depends also on how large the expenditure is relative to the effort necessary to reduce it. For businesses, this effect is determined by the absolute size of the expenditure; for households, it is determined by the size of the expenditure relative to the available income. The *expenditure effect* states that buyers are more prices sensitive when the expenditure is larger, either in dollar terms or as a percentage of household income. The more a buyer spends, the greater the gain from carefully evaluating the expenditure and attempting to find a better deal. This explains why the same person will sometimes shop at an expensive convenience store (for a small purchase) but be very sensitive to price when deciding where to go for the weekly shopping excursion. This partially explains why heating insulation costs much more when sold to maintenance men in lots of twenty-five feet than when sold to building contractors by truckloads of tens of thousands of feet. At the other extreme, small "impulse purchases" are simply not worth any effort to ensure that the price is a good deal. Consequently, percentage price differences across suppliers are often very large.

The effect of the expenditure size on price sensitivity is confounded in consumer markets by the effect of income. A family with five children may spend substantially more on food than a smaller family, yet still be less price sensitive if the cost of food accounts for a smaller portion of the large family's higher income. This relationship between a buyer's price sensitivity and the percentage of income devoted to the product results from the trade-off buyers must make between conserving their limited income and conserving the limited time they have to shop. Higher-income buyers can afford a wider variety of goods but cannot always afford more time to shop for them. Consequently, they cannot afford to shop as carefully as lower-income buyers, and so they accept higher prices as a substitute for time spent shopping.

The expenditure size relative to income is also a constraint on both a business's and a household's primary demand for a product. A young man may long

for a sports car, believing that a Porsche clearly has differentiating attributes that justify its premium price relative to similar cars. An economic value estimation of sports cars would reveal his decided preference and belief that the Porsche offers a "good value" relative to other sports cars. At his low income, however, he is not making purchase decisions among competing sports cars. Expenditures in other purchase categories (housing, food, and education) are of higher importance than a sports car, and those categories currently consume his income. Until his income rises, or the price of sports cars becomes much less, his brand preference within the category is not relevant.

5. End-Benefit Effect

An individual purchase is often one of many that a buyer makes to achieve a single benefit. Cream cheese is one of several products that a cook must buy to make a cheesecake. Software is just one component of a computer system, the cost of which may be minor compared to the cost of processor, modem, data storage, etc. The relationship of a purchase to a larger benefit is the basis of the *end benefit effect*, which can be divided into two parts: the derived demand and the price proportion. *Derived demand* is the relationship between a desired end benefit and the buyer's price sensitivity for one of the products that contributes toward achieving that end benefit. The more sensitive buyers are to the cost of the end benefit, the more sensitive they will be to the price of products that contribute to that end benefit. In the examples above, the more price sensitive the buyer is about the decision to make a cheesecake or build a computer system, the more price sensitive she will be to the cost of cream cheese or disk storage devices. *Price proportion cost* refers to the percent of the total cost of the end benefit accounted for by the product's price. The smaller the proportionate share accounted for, the less sensitive the customer will be to price differences.

Derived demand is most obvious in business markets. The more (less) price sensitive is the demand for a company's own product, the more (less) price sensitive that company will be when purchasing supplies. A manufacturer of office furniture purchases sheet steel to make desks. The more desks it can sell the more steel it will buy. If desk buyers were highly price sensitive, any attempt to pass on steel price increases to the price of desks would cause a large reduction in sales. Consequently, the high price sensitivity of desk buyers would force the desk manufacturer to be highly sensitive to the cost of its desks and, therefore, to the price of steel.

Imagine how the manufacturer's purchase behavior would change, however, if booming demand were to cause an order backlog to lengthen and customers to lose leverage in negotiating desk prices. Since the manufacturer could now more easily pass on added costs to the customer, its goal in purchasing would become less

to save money on supplies and more to ensure on-time and defect-free deliveries to keep the manufacturing process running smoothly. It is essential for salespeople in business markets to understand the end benefit that drives a customer's purchase decision (is it cost minimization, maximum output, quality improvement, civic mindedness) in order to infer the importance of price in the purchase decision.

The relationship between price sensitivity for a product and for the end benefit to which it contributes is not simply an economic phenomenon. There is a strong psychological component that depends on how a buyer perceives the absolute price, or price difference, in proportion to the total cost of the end benefit.

To fully appreciate the marketing implications of the end-benefit effect, managers need to recognize that it is both an economic and a psychological phenomenon. Consider how you would react if, after celebrating a very special occasion at a nice restaurant, your beloved paid for it with a two-for-one discount coupon. Unless you are an economist, this action would probably be seen as rather unromantic. Most people think it tacky to make choices based on price when an end benefit is emotionally important to them. Moreover, one must also recognize that the "total cost" of the end benefit need not be only monetary. Dieters are less sensitive to price than non-dieters when treating themselves to chocolates or ice cream because the dollar expenditure is only a small part of the total cost (both monetary and non-monetary) that they pay for this treat. The psychological aspects of this effect make it an excellent target for promotional activity. Once a brand is established in customers' minds as somehow "better", advertisers can increase the value of that perceived difference by relating it to end benefits to which the customer already attaches a high value.

6. Shared-cost Effect

Although the portion of the benefit accounted for by the product's price is an important determinant of price sensitivity, so also is the portion of that price actually paid by the buyer. People purchase many products that are actually paid for in whole or in part by someone else. Insurance covers a share of the buyer's cost of a doctor's visit or a prescription drug. Tax deductions cover a share of the cost of publications, educational seminars, and travel related to one's profession. Businesses usually compensate employee travelers for all or part of their travel and entertainment expenses.

Fairness Effect The concept of a "fair price" has bedeviled marketers for centuries. In the Dark Ages, merchants were put to death for exceeding public norms regarding the "just price". In the more recent dark history of Communism, those who "profiteered" by charging more than the official prices those very prices at

which the state was unable to meet demand-were regarded as criminals. Even in modern market economies, "price gougers" are often criticized in the press, hassled by regulators, and boycotted by the public. Consequently, it is well worth a marketer's time to understand and attempt to manage this phenomenon.

Buyers are more sensitive to a product's price when it is outside the range that they perceive as "fair" or "reasonable" given the purchase context. But what is fair? Managers should note that the concept of fairness appears to be totally unrelated to issues of supply and demand. It is related to perceptions of the seller's profit, but not entirely. Oil companies have often been accused of gouging, even when their profits are below average. In contrast, popular forms of entertainment (for example, Disney World, state lotteries) are very profitable and expensive, yet their pricing escapes widespread criticism. Recent research seems to indicate that perceptions of fairness are more subjective, and therefore more manageable, than one might otherwise have thought. Buyers apparently begin by making an inference about the seller's likely margin relative to what they expect the seller earned in the past, or relative to what others earn in similar purchase contexts. The effect of margin on fairness is strongly mitigated, however, by another factor: the inferred motive of the seller. Explaining the action with a "good" motive makes the price more acceptable than a "bad" motive. Finally, the research indicates that companies with good reputations are much more likely to be given the benefit of the doubt that their pricing decisions have good underlying motives, while those with unpopular reputations are likely to find their motives suspect.

7. The Framing Effect

The preceding discussion about prices and price increases being more objectionable for "necessities" follows from a stream of research called *prospect theory*, which has many important implications for managing price sensitivity. The essential idea of prospect theory is that people "frame" purchase decisions in their minds as a bundle of gains and losses. Moreover, how they frame those decisions affects how attractive they perceive a choice to be. The *framing effect* states that buyers are more price sensitive when they perceive the price as a "loss" rather than as a forgone "gain," and that they are more price sensitive when the price is paid separately rather than as part of a bundle.

Many marketing implications of prospect theory have been suggested that seem consistent with both common observation and controlled research:

- To make prices less objectionable, make them opportunity costs (gains forgone) rather than out-of-pocket costs. Banks often waive fees for checking accounts in return for maintaining a minimum balance. Even when the interest forgone on the funds in the account exceeds the charge for checking, most people choose the minimum balance option.
- When your product is priced differently to different customers and at different times, set the list price at the highest level and give most people discounts. This type of pricing is so common that we take it for granted. Colleges, for example, charge only a small portion of customers the list price and give everyone else discounts (a.k.a. scholarships). To those who pay at or near the full price, the failure to receive more of a discount (a gain forgone) is much less objectionable than if they were asked to pay a premium because they are not star students, athletes, or good negotiators.
- Unbundle gains, bundle losses. Many companies sell offerings that consist of many individual products and services. For example, a printing company not only prints brochures but helps design the job, matches colors, schedules the job to meet the buyer's time requirements, etc. To maximize the perceived value, the seller should identify each of these as a separate product and identify the value of each one separately (unbundle the gains). However, rather than asking the buyer to make individual expenditure decisions, the seller should identify the customer's needs and offer a package price to meet them (bundle the loss). If the buyer objects to the price, the seller can take away a service, which will then make the service feel like a stand-alone "loss" that will be hard to give up.

Anyone who thinks only in terms of objective economic values will consider these principles far-fetched. One might argue that buyers in these cases could easily think of the same choices as entirely different combinations of "gains" and "losses". That is precisely the point that prospect theorists make. There are many different ways to frame the same transactions, and each way implies somewhat different behavior. Researchers have presented research subjects with many objectively identical choices, changing only the framing of the presentation. They have found that changing how people think about the choice in terms of "gains" and "losses" consistently and predictably changes the choices they make.

BIBLIOGRAPHY

1. Tureac, C. E., *Management*,"Danubius" Academic Foundation Publishing-House, Galati, 2007;
2. Turtureanu, A., Ioan, A. C., *Current Management, From Theory to Mathematical Holding*, Galati, Editura Sinteze, 2005.

Miscellaneous

COMBINED DEEP AND SHALLOW KNOWLEDGE IN A UNIFIED MODEL FOR DIAGNOSIS BY ABDUCTION

Professor Viorel Ariton, PhD
Reasearch Assistant Eng. Florin Postolache, PhD in progress
„Danubius"University from Galaţi

Abstract*: Fault Diagnosis in real systems usually involves human expert's shallow knowledge (as pattern causes-effects) but also deep knowledge (as structural / functional modularization and models on behavior). The paper proposes a unified approach on diagnosis by abduction based on plausibility and relevance criteria multiple applied, in a connectionist implementation. Then, it focuses elicitation of deep knowledge on target conductive flow systems – most encountered in industry and not only, in the aim of fault diagnosis. Finally, the paper gives hints on design and building of diagnosis system by abduction, embedding deep and shallow knowledge (according to case) and performing hierarchical fault isolation, along with a case study on a hydraulic installation in a rolling mill plant.*

Keywords: *Faulty Diagnosis,* abduction, *plausibility criteria, relevant criterion*

Jel Classification*: Other Special Topics*

1. INTRODUCTION

Real systems are so complex that someone's efforts on detailed modeling fail. So, diagnosis (in technical, medical or economical domains) performed by human diagnosticians, often relies on incomplete, imprecise and uncertain knowledge. Human experts think in terms of *discrete* pieces: events, modules, causes and effects - all as separate knowledge pieces. Human concepts are also

qualitative – regarding relations between causes and effects. Designers and practitioners cope with complexity of real systems by means of physical, functional and behavioral units.

Diagnostic problem solving is abductive problem solving; human diagnostician's way involves shallow knowledge – regarding associations between causes and effects from practice, and deep knowledge – regarding causal links from laws in the domain.

The paper proposes a unified model for diagnosis by abduction with straight forward connectionist implementation, able to embed deep and shallow knowledge of human experts on the target system's faulty behavior, again computational issues included. The study that follows integrates concepts from means-end and bond-graphs modeling, in the effort to embed deep and shallow knowledge in a diagnosis system based on abduction.

2 UNIFIED MODEL FOR DIAGNOSIS BY ABDUCTION

Abduction means finding causes as explanation of effects observed in the target system. This chapter proposes a unified model for diagnosis by abduction, based on *plausibility* of causes from effects and *relevance* of causes. Plausibility embeds shallow and deep knowledge on cause-effects relations, relevance embeds deep knowledge on causes, related to physical and functional structures and to behavioral aspects of the target system.

2.1 Characteristics of abductive problem solving

Abductive reasoning in fault diagnosis considers the *cause* as single or multiple fault explaining *effects* appeared and observed by instance manifestations. Diagnosis in real systems faces a huge number of causes, due to various sources (equipment, environment human operator) and to various combinations of faults. On the other hand, the effects-to-faults links are complicated, while effects may enter, for example, conjunction or disjunction grouping when evoking faults, also interaction between causes when provoking some effects. [5] proposes four categories of abduction problems:

- *independent* abduction problems - no interaction exists between causes;
- *monotonic* abduction problems - an effect appears if cumulative causes appear;
- *incompatibility* abduction problems – pair of causes are mutually exclusive;

- *cancellation* abduction problems – pair of causes cancel some effect, otherwise explained separately.

[4] has a sound approach on abductive problem solving based on neural networks adapted to abductions problems above. They introduced a fifth category:

- open abduction problems - when observations consist of three sets: present, absent and unknown observations.

Human diagnostician usually master target systems structure and behavior complexity dealing with discrete pieces of knowledge: modules and components on physical structure, then process ends and component roles on functional structure. Regarding diagnosis, he or she employs other discrete pieces – faults and manifestations, which have truth values attached and refer to physical and functional units in a qualitative manner.

Various links between effects and causes (as reversed causal relation) commonly get a connectionist computational model, suited to abduction. Diagnosis applications meant for real complex systems exploits the great number of effects-to-faults patterns, obtained from human diagnostician's practice or from experiments, and embeds that shallow knowledge by training artificial neural networks. Deep knowledge – on causes and effects as in abduction problems above, may enter various dedicated processing (as in [4]).

2.2 Abductive problems solving by plausibility and relevance

Direct relations between effects and causes represent *plausibility criteria* [5]. From the set of all plausible causes only a subset represent actual causes, usually obtained through a parsimonious principle. [6] considers the minimum cardinality as a *relevance criterion* and applies it to the set of plausible faults to obtain the diagnostic subset.

2.2.1 Cause isolation by relevance

Plausibility criteria detects causes (e.g. faults), while relevance criteria isolate them. The paper extends the concept of relevance and makes it effective in Fault Detection and Isolation (FDI).

Relevance assumes some grouping of causes followed by selection of most plausible item from the group (in [1] called *relevance group*). For example, all faults occurring at a physical component form a group, only one likely to be the cause of

effects appeared. Following the minimum cardinality principle over the structure, if one fault is relevant there is a single fault diagnosis and if certain number of faults there is performed multiple fault diagnosis.

The concept of relevance is useful when fault diagnosis relies on expert's deep knowledge, when he or she applies different grouping criteria to faults according to deep knowledge in the domain. Hence, relevance is effective not only regarding the minimum cardinality principle over the structure but also regarding some phenomena happening in the target system and domain. For example, while relevance criterion over structure states "a component is unlikely to have more than one fault at a time", in conductive flow systems another relevance criterion may apply "leakage is unlikely to be caused by more than one fault at a time". Relevance involves first grouping causes, then selecting the most relevant by some processing – for example sorting causes by plausibility.

2.2.2. Plausibility and relevance in a connectionist approach

As a general idea, abductive problem solving proceeds by multiple applying the two functions:

- *Plausibility* (*P_CRITERIA, EFFECTS*) which output is the set of all plausible *CAUSES*, activated from instance *EFFECTS* according to plausibility criteria *P_CRITERIA*;
- *Relevance* (*R_CRITERIA, CAUSES*) which output is a subset of *CAUSES* from the set of the plausible ones, in groups and relevance criteria according to *R_CRITERIA.*

Various *P_CRITERIA* and *R_CRITERIA* may apply sequentially to effects and causes until a final set of *CAUSES* have truth values of highest level achievable. If cardinality of the final set of *CAUSES* is 1 then one deals with single fault diagnosis, else with multiple fault diagnosis.

In a computational model using Artificial Neural Networks (ANN) plausibility criteria get implemented in forward excitatory links from *EFFECTS* to *CAUSES* and relevance criteria get implemented in competing links between *CAUSES*. In ANN implementation of diagnosis, both effects and faults get logical truth values, while in the incomplete and imprecise environment they may get following meanings: effects "almost" appeared, and causes "possibly" occurred. Links between effects and causes enforce or reduce causes' truth values, toward the diagnostic – that is the set of most plausible and relevant causes.

However, ANN architecture must be adapted to comply with general types of abduction problems above, also to conjunction / disjunction grouping of effects to causes. In this respect, human diagnostician way of acting is again helpful, while plausibility and relevance get certain logical meanings from his or her point of view, as shown below.

2.2.3. Characteristics of plausibility and relevance

When activating causes form actual effects plausibility criteria should exhibit *qualitative* and *logical* features, for example when activating causes even their effects are not certain (i.e. as long as effects truth value grows, the cause truth value grows), or when cause activation depends on conjunction of some effects. Relevance criteria should exhibit *quantitative* features, while causes have to be compared to select the relevant one. In the computational model for abductive problem solving:

- plausible causes result from qualitative or logical processing that activate all causes from given set of effects;
- relevant causes result from quantitative processing that selects causes from the plausible set if exhibit a given certainty degree (greater than the threshold value).

While computational model deals with numbers, the two criteria should handle them adequately: numbers involved in plausibility criteria should suffer "*logical overload*" to allow conjunction / disjunction of effects to causes (and between causes) and numbers involved in relevance criteria assess the *degree* causes may belong to the diagnostic set.

The "logical overload" of numbers is a meaning attached to a range of values, similar to fuzzy truth values attached to elements in fuzzy subsets. Cardinality of partition, over the universe of discourse of a numerical variable *V*, may take the values: 2 – if processing refers to classical logical approach (truth values 0 and 1), 3 or more – if processing refers to Lukasiewicz or to Zadeh logic, depending on horizontal (α-cuts) or vertical (continuous) representation of the fuzzy subsets.

An example of logical overload of numbers is the following: if the input of a fault-neuron from a manifestation-neuron is greater than 0.5 (doubt threshold) then the link is declared as "important" and enters the fault neuron (added to the other inputs), else it is "not important" hence blocked (set to 0).

2.3. Connectionist model of abduction by plausibility and relevance

In the presented approach, the ANN architecture for abductive problem solving is not a particular one; the only restrictions that apply are: the two layers *EFFECTS* and *CAUSES* are neighbour causes (because of possible conjunctions of effects to a fault – see §2.3.1). Plausibility criteria are forward links between *EFFECTS* and *CAUSES*, relevance criteria form various grouping of *CAUSES* then provoke competitions inside the relevance group. ANN architecture as Adaline, Perceptron or Counterpropagation, etc. are suited to implement the presented approach on abduction.

2.3.1 Neural models of plausibility

Let consider a cause C_i as a neuron that observes general equation for neuron activation by forward excitatory link from the layer of effects E_j (see Figure 1. a):

$$C_i = f(\Sigma\, w_{ji} \cdot E_j + \theta_i) \qquad (1)$$

If both cause and effects get truth values, i.e. $C_i \in [0,1]$ and effects $E_j \in [0,1]$, then a link with weight w enforces the cause truth value at some effects. Cause neuron truth value C_i indicates how plausible is that cause in the context of actual effects values E_j. However, the above equation should also comply to plausibility criteria where effects enter a conjunction first, then attack the neuron's input.

In the presented approach, an input of cause-neuron get "logical overload" to allow logical processing (e.g. conjunction) required by plausibility criteria. After the training phase the weights w get certain values and the an actual input at cause neuron C_i in recall phase will be $I_{ij} = w_{ij} \cdot E_j$. If the effect is not certain (E_j<0.5) then input is $I_{ij} \le w_{ij}/2$, hence:

if $I_{ij} > w_{ij}/2$ *then* I_{ij} = "important" *else* I_{ij} = "not important" (2)

It is now possible to perform logical aggregation on effects and causes. Neural model of plausibility is the *site* that performs the aggregation of input effects as follows (see Figure 1):

- *disjunctive aggregation* – performed by default through cumulative processing of effects E at case-neuron input I:

$$I_i = \Sigma\, w_{ij} \cdot E_j. \quad (3)$$

- *conjunctive aggregation* – performed by the "*conjunction site*", see Figure 1. a, and the truth table; output *O* of the site observes the rule:

$$if\ I_1 > w_1/2\ AND\ I_2 > w_2/2\ then\ O = I_1 + I_2\ \ else\ O = 0 \quad (4)$$

- *negation* – performed by the "*negation site*", see Figure 1. b, and the truth table; output *O* of the site observes the rule:

$$O = w_1 - I_1 \quad (5)$$

The original architecture of ANN is changed by the sites added to cause-neurons that require logical aggregation.

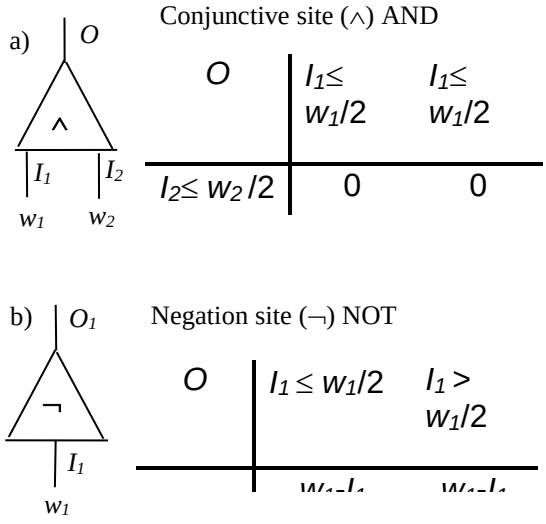


Figure 1. *Neural sites for logical aggregation of effects to causes.*

Note that added sites do not disturb or change the original running of the initial ANN, while they do not change either the training procedure or the values *w* of weights. For example, if two effects enter a conjunction aggregation, the input pattern for training such situation presents the two inputs with truth values greater than doubt value (0.5), while that pattern comply the real situation (both input effects are important); at recall phase it worth to activate the fault only if both actual effects are important.

2.3.2 Neural models for abduction problems

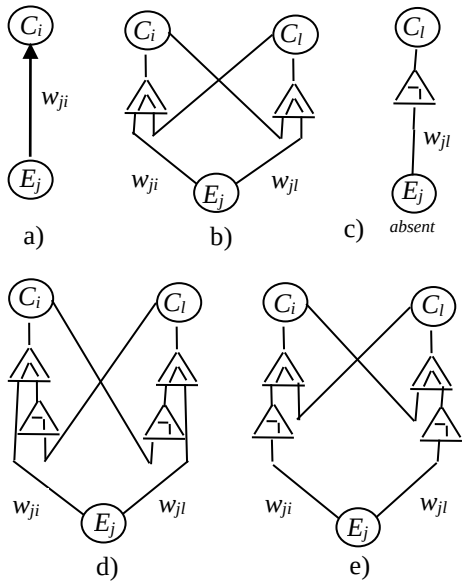


Figure 2. *Abduction problem solving using neural network models for plausibility criteria*

Neural (sites) models for the five abduction problems in the literature are depicted in Figure 2. and solve each category from §2.1 as follows:

a) For *independent* abduction problems – excitatory links apply directly from effect E_j to corresponding cause C_i (see Figure 2. a. If there exist also conjunction grouping of effects to the cause, conjunction site(s) get "mounted" and entering the default disjunctive grouping to neuron input.

b) For *monotonic* abduction problems – causes C_i and C_l evoking both the same effect E_j, suffer conjunction with one-another and with the common effect through conjunction sites as in Figure 2. b:

$$(C_i \leftarrow C_l \; AND \; E_j) \; AND \; (C_l \leftarrow C_i \; AND \; E_j) \qquad (6)$$

c) For *incompatibility* abduction problems – the pair C_i and C_l of causes are mutually exclusive, i.e. one is active if the other one is not, both evoking the same effect E_j. The pair of causes suffers conjunction with negation of another one conjunction with the common effect as in Figure 2. d:

$$(C_i \leftarrow NOT\ C_l\ AND\ E_j)\ AND\ (C_l \leftarrow NOT\ C_i\ AND\ E_j) \qquad (7)$$

d) For *cancellation* abduction problems – the pair C_i and C_l of causes are mutually exclusive, i.e. one is active if the other one is not, both evoking the same effect E_j. The pair of causes suffers conjunction with negation of another one conjunction with the common effect as in Figure 2. e:

$$(C_i \leftarrow NOT\ C_l\ AND\ E_j)\ AND\ (C_l \leftarrow NOT\ C_i\ AND\ E_j) \qquad (8)$$

e) For *open* abduction problems – the only problem is dealing with absent effects: cause C_i is activated if no effect E_j *exists*, see Figure 2. c:

$$C_i \leftarrow NOT\ E_j \qquad (9)$$

Original ANN architecture for abductive problem solving is changed adding sites specific to each abduction problem, adequate to causes and effects in concern. However, similar to final note at §2.3.1, the ANN running is not changed – regarding the training procedure and values of weights obtained.

2.3.3 Neural models of relevance

A relevance criterion usually observes minimal cardinality of *CAUSES* over criterion's specific relevance group. In general, relevance involves three stage processing:

i) Consider all plausible causes belonging to relevance group.
ii) Start competition between causes inside relevance group.
iii) Select cause(s) for diagnostic set, observing an ordinal property of causes and some selection threshold.

Neural model of relevance is competition between causes. Computationally, it may consist from sorting all causes in the relevance group, then selecting the one(s) with higher degree according to a maximum number (e.g. 1 if single fault diagnosis), or a "relevance value" (e.g. minimum activation of causes – if they exceed the doubt value 0.5). For example, if the ordinal property for sorting is plausibility of causes (truth values of *CAUSES*), then the sorting procedure is applied to all causes in the relevance group - not only to plausible ones, while those not plausible have the lowest degree. So, competition proceeds always over the entire set of *CAUSES* in the relevance group.

3 DEEP AND SHALLOW KNOWLEDGE IN DIAGNOSIS

Knowledge elicitation is a very important phase in diagnosis system design, while it involves information on various causes and effects, on physical structure and on normal and faulty behavior of the target system in real life. Any approach on diagnosis depends on how knowledge covers spaces of causes, effects and their relations; otherwise, one gets open spaces and incomplete knowledge leads to inaccurate diagnosis. When the target system is a *conductive flow system* (CFS) diagnosis is more difficult due to propagated effects throughout the system.

Few works refer to methodical procedures to guide knowledge elicitation, and fewer to generic models suited to control and guide knowledge covering for diagnosis purposes. [3] proposes knowledge pieces suited to cover faulty behavior of CFSs based on means-end modeling approach and bond graphs, and [2] presents a CAKE (Computer Aided Knowledge Elicitation) tool for methodical covering of structural and behavioral complexity of a target CFS.

Present chapter stresses main directions to extract deep knowledge on structure and behavior of conductive flow systems which perform simultaneously multiple functions – further denominated *Multifunctional Conductive Flow Systems* (MCFSs), and the ways such knowledge is represented and become plausibility and relevance criteria for diagnosis by abduction.

3.1 Abstraction levels for structure and behavior

It is commonly accepted that discrete pieces in physical and functional structure of a real target system is only an abstraction that requires also models for continuous behavior; the entire model obtained is a *hybrid dynamic model* (as discussed in [7]). In this view, deep knowledge on the target MCFS refers to:

- physical and functional units, from means-end modeling perspective – as Discrete Event System abstraction;
- bond graph components and junctions, from bond graph modeling perspective – as Continuous System abstraction required to assess abnormal behavior of structural units.

For CFSs bond graphs represent powerful modeling means, as they not only capture essential ideas from Kirchkoff''s laws but, additionally, offer a proper modularization of the target system's model, in a general conceptualization.

3.1.1 Physical and functional structure

From means-end point of view the *module* is a network of *components*, and the entire target *MCFS* is a network of modules. Modules accomplish specific *ends* during specific *activities* through components *flow functions* as in [8]. Each module may accomplish more ends, provided one end attained during one activity; each components may have more functions but only one during one activity of the superset module.

From bond graph point of view, modules correspond to bond graph junctions. [3] proposes three generic flow functions that correspond to bond graph primitive components, so reducing them to a meaningful subset for diagnosis purposes:

- flow transport function (*ftf*) – *R* component; when faulty, directly affects propagation of power flow along paths in the target CFS;
- flow storing function (*fsf*) – *C* and *I* components; when faulty, directly affect time delays in the running process;
- flow processing function (*fpf*) – *TR* and *GY* components; when faulty, directly affect the ends of modules.

3.1.2 Faulty behavior structure

Fault is a physical non-conformity occurred at component level, opposed to designed specifications from producer. Fault's name often suggests a disorder or a physical damage so, it reflects knowledge incompleteness about component structure. The set of all "known" faults should be decided at elicitation phase; some of them indicate a specific damage, some – a class of damages.

Manifestation is a piece of knowledge assessing values of an observed variable at component, during a certain activity of the superset module. Manifestation is a linguistic variable with truth values for normal (*no*) or "too low" (*lo*), "too high" (*hi*) linguistic values. Some manifestations arrive by sensors (from continuous or binary variables), some by human operators tests (from human senses – as adjectives, or from test points – as numbers) on observed variables in the process. Manifestations may refer to primary effects or to secondary effects.

Anomaly or symptom is a piece of knowledge obtained from a set of manifestation by some processing, and deposits deep knowledge in the domain, so helpful in diagnosis (see below).

3.1.3 Generic anomalies in the faulty behavior

To each generic flow function a generic anomaly is attached:

i) *Process anomaly* (AnoP) – means deviation from the normal value (e.g. "too high" or "too low") of an end-variable; it refers to transformations the flow undergoes.
ii) *Transport anomaly* (AnoT) - means changes on flow variables or on inner structure of component, relative to flow transport along flow paths.
iii) *Store anomaly* (AnoS) – refers to deviation from the normal value for the delay specific to storing (capacitor-like) or inertial (inductance-like) component (see §2.3.).

Note that only transport anomalies refer to propagated effects, while process and store anomalies are located at component showing corresponding flow function *fpf* or *fsf* as above. If there is a definite set of transport anomalies located at faulty component, then they get meanings of primary effects.

[3] presents signatures with manifestations at effort and flow (bond graph) variables in 1-junction and 0-junction, specific to transport anomaly occurred in the junction.

3.1.4 Orthogonal transport anomalies

It works on fault diagnosis deal with concepts as "leakage" or "obstruction". [3] defines a set of four orthogonal transport-anomalies for bond graph components, as follows:

a) *Obstruction* – change of resistance parameter (increase), without flow path modification, e.g. clogged pipe.
b) *Tunneling* – change of resistance parameter (decrease), without flow path modification, e.g. broken-through pipe.
c) *Leakage* – structure change (balance too low on flow), involving flow path modification, e.g. whole in pipe.
d) *Infiltration* – structure change (balance too high on flow), involving flow path modification, e.g. flow injection.

Transport anomalies are orthogonal in pairs (obstruction to tunneling and leakage to infiltration), each pair orthogonal to the other. A fault causes a unique transport anomaly that appears at respective component and, by default, at module it

belongs. Thus, transport anomaly is a primary effect located at module level, hence isolating it means isolating the faulty module.

Each type of transport anomaly has a specific signature – regarding deviations for bond graph junctions.

3.2 Guidelines on knowledge embedding in plausibility and relevance criteria

The main problem raised on diagnosis by abduction in the proposed approach is deep and shallow knowledge elicitation and embedding in the neural network for diagnosis.

During elicitation phase, knowledge engineer discriminates:

- *physical structure* – i.e. modules and components;
- *functional structure* – i.e. activities for modules and flow functions for components, bond graph junctions for interconnected modules and bond graph components with specific parameters for corresponding flow functions;
- *behavioral structure* – i.e. faults, manifestations and flow anomalies (processing, store, transport).

Note that components result from hierarchical decomposition of physical structure according to the accepted granularity of fault isolation, that is location units for faults may also have structure.

Plausibility criteria embed shallow knowledge as patterns of non-propagated manifestations-to-faults (e.g. color, position) and anomalies-to-faults. Deep knowledge refers to conjunction and abduction problems related to manifestations and certain faults.

Relevance criteria involve modularization of faults according to deep knowledge on physical and functional structure and on anomalies they provoke (in the given structural unit).

It worth stressing that shallow knowledge for plausibility is obtained for each module separately. So, practical survey rather experiments on real complex systems seem realistic (in technical and economical domains), while they are much easier performed and less combinatorial burden occur than for the entire system.

3.3. Abduction procedure for diagnosis

All discrete concepts resulted from elicitation phase should enter in ANN structure for diagnosis by abduction. So, all units from behavioral structure become neurons: manifestations on input layer, faults on output layer and anomalies on an intermediate level (activated by manifestations and attacking faults). All behavioral units attached to a module belong to a separate neural network (ANN). Links between neurons get weights by training procedure (from shallow knowledge) and sites from deep knowledge, all according to plausibility criteria stated by human diagnostician at elicitation phase.

All units from physical and functional structures become relevance groups related to relevance criteria at elicitation phase.

For proper diagnosis, each component (as final location in fault isolation) have attached the "normal" *CAUSE*, beside all faults at component in concern. So, to the set F_0, F_1,… F_{n-1} of neurons indicating faults, it is added the F_n neuron – assessing the truth value of normal running. It is important to exist a F_n neuron because *NORMAL* situation enters relevance competition with *FAULTY* situation. So, before finding the cause when faulty situation occurred, diagnosis system should asses if the target system is *FAULTY* (i.e. it performs fault detection).

To asses *FAULTY* situation a relevance criterion is applied over all decisions F_0 to F_{n-1} and F_n as follows:

$$\textit{if} \quad \exists F_i > 0.5 \ (\mathrm{i} = 1..\mathrm{n}-1) \ \wedge \ \sum_{i=0}^{n-1} F_i > n \cdot F_n \quad \textit{then FAULTY} \quad (10)$$

in words: if any of activated faults have truth values greater than the "doubt value" and the relative level of the *NORMAL* situation is greater than all current (activated) faults, then the *FAULTY* situation is credited.

Diagnosis is performed in hierarchic and sequential manner, detecting transport anomaly at module, then isolating fault(s) by abduction through multiple plausibility and relevance criteria:

1) *faulty module isolation* – by plausibility and relevance of transport anomalies possibly occurred based on signatures in junctions of the system's bond graph model (see [2]);

2..n-1) *fault isolation* – proceed by sequential application of a given sets of plausibility and relevance criteria, specific to module detected in stage 1;

n) *diagnostic* – fault(s) obtained after assessing faulty situation versus normal situation at module, by relevance as in (10).

Because modules of target MCFS simultaneously accomplish ends (independent from one another), combinations of activities raise to a huge number. In the hierarchic way proposed, diagnosis relies only on shallow knowledge and deep knowledge at module level, then on groups of modules in bond graph junctions.

4 CASE STUDY ON A HYDRAULIC INSTALLATION

Fault diagnosis was meant for a simple hydraulic installation in a rolling mill plant (see Figure 3), comprising three modules: Supply Unit (pump, tank and pressure valve), Hydraulic Brake (control valve, brake cylinder) and Conveyor (control valve, self, the conveyor cylinder). For the 20 faults to 8 components considered, manifestations come from sensors as *lo*, *no*, *hi* values (2 flow-rate, 4 pressure, 5 temperature), 8 binary values (cylinders at left/right ends and open/shut valves) also 10 linguistic manifestations from operator observed variables (for noise and oil-mud). Software architecture exhibit 6 ANN perceptron blocks – 2 per module.

The three modules – corresponding to Hydraulic Brake, Carrier and Oil Supply, are all bond graph 1-jonctions (if considering components on the loop for each) and they enter a 0-junction, corresponding to the entire hydraulic MCFS. Modules evolve (somehow) independently those with hydraulic cylinders in 4 activities and the third with 2 activities.

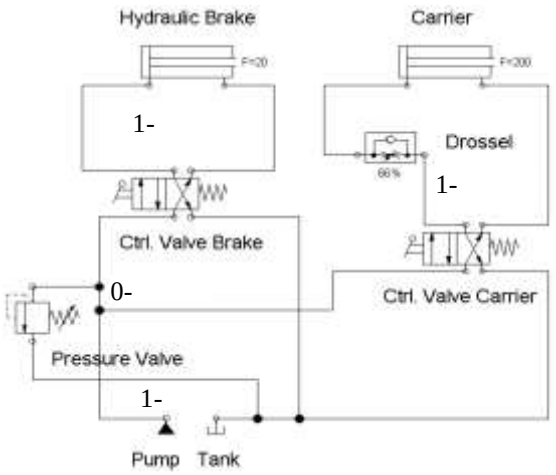


Figure 3. *Hydraulic installation under fault diagnosis.*

Figure 4 presents the diagnostic for 20 simulated faults in the example hydraulic installation and the maximum number of successive activities in which the diagnosis system is able to properly indicate the fault; additional observations supplied by human operator count as distinct activities.

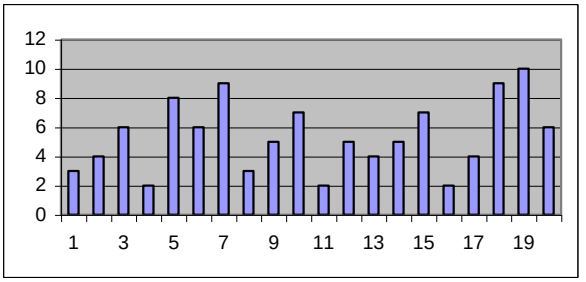


Figure 4. *20 faults and the number of activities in which they are properly recognized.*

Diagnosis performed on the target hydraulic system applied plausibility criteria from human diagnostician concerning patterns of manifestations-to-faults from practice and deep knowledge on specific transport anomalies for the faults in concern. Deep knowledge for relevance criteria refer to physical structure and to transport anomalies shared by faults.

5 CONCLUSION

Diagnosis is a difficult task in real life, while it is often performed on open spaces of causes and effects, in an incomplete and imprecise knowledge milieu. Human diagnostician performs diagnosis by abduction; abductive reasoning itself is a challenge for philosophy, science and practice.

The paper proposes a unified model for diagnosis by abduction, based on plausibility and relevance criteria on causes. It allows connectionist implementation through various artificial neural network types – if adequate to implement plausibility by excitatory links between effects and causes, and relevance by competition in special groups of causes; all effects and causes become neurons with graded levels of truth – regarding evidence of effects and certainty of cause, respectively.

The unified model for diagnosis by abduction is simpler than the one proposed by [4], and offers also natural meanings for human diagnosticians interested on practical implementation in technical or economical domains.

The unified for abductive problem solving model is fully functional for all categories of abduction problems, also for disjunctive and conjunctive grouping of effects to a cause. It is meant to embed shallow and deep knowledge from human diagnostician in the way he or she actually does in practice and the connectionist model

The paper presents also hints on knowledge elicitation of deep and shallow knowledge on the class of multifunctional conductive flow systems (MCFSs), i.e. systems that perform simultaneously multiple functions, based on (multiple) flow conduction. Such systems are often met in industry but also in other domains of real life. So, along with the diagnosis model by abduction the paper offers design guidelines for computational model of an automated diagnosis system. Application in simulated environment shows good performance, of diagnostic, however strongly dependent on available knowledge.

REFERENCES

1 Ariton, V., Ariton D., *A General Approach for Diagnostic Problems Solving by Abduction*, 4th IFAC Symposium on Fault Detection Supervision and Safety, Budapest , pp. 446-451;
2. Ariton, V., Baciu, C., *Knowledge Elicitation And Case Tool For Fault Diagnosis In Multifunctional Conductive Flow Systems*, 5th World Multiconference on Systemics, Cybernetics and Informatics (SCI2002), Electronics, Informatics and Control Systems – Invited Session, Organizer: Volume XXII, Olrando, Florida, USA, 2002;
3. Ariton V., *Deep and shallow knowledge in fault diagnosis*, Lecture Notes in Artificial Intelligence LNAI 2773, Palade, V., Howlett, R. J., Lakhmi J. (Eds), Springer Verlag - 7th International Conference , KES2003, Oxford, UK, pp.748-755, 2003;
4. Ayeb B., Wang S., Ge J. (1998), *A Unified Model for Abduction-Based Reasoning*, IEEE Trans. on Syst. Man and Cyberneticcs - Part A: Systems nad Humans, 28, no.4. 1998, pp.408-424;
5. Bylander T., Allemang D., Tanner M. C., Josephson J, R., (1991). *The computational complexity of abduction*, Artificial Intelligence 49, Elsevier 1991, pp. 25-60;
6. Konolige K. (1992). *Abduction versus closure in causal theories*, Artificial Intelligence, 53, Elsevier1992, pp. 255-272;
7. Mosterman P.J. *Hybrid Dynamic Systems: A hybrid bond graph modeling paradigm and its application in diagnosis*, Ph.D. Thesis, Vanderbilt University, 1997;

8. Larsson, J. E., *Knowledge-based methods for control systems*, PhD Thesis Dissertation, Lund - Sweden, 1992.

THE SPEECH ACT THEORY BETWEEN LINGUISTICS AND LANGUAGE PHILOSOPHY

Liviu-Mihail MARINESCU
„Danubius" Universitaty from Galati

Abstract: *Of all the issues in the general theory of language usage, speech act theory has probably aroused the widest interest. Psychologists, for example, have suggested that the acquisition of the concepts underlying speech acts may be a prerequisite for the acquisition of language in general, literary critics have looked to speech act theory for an illumination of textual subtleties or for an understanding of the nature of literary genres, anthropologists have hoped to find in the theory some account of the nature of magical incantations, philosophers have seen potential applications to, amongst other things, the status of ethical statements, while linguists have seen the notions of speech act theory as variously applicable to problems in syntax, semantics, second language learning, and elsewhere.*

Keywords*: speech act theory, presupposition, implicature, deixis*

Jel Classification: *Y50- Further reading*

1. Prefatory View

Of all the issues in the general theory of language usage, **speech act theory** has probably aroused the widest interest. Psychologists, for example, have suggested that the acquisition of the concepts underlying speech acts may be a prerequisite for the acquisition of language in general, literary critics have looked to speech act theory for an illumination of textual subtleties or for an understanding of the nature of literary genres, anthropologists have hoped to find in the theory some account of the nature of magical incantations, philosophers have seen potential applications to, amongst other things, the status of ethical statements, while linguists have seen the

notions of speech act theory as variously applicable to problems in syntax, semantics, second language learning, and elsewhere. Meanwhile in linguistic pragmatics, speech acts remain, along with **presupposition** [1] and **implicature**[2] in particular, one of the central phenomena that any general pragmatic theory must account for.

Given the widespread interest, there is an enormous literature on the subject, and this paper is not meant to examine all the work within linguistics, let alone a small fraction of the technical literature within language philosophy.

2. J.L. Austin's Brand New Ideas – A Huge Step ahead Logical Positivism. From Austin to Searle

To start with the very beginning, one might notice that issues of truth and falsity have always been of central interest throughout much of the literature focussed on **deixis**[3], **presupposition** and **implicature**. Indeed those issues derive

[1] The term points out what a speaker or writer assumes that the receiver of the linguistic message already knows.
For example:
speaker A: *What about inviting Simon tonight*?
speaker B: *What a good idea; then he can give Monica a lift*.
Here, the presuppositions are, amongst others, that speaker A and B know who Simon and Monica are, that Simon has a vehicle, most probably a car, and that Monica has no vehicle at the moment. Children often presuppose too much. The may say:
...and he said "let's go" and we went there.
even if the hearers do not know who *he* is and where *there* is.

[2] This linguistic concept is connected to conversational maxims i.e. those unwritten rules about conversation which people know and which influence the form of conversational exchanges. For example in the following exchange:
A: *Let's go to the movies.*
B: *I have an examination in the morning*.
B's reply might appear not to be connected to A's remark. However, since A has made an invitation and since a reply to an invitation is usually either an acceptance or a refusal, B's reply is here understood as an excuse for not accepting the invitation (i.e. a refusal). B has used the "maxim" that speakers normally gives replies which are relevant to the question that has been asked. The linguist and philosopher Grice has suggested that there are four conversational maxims: a). the maxim of quantity: give as much information as needed; b). the maxim of quality: speak truthfully; c). the maxim of relevance: say things that are relevant; d). the maxim of manner: say things clearly and briefly. The use of conversational maxims to imply meaning during conversation is called ***conversational implicature***, and the "co – operation" between speakers in using the maxims is sometimes called the ***co – operative principle***.

[3] The concept of ***deixis*** points out those words or phrases – called ***deictic*** – which directly relate an utterance to a time, place or person.
Examples of deictic words in English are:
here and ***there***, which refer to a place in relation to the speaker:
The letter is <u>here</u>. (near the speaker)
The letter is <u>over there</u>. (farther away from the speaker)

much of their interest from the way in which they remind us of the strict limitations to what can be captured in a truth – conditional analysis of sentence meaning. Nevertheless in the 1930s there flourished what can now be safely treated as a linguistic and philosophical excess, namely the doctrine of **logical positivism**, a central tenet of which was that unless a sentence can, at least in principle, be *verified* (i.e. tested for its truth and falsity), it was strictly speaking *meaningless.* Of course it followed that most ethical, aesthetic and literary discourses, not to mention everyday utterances, were simply meaningless. But rather than being seen as a *reductio ad absurdum*, such a conclusion was reviewed by proponents of logical positivism as a positively delightful result **(**see the marvelously prescriptive work by Ayer (1936)**)**[1], and the doctrine was pervasive in philosophical circles of the time. It was this movement **(**which Wittgenstein had partly stimulated in his *Tractus – Logico – Philosophicus* (1921)**)** that the later Wittgenstein was actively attacking in *Philosophical Investigations* with the well known slogan "meaning in use", and the insistence that utterances are only explicable in relations to the activities, or **language – games**, in which they play a role.

It was in the same period, when concern with verifiability and distrust of the inaccuracies and vacuities of ordinary language were paramount, that J.L. Austin launched his theory of speech acts. There are strong parallels between the latter Wittgenstein's emphasis on language usage and language games and Austin's insistence that "the total speech act in the total speech situation is the *only actual* phenomenon which, in the last resort, we are engaged in elucidating."[2] Nevertheless, Austin appears to be largely unaware of, and probably quite uninfluenced by, Wittgenstein's later work, and we may treat Austin's theory as autonomous.

In the set of lectures that were posthumously published as *How to Do Things with Words,* Austin set about demolishing, in his mild and urbane way, the view of language that would place truth conditions as central to language understanding. His method was this:

First, he noted that some ordinary language declarative sentences, contrary to logical positivist assumptions, are not apparently used with any intention of making true or false statements. These seem to form a special class, and are illustrated below:

[1] Ayer, A.J., *Language, Truth and Logic*, Victor Gollancz. London, 1936
[2] Austin, J.L., *How to Do Things with Words*, Clarendon Press, Oxford, 1962

(1) I bet you six pence it will rain tomorrow
I hereby christen this ship the H.M.S. Flounder
I declare war on Zanzibar
I apologize
I dub thee Sir Walter
I object
I sentence you to ten years of hard lobour
I bequeath you my Sansovino
I give my word
I warn you that trespassers will be prosecuted

The peculiar thing about these sentences, according to Austin, is that they are not used to say things, i.e. describe states of affairs, but rather actively to *do* things. After you've declared war on Zanzibar, or dubbed Sir Walter, or raised an objection, the world has changed in substantial ways. Further, you cannot assess such utterances are true or false – as illustrated by the bizarre nature of the following exchange:

(2) A: *I second the motion.*
B: *That's false.*
(3)A: *I dub thee Sir Walter.*
B: *Too true.*

Austin termed these peculiar and special sentences ***performatives***, and contrasted them to statements, assertions and utterances like them, which he called ***constatatives***.

He then went on to suggest that although, unlike constatatives, performatives cannot be true or false (given their special nature, the question of truth and falsity simply does not arise), yet they can go wrong. He then set himself the task of cataloguing all the ways in which they can go wrong, or be ***infelicitous*** as he put it. For instance, suppose I say *I christened this ship the H.M.S. Flounder*, I may not succeed in so christening the vessel if, for instance, it is already named otherwise, or I am not an appointed namer, or there are no witnesses, slipways, bottles of champagne, etc. Successfully naming a ship requires certain institutional arrangements, without which the action that the utterance attempts to perform is simply null and void. On the basis of such different ways in which a performative can fail to come off, Austin produced a typology of conditions which performatives must meet if they are to succeed or be ***felicitous***. He called these conditions ***felicity conditions***, and he distinguished three main categories:

(4) A. (i) There must be a conventional procedure having a conventional effect;

(ii) The circumstances and persons must be appropriate, as specified in the procedure.

B. The procedure must be executed (i) correctly and (ii) completely
C. Often, (i) the persons must have the requisite thoughts, feelings and intentions, as specified in the procedure, and (ii) if consequent conduct is specified, then the relevant parties must do so

As evidence of the existence of such conditions, consider what happens when some of them are not fulfilled. For example, suppose, a British citizen says to his wife:

(5) *I hereby divorce you*

He will not thereby achieve a divorce, because there simply is no such procedure (as in A (i)) whereby merely by uttering (5) divorce can be achieved. In contrast in Muslim cultures there is such a procedure, whereby the uttering of a sentence with the import of (5) three times consecutively does thereby and *ipso facto* constitute a divorce. As an illustration of a failure of condition A (ii), consider a clergymen baptizing the wrong baby, or the right baby with the wrong name, or consider the case of one head of state welcoming another, but addressing the attendant bodyguard in error. As for condition B, the words must be conventionally correct and complete. Finally, the violations of the C conditions are insincerities: to advise someone to do something when you really think it would be advantageous for you but not for him, or for a juror to find a defendant guilty when he knows him to be innocent, would be to violate condition C (i). And to promise to do something which one has no intention whatsoever of doing would be a straightforward violation of C (ii).

Austin notes that these violations are not of equal stature. Violations of A and B conditions give rise to **misfires** as he puts it – i.e. the intended actions simply fail to come off. Violations of C conditions on the other hand are **abuses**, not so easily detected at the time of the utterance in question, with the consequence that the action is performed, but infelicitously or insincerely.

On the basis of these observations Austin declares that (a) some sentences, performatives, are special: uttering them *does* things, and does not merly say things (report states of affairs); and (b) these performative sentences achieve their corresponding actions because there are specific *conventions* linking the words to

institutional procedures. Performatives are, if one likes, just rather special sorts of ceremony. And unlike constatatives, which are assessed in terms of truth and falsity, performatives can only be assessed as felicitous or infelicitous, according to whether their felicity conditions are met or not.

But Austin is playing cunning: given this much, he has his wedge into the theory of language and he systematically taps it home. Readers of *How to Do Things with Words* should be warned that there is an internal evolution to the argument, so that what is proposed at the beginning is rejected by the end. Indeed what starts off a theory about some special and peculiar utterances – performatives – ends up as a general theory that pertains to all kinds of utterances. Consequently there are two crucial sliding definitions or concepts: firstly, there is a shift from the view that performatives are a special class of sentences with peculiar syntactic and pragmatic properties, to the view that there is a general class of performative utterances that includes both **explicit performatives** (the old familiar class) and **implicit performatives**, the latter including lots of other kinds of utterances if not all. Secondly, there is a shift from the dichotomy performative / constatative to a general theory of **illocutionary acts** of which the various performatives and constatatives are just special sub – cases. Let us take these two shifts in order, and review Austin's arguments for the theoretical 'sea – change', as he puts it.

If the dichotomy between performatives and constatatives is to bear the important load that Austin indicates, namely the distinction between truth – conditionally assessed utterances and those assessed in terms of felicity, than it had better be possible to tell the difference – i.e. to characterize performatives in independent terms. Austin therefore teases us with an attempt to characterize performatives in linguistic terms. He notes that the paradigm cases, as in (1) above, seem to have the following properties: they are first person indicative active sentences in the simple present tense. This is hardly surprising, since, if in uttering a performative the speaker is concurrently performing an action, we should expect just those properties. Thus we get the contrast between the following sentences: only the first can be uttered performatively.

(6) a. I bet you five pounds it'll rain tomorrow.
b. I am betting you five pounds it'll rain tomorrow.
c. I did bet you five pounds it'll rain tomorrow.
d. He bets you five pounds it'll rain tomorrow.

The progressive aspect in (6b) renders that (most probably) a reminder, as does the third person in (6d), while the past tense in (6c) indicates a report; none of these constatatives seems, then, to be capable of doing betting, unlike the performative (6a).

Austin's work is, however, not easy to summarize as it is rich with suggestions that are not followed up, and avoids dogmatic statements of position. Of the large amount of philosophical work that it has given rise to, one development in particular is worth singling out, i.e. the very influential doctrine of J.R. Searle.

In general, Searle's theory of speech acts is just Austin's systematized, in part rigidified, with sallies into the general theory of meaning, and connections to other philosophical issues. Austin thought that one could come to an interesting classification through taxonomy of performative verbs, but Searle seeks some more abstract scheme based on felicity conditions. In fact, he proposes that there are just five basic kinds of action that one can perform in speaking, by means of the following five types of utterance:

1. **representatives**, which commit the speaker to the truth of the expressed proposition (paradigm cases: asserting, concluding, etc.)

2. **directives**, which are attempts by the speaker to get the addressee to do something (paradigm cases: requesting, questioning)

3. **commissives**, which commit the speaker to some future course of action (paradigm cases: promising, threatening, offering)

4. **expressives**, which express a psychological state (paradigm cases: thanking, apologizing, welcoming, congratulating)

5. **declarations**, which effect immediate changes in the institutional state of affairs and which tend to rely on elaborate extra – linguistic institutions (paradigm cases: excommunicating, declaring war, christening, firing from employment)

To Searle, as with Austin, the illocutionary act is directly achieved by the conventional force associated with the issuance of a certain kind of utterance in accord with a conventional procedure. In contrast, a **perlocutionary act** is specific to circumstances of issuance, and is therefore not conventionally achieved just by uttering that particular utterance, and includes all those effects, intended or unintended, often indeterminate, that some particular utterance in some particular situation may cause.

3. A Last Annotation. Instead of Final Judgment

To squeeze all that goes under the label of **speech act theory** within the confines of a linguistic paper like this one would be not only impossible – as the numerous volumes dedicated to this topic could not cover its plenitude of

significance – but also undesirable. Consequently, this paper is quite conservative in scope and approach, and verges upon only what is of an utmost importance in speech act theory.

LIST OF REFERENCES

1. Appel, R., Muysken, P., *Language Contact and Bilingualism*, Arnold, London, 1987;
2. Asher, R. A., *The Encyclopaedia of Language and Linguistics*, Penguin Press, London, 1994;
3. Austin, J. L., *How to Do Things with Words*, Clarendon Press, Oxford, 1962;
4. Ayer, A. J., *Language, Truth and Logic*, Victor Gollancz. London, 1936;
5. Berry, M., *An Introduction to Systemic Linguistics: Structures and Systems*, Batsford, London, 1975;
6. Bolinger, D., *Aspects of Language*, Hacourt, Brace & World, New York, 1975;
7. Clyne, M., *Perspectives on Language Contact*, The Hawthorn Press, Melbourne, 1972;
8. Dakin, J., *The Language Laboratory and Language Learning*, Longman, London, 1973;
9. Edwards, JR., *Language and Disadvantage*, Edward Arnold, London, 1978;
10. Ellis, R., *Understanding Second Language Acquisition*, Oxford University Press, 1985;
11. Firth, J. R., *Papers in Linguistics 1934 – 1951*, London, 1964;
12. Halliday, M. A., Hasan, R., *Cohesion in English*, Longman, London, 1976;
13. Halliday, M. A.; Hasan, R., *Language, Text and Context: Aspects of Language in a Semiotic Perspective*, Oxford University Press, Oxford, 1989;
14. Harmer, Jeremy, *The Practice of English Language Teaching*, Longman Group UK, 1991;
15. Hudson, R., *Sociolinguistics*, Cambridge University Press, Cambridge, 1981;
16. Krashen, S. D., *The Input Hypothesis: Issues and Implications*, Longman, London, 1985;
17. James, C., *Contrastive Analysis*, Longman, London, 1980;
18. Oxford, R. L., *Language Learning Strategies*, Newbury House, New York, 1990;
19. Richards, J. C., *The Language Teaching Matrix*, Cambridge University Press, 1990;
20. Sinclair, J. McH., Brasil, D., *Teacher Talk*, Oxford University Press, Oxford, 1982;
21. Titone, R., *Teaching Foreign Language; An Historical Sketch*, Georgetown University Press, Washington, 1968.