

Mathematical and Quantitative Methods**The Extreme of a Function Subject
to Restraint Conditions****Cătălin Angelo Ioan¹, Gina Ioan²**

Abstract: Many papers treat the classical problem of the determination function's extreme points subject to equality type restrictions. The well-known method of Lagrange's multipliers gives necessary conditions but not sufficient. In this paper, it is shown that in additional hypotheses (like the linearity) the nature of a stationary point remains the same for the restricted function and for the initial one.

Keywords: Lagrange; multipliers; extreme

JEL Classification: C65; C81; C82

1 Introduction

In microeconomic theory, very often it is used the method of Lagrange multipliers in order to determine the extremes of a function whose variables satisfy additional restrictions. Unfortunately, many times, it is considered that the solutions of the system of equations derived from the method of Lagrange multipliers, are automatically extreme points. What is certain is that the method of Lagrange multipliers involve necessary conditions for the extreme, but not sufficient.

Given the importance of this method for a range of economic applications, such as the optimum choice when the consumer is given his income, the problem of the minimizing of the income providing a constant utility, minimizing the cost function under conditions of constant output or maximizing profit under restrictive conditions, we shall broach the problem of extreme points under additional

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relationships, without using Lagrange multipliers, obtaining, finally, sufficient conditions of extreme.

Let D an open subset in \mathbf{R}^n and a function on $D: U: D \subset \mathbf{R}^n \rightarrow \mathbf{R}$, C^2 -differentiable on D such that: $(x_1, \dots, x_n) \rightarrow U(x_1, \dots, x_n) \in \mathbf{R}$ for any $(x_1, \dots, x_n) \in D$.

Let also consider the restrictions: $g_k(x_1, \dots, x_n) = 0$, $k = \overline{1, m}$, $m < n$ Let suppose that

$$\frac{D(g_1, \dots, g_m)}{D(x_{n-m+1}, \dots, x_n)} \neq 0.$$

From the implicit function theorem, there exist locally the family of functions φ_k , $k = \overline{1, m}$, such that: $x_{n-m+k} = \varphi_k(x_1, \dots, x_{n-m})$ and

$$\frac{\partial \varphi_k}{\partial x_i} = - \frac{\frac{D(g_1, \dots, g_m)}{D(x_{n-m+1}, \dots, x_n)}}{\frac{D(g_1, \dots, g_m)}{D(x_{n-m+1}, \dots, x_n)}}, k = \overline{1, m}.$$

Substituting $x_{n-m+1} = \varphi_1(x_1, \dots, x_{n-m}), \dots, x_n = \varphi_m(x_1, \dots, x_{n-m})$ in U , we find that the function $u = U(x_1, \dots, x_{n-m}, \varphi_1(x_1, \dots, x_{n-m}), \dots, \varphi_m(x_1, \dots, x_{n-m}))$ is the restriction of U at

$$D_1 = \{ (x_1, \dots, x_n) \in D \mid g_k(x_1, \dots, x_n) = 0, k = \overline{1, m} \}.$$

In what follows we shall inquire into the existence of a locally extreme of u , that is the locally extremum of U subject to the restrictions $g_k = 0$, $k = \overline{1, m}$.

This problem it is classical and it is solved with Lagrange's multipliers method. The problem arises from the fact that this method does not gives sufficiently conditions for the extreme.

2 Main Theorem

In what follows, we shall compute the second differential of u .

$$(1) \frac{\partial u}{\partial x_i} = \frac{\partial U}{\partial x_i} + \sum_{k=1}^m \frac{\partial U}{\partial x_{n-m+k}} \frac{\partial \varphi_k}{\partial x_i} \text{ and}$$

$$(2) \frac{\partial^2 u}{\partial x_i \partial x_j} = \frac{\partial^2 U}{\partial x_i \partial x_j} + \sum_{k=1}^m \frac{\partial^2 U}{\partial x_i \partial x_{n-m+k}} \frac{\partial \varphi_k}{\partial x_j} + \sum_{k=1}^m \frac{\partial^2 U}{\partial x_j \partial x_{n-m+k}} \frac{\partial \varphi_k}{\partial x_i} + \sum_{k,p=1}^m \frac{\partial^2 U}{\partial x_{n-m+p} \partial x_{n-m+k}} \frac{\partial \varphi_k}{\partial x_i} \frac{\partial \varphi_p}{\partial x_j} + \sum_{k=1}^m \frac{\partial U}{\partial x_{n-m+k}} \frac{\partial^2 \varphi_k}{\partial x_i \partial x_j}$$

The second differential is:

$$\begin{aligned} d^2u &= \sum_{i,j=1}^{n-1} \frac{\partial^2 u}{\partial x_i \partial x_j} dx_i dx_j = \sum_{i,j=1}^{n-m} \frac{\partial^2 U}{\partial x_i \partial x_j} dx_i dx_j + 2 \sum_{i,j=1}^{n-m} \sum_{k=1}^m \frac{\partial^2 U}{\partial x_i \partial x_{n-m+k}} \frac{\partial \varphi_k}{\partial x_j} dx_i dx_j + \\ &\sum_{i,j=1}^{n-m} \sum_{k,p=1}^m \frac{\partial^2 U}{\partial x_{n-m+p} \partial x_{n-m+k}} \frac{\partial \varphi_k}{\partial x_i} \frac{\partial \varphi_p}{\partial x_j} dx_i dx_j + \sum_{i,j=1}^{n-m} \sum_{k=1}^m \frac{\partial U}{\partial x_{n-m+k}} \frac{\partial^2 \varphi_k}{\partial x_i \partial x_j} dx_i dx_j = \\ &\sum_{i,j=1}^{n-m} \frac{\partial^2 U}{\partial x_i \partial x_j} dx_i dx_j + 2 \sum_{k=1}^m \left(\sum_{j=1}^{n-m} \frac{\partial \varphi_k}{\partial x_j} dx_j \right) \sum_{i=1}^{n-m} \frac{\partial^2 U}{\partial x_i \partial x_{n-m+k}} dx_i + \\ &\sum_{k,p=1}^m \frac{\partial^2 U}{\partial x_{n-m+p} \partial x_{n-m+k}} \left(\sum_{i=1}^{n-m} \frac{\partial \varphi_k}{\partial x_i} dx_i \right) \left(\sum_{j=1}^{n-m} \frac{\partial \varphi_p}{\partial x_j} dx_j \right) + \sum_{k=1}^m \frac{\partial U}{\partial x_{n-m+k}} \sum_{i,j=1}^{n-m} \frac{\partial^2 \varphi_k}{\partial x_i \partial x_j} dx_i dx_j = \\ &\sum_{i,j=1}^n \frac{\partial^2 U}{\partial x_i \partial x_j} dx_i dx_j - 2 \sum_{i=1}^{n-m} \sum_{k=1}^m \frac{\partial^2 U}{\partial x_i \partial x_{n-m+k}} dx_i dx_{n-m+k} - \sum_{k=1}^m \sum_{p=1}^m \frac{\partial^2 U}{\partial x_{n-m+k} \partial x_{n-m+p}} dx_{n-m+k} dx_{n-m+p} + \\ &2 \sum_{k=1}^m \left(\sum_{j=1}^{n-m} \frac{\partial \varphi_k}{\partial x_j} dx_j \right) \sum_{i=1}^{n-m} \frac{\partial^2 U}{\partial x_i \partial x_{n-m+k}} dx_i + \sum_{k,p=1}^m \frac{\partial^2 U}{\partial x_{n-m+p} \partial x_{n-m+k}} \left(\sum_{i=1}^{n-m} \frac{\partial \varphi_k}{\partial x_i} dx_i \right) \left(\sum_{j=1}^{n-m} \frac{\partial \varphi_p}{\partial x_j} dx_j \right) + \\ &\sum_{k=1}^m \frac{\partial U}{\partial x_{n-m+k}} \sum_{i,j=1}^{n-m} \frac{\partial^2 \varphi_k}{\partial x_i \partial x_j} dx_i dx_j = \\ &\sum_{i,j=1}^n \frac{\partial^2 U}{\partial x_i \partial x_j} dx_i dx_j + 2 \sum_{k=1}^m \left(-dx_{n-m+k} + \sum_{j=1}^{n-m} \frac{\partial \varphi_k}{\partial x_j} dx_j \right) \sum_{i=1}^{n-m} \frac{\partial^2 U}{\partial x_i \partial x_{n-m+k}} dx_i + \\ &\sum_{k,p=1}^m \frac{\partial^2 U}{\partial x_{n-m+p} \partial x_{n-m+k}} \left[\left(\sum_{i=1}^{n-m} \frac{\partial \varphi_k}{\partial x_i} dx_i \right) \left(\sum_{j=1}^{n-m} \frac{\partial \varphi_p}{\partial x_j} dx_j \right) - dx_{n-m+k} dx_{n-m+p} \right] + \\ &\sum_{k=1}^m \frac{\partial U}{\partial x_{n-m+k}} \sum_{i,j=1}^{n-m} \frac{\partial^2 \varphi_k}{\partial x_i \partial x_j} dx_i dx_j \end{aligned}$$

Because $x_{n-m+k} = \varphi_k(x_1, \dots, x_{n-m})$ we find that: $dx_{n-m+k} = \sum_{i=1}^{n-m} \frac{\partial \varphi_k}{\partial x_i} dx_i$ therefore:

$$d^2u = \sum_{i,j=1}^n \frac{\partial^2 U}{\partial x_i \partial x_j} dx_i dx_j + \sum_{k=1}^m \frac{\partial U}{\partial x_{n-m+k}} \sum_{i,j=1}^{n-m} \frac{\partial^2 \varphi_k}{\partial x_i \partial x_j} dx_i dx_j$$

We can write therefore:

$$(3) d^2u = d^2U + \sum_{k=1}^m \frac{\partial U}{\partial x_{n-m+k}} d^2\varphi_k .$$

Theorem

Let D an open subset in \mathbf{R}^n and a function on D : $U: D \subset \mathbf{R}^n \rightarrow \mathbf{R}$, C^2 -differentiable on D such that: $(x_1, \dots, x_n) \rightarrow U(x_1, \dots, x_n) \in \mathbf{R}$ for any $(x_1, \dots, x_n) \in D$. Let also consider the restrictions: $g_k(x_1, \dots, x_n) = 0$, $k = \overline{1, m}$, $m < n$ with $\frac{D(g_1, \dots, g_m)}{D(x_{n-m+1}, \dots, x_n)} \neq 0$ and φ_k such that:

$$x_{n-m+k} = \varphi_k(x_1, \dots, x_{n-m}), \quad k = \overline{1, m} .$$

- a) If $\frac{\partial U}{\partial x_{n-m+k}} \geq 0$, $d^2\varphi_k$ – positive half-definite for any $k = \overline{1, m}$ and d^2U – positive definite then d^2u is positive definite;
- b) If $\frac{\partial U}{\partial x_{n-m+k}} \geq 0$, $d^2\varphi_k$ – negative half-definite for any $k = \overline{1, m}$ and d^2U – negative definite then d^2u is negative definite;
- c) If $\frac{\partial U}{\partial x_{n-m+k}} \leq 0$, $d^2\varphi_k$ – positive half-definite for any $k = \overline{1, m}$ and d^2U – negative definite then d^2u is negative definite;
- d) If $\frac{\partial U}{\partial x_{n-m+k}} \leq 0$, $d^2\varphi_k$ – negative half-definite for any $k = \overline{1, m}$ and d^2U – positive definite then d^2u is positive definite.

Corrolary 1

If φ_k are affine functions for any $k = \overline{1, m}$ then $d^2u = d^2U$ that is the nature of stationary points are the same for the function and for those with restrictions.

Proof If φ_k are affine functions then $d^2\varphi_k = 0$ for any $k = \overline{1, m}$.

3 Application

One spectacular application is in the economical theory of consumer. Let a function of utility U which is supposed to be of class C^2 and concave, that is d^2U is negative definite. If for the goods G_1, \dots, G_n with the corresponding prices p_1, \dots, p_n it is a limited budget V , the problem is to determine the optimal distribution in order to maximize the utility.

In this case, if we note with x_1, \dots, x_n the desired quantities of G_1, \dots, G_n , we have the restriction $p_1x_1 + \dots + p_nx_n = V$ therefore: $x_n = \frac{V}{p_n} - \frac{p_1}{p_n}x_1 - \dots - \frac{p_{n-1}}{p_n}x_{n-1}$. The restriction of U related at this condition is therefore: $u = U\left(x_1, \dots, x_{n-1}, \frac{V}{p_n} - \frac{p_1}{p_n}x_1 - \dots - \frac{p_{n-1}}{p_n}x_{n-1}\right)$. Because U is concave, follows that if u has a stationary point this is a maximum point.

4 References

- Dixit, A.K. (1990). *Optimization in Economic Theory*. New York: Oxford University Press.
- Ioan, C.A. (2009). A method of the determination of an acquisition program for the maximization of the total utility. *The 4th International Conference European Integration – Realities and Perspectives*. Galati: Editura Universitara Daubius.
- Ioan, C.A. & Ioan, G. (2010). *Applied mathematics in micro and macroeconomics*. Galati: Sinteze.
- Ioan, C.A., Ioan, G. (2010). *Mathematical models and methods in economical activity*. Galati: Sinteze.
- Mas-Collel, A.; Whinston, M.D. & Green, J. R. (1995). *Microeconomic Theory*. New York: Oxford University Press.
- Pindyck, R.S. & Rubinfeld, D.L. (1996). *Microeconomics*. Prentice-Hall International.

Globalization and Economic Growth: Evidence from Pakistan

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Abstract: The process of globalization has positives as well as negative impacts on any society, which differ from country to country. The negative effects can be countered by adopting the policies that contribute in economic development. This study attempts to review the nature of relationship between economic growth and globalization in Pakistan for the period 1980-2009 by employing the time series data. Co-integration and error correction technique are use to determine the long run effect of globalization on economic growth. The results indicate that globalization can be a useful tool for economic growth for a developing country like Pakistan.

Keywords: globalization; economic growth; co-integration; error correction

JEL Classification: C22; F01; F4

1. Introduction

Globalization has diverse meanings and concepts, which depend upon one who is discussing about it. Any authentic and precise definition of globalization is unavailable. One can say that, the globalization is a process, which include trade openness, technological advancement, open access of financial and good markets and also disease, ailment and pollution. Alternatively,"the globalization is a process which interlinks the people of different culture and societies and strengthens their economies.

The continuous universal trend towards the free flow of financial infusorians across the worlds expresses globalization as a process which revolutionizes the global financial structure. It provides economic sovereignty and prompt competition

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invigorating globalization to raise the living standard of people working in the global trade environment. It is a socio-economic term that, now a days used as synonym for the economic growth of a country. In simple terms, different societies, economies, traditions, and culture integrate with each other through a continuous process. According to World Economic Forum, "the world is transmitted in a place where the fast eat the slow from the world where the big eat the small." There is a consensus about the improved living standards of people almost all the economic and political analysts have the same opinion about that. With the introduction of improved technology in the international markets, the demand and utilization of commodities has increased substantially.

Financial sector is observing high infusions by addition of people in the world's trade. Another aspect that can be considered as a positive effect of globalization is the reduction in inflation rate. It is because of increasing competition, which forced the producer to stabilize their prices. Technological enhancement and efficiency growth are the supplementary benefits of globalization.

Globalization has become the new slogan in the world's economy, leading the globe since the last two decades. MNC's started playing a crucial role in the progress of developing countries by investing in different sectors of their economies. To a certain extent, the effects of globalization have been sufficient on the developing countries. It is providing a variety of openings for the developing economies. However, Globalization has also created many problems like inequality, instability in financial market, worse environmental condition. One of the most important and depressing features of globalization is that, a greater proportion of third world nations stayed out of the whole limelight. Trade liberalization is the major step toward globalization. Third world countries have not gain any thing from this because of the barriers to exports raised by developed countries over the small underdeveloped countries.

Even though Pakistan has suffered from unstable economic system from the time of its emergence on earth, During 1970s Pakistan's economy suffered immensely because of number of events, first the war with India and separation of West Pakistan (now Bangladesh) adversely affect the Pakistan's economy. In 1972 the devaluation, then the international oil price shock of 1973 substantially increased the imports bill, were the most notable events during 1970s. The economic condition improves to some extent because of opening of Middle East markets and remittance from abroad. Pakistan started economic reforms in the beginning of 1980's in coordination with IMF and World Bank to improve the effectiveness of the economy by involving the private investor in economic development, Price deregulation, and denationalization of industry, trade liberalization, and expansion in exports. The process of trade openness started during the first half of 1990's to transmit the close economic system to open economy. The Government has taken a number of steps to reform the Pakistan's economic system, includes, foreign

exchange liberalization, privatization, and trade liberalization also welcomed foreign investors to invest in different sectors of Pakistan.

Pakistan has made considerable efforts for economic integration with the world by trade liberalization, foreign investment and other macroeconomic strategies. Recognizing the increasing tendency of poverty during the last decade of previous century, the government of Pakistan implemented the strategies for poverty diminution at the beginning of 21 century. Steps are taken to reduce poverty trend by implementing macroeconomic policies and investment in human capital formation to foster the economic and social development and poverty reduction.

The objective of this study is to determine the nature of relationship between globalization and economic growth for Pakistan for the period 1980-2009.

1.1. Globalization and the Asian Growth Experience

Asian region is among the rapidly growing region of the world. East and south Asian countries are mainly responsible for such a remarkable growth in the region. Countries like China and Korea showing double-digit GDP growth for few years. China is having an average GDP growth of 9.3% for past 2 decades, whereas the overall Asian region is showing a growth rate of about 5.2% during last 3 decades. East Asian showed comparatively better growth figures as compare to south Asian, simply because it includes courtiers like China and Korea. The other reasons include the amalgamation with the world economy and skilled human resource.

Economies like Sri Lanka and Pakistan have huge potential but caught badly in balance of payment and debt burden problems. Weak industrial structure and stagnant growth in agricultural sector causes this problem to further increase. Economies like India and few others in South Asia succeeded well in developing a balanced policy for integrated growth in industrial as well as agricultural sector. The East Asian economies facilitated private investment and tried hard to create "Crowd in rather than Crowd out effect" therefore the technological progress and rapidly growing labour productivity is the clear outcome of such well-designed policies.

Furthermore, different strategies were adopted by both regions of Asia for of social-economic development and poverty reduction. The East Asian countries have tried to approach the problem in a more scientifically. The East Asian countries focus on human capital accumulation by investing in education, training, and skill development and the proper utilization of skilled labor by increasing employment opportunities. On the other hand, The South Asian countries, rather than focusing on the issues of employment, poverty, education, and energy more scientifically, pay more attention on subsidizing the different sector of economy, which in the long run effects the economic growth very badly.

1.2. The Case of Pakistan in Globalizing World

Situation of Pakistan's economy is very unstable from the beginning. However, the poverty and inequalities grew rapidly during the period of 1990's. According to UNDP report, the poverty rate climbed up to 32.6% between the periods of 1998 to 1999. 13.4% of Pakistan's population lived under poverty line in 2002 and almost 66% people lived with the earning of 2 dollar in a day. According to The reports, World Bank's task force on food security the head Count Ratio (HCR) about poverty trends in Pakistan was 33.8 percent in year 2008 and increased to 36.1 percent in year 2009. About 62 million people in Pakistan were living under poverty line in year 2008-2009. According to statistics of central intelligence agency (CIA), the GINI index for Pakistan was 30.6 in the year 2008 and 41 in year 2009, and the share of income of house hold consumption, lowest 10%: 3.9% and highest 10%: 26.5% (2005). The life expectancy at birth increased from 57 years in 1980 to 65.63 years during the year 2010. Human capital formation that comprise of education and health sector, has been a biggest challenge for Pakistan but only 3.65 % GDP was spent in development of education and health sector in Pakistan in year 2009. During the year 2010, the government expenditure on education was 2.9 as a percentage of GDP and health expenditure as percentage of GDP were 2.4%.

In the era of globalization, economic integration and technological advancement the Pakistan's trade is playing a minor role in growth as in the year 1980 the trade percentage of GDP was 36.57 % and in year 2009 remained at 33.39%. Trends of Direct foreign investments inflow as percentage of GDP in Pakistan witnessed moderate changes over the years. In the year 1980 the investment inflow was 0.26% in the year 2000 was 0.41% and in 2009 1.47% in comparison with the economical dimensions and the demographical potential of the country. If Pakistan can control its economic and natural assets in order to enhance its exports to the global market, it would enhance and escalate the economic development and domestic production. In addition, for Pakistan, it is crucial to find a way to compete in the international trade markets, to invite foreign investments, technological and industrial progress is as important as the development of human capital, and it is the need of time that Pakistan should focus in all these areas to improve its the economic condition.

2. Literature Reviewed

(Dollar, D, and A. Kray, 2000) studied the trade and growth relation and their effects on poverty, recommend, "Trade is good for growth and growth is good for the poor.

(Bhanmurthy & Mitra, 2006), analyzes the globalization, growth and poverty relationship, concluded that the growth effect on poverty in India is a lot higher than the inequality.

(Kakwani, Prakash & Son, 2000) present their findings about the relationship of growth and inequality by investigating the behavior of growth toward poor. Indicate that the economic development interlinked with income inequality and poverty.

(Kuznet, 1955, 1963) stressed out that income inequalities were worse at the start when economic growth took place and with the progress in industrialization and technology. (Kravis 1960) and (Oshima, 1962) supported the finding of kuznet.

(Ravallion, M and Chen 1997) presented the evidence of reduction in income inequality with economic development. They conclude that the inequalities and economic growth have significant negative relationship with each other.

According to (Streeten, 1999), globalization has produced various chances for a few numbers of countries and individuals. HRD indicators like adult literacy rate and life expectancy have been improved during the last 20 years.

(Nasim, 1998) argue about the association between economic development and globalization from the technological viewpoint in South Asian and south East Asian countries, also shed light on the importance of technological progress and industrial development for economic wellbeing.

3. Data and Methodology

This study investigates long-run relationship between globalization, and economic growth in context of the Pakistan for the years 1980 to 2009. The variables include "GDP growth rate, foreign direct investment inflow, population growth rate, real effective exchange rate, life expectancy, government expenditure on education as percentage of GDP, trade % of GDP. GDP growth rate has used as a measure of economic growth, government expenditure on education as percentage of GDP is used as a measure of human capital, and trade percentage of GDP is used as a proxy of trade liberalization. Data is taken from IFS CD ROM.

To measure the long run relationship between the variables it is necessary that the data should be integrated of same order, to check the order of integration the "ADF" unit root test has been employed whereas Johnson co-integration test has been used

to determine the long run relationship between variables. Following log linear model is estimated.

$$\text{LNY} = \alpha + \beta_1 \text{LNEXR} + \beta_2 \text{LNTOP} + \beta_3 \text{LNPG} + \beta_4 \text{LNFDI} + \beta_5 \text{LNHCEP} + \varepsilon_i$$

Where:

Ln = Natural Logarithm

Y = Annual Growth Rate of GDP

FDI = Foreign Direct Investment Inflow

TOP = Trade openness (Total Import and Exports As % Of GDP)

PG = Population Growth Rate

HCEP = Human Capital (% of Public Expenditure on Education and Health)

EXR = Real Effective Exchange Rate

ε = Error Term

4. Calculations and Results

This study intended to scrutinize the relationship between economic growth and globalization in Pakistan from 1980-2006. Time series data usually have a tendency to be non-stationary, and the estimated regression results may indicate spurious results. To insure that the data is stationery and to determine the order of integration between variables test of unit root is, there are several unit root test available to solve the problem of stationerity, we have used "ADF unit root test" at level and at first difference. The results in table-1 indicate that all the variables are non-stationery at level, thus carry unit root. When the unit root is tested at first difference estimates show non-stationerity at, which means the problem of unit root has been removed and the variables are integrated of order 1, I(1).

Table 1. ADF Unit Root Test

Variables	Level	1st Difference	Result
LNY	-3.487657	-6.961929	I(1)
LNTOP	-3.266110	-6.146234	I(1)
LNHCEP	-2.456078	-5.232878	I(1)
LNEXR	-1.829914	-5.417473	I(1)
LNPG	-3.122054	-5.365014	I(1)
LNFDI	-3.502897	-4.750253	I(1)

NOTE: the data is stationery at 5% significance level, checked at both the level as well as on first difference.

Since the variables in the model are non-stationery and are integrated of same order we now apply the johansen co-integration test to determine the long term relationship between the variables. The "VAR" method is used to determine the optimal the lag length and stability condition. The "FPE, AIC AND SC" criteria determine the leg length and support the "lag 1" as the best choice. The johansen co-integration results in table-2(a) and table-2(b) presents both the trace and maximum eigenvalues. The trace statistics indicates four co-integrated equations and maximum eigenvalue identify two co-integrating relationships. Thus, we conclude that all the variables are co-integrated and have long-run relationship with each other.

Table 2. Johansen Multivariate Co-integration Test

(a)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.860212	163.0457	107.3466	0.0000
At most 1 *	0.747051	107.9521	79.34145	0.0001
At most 2 *	0.665891	69.46418	55.24578	0.0017
At most 3 *	0.573390	38.76811	35.01090	0.0189
At most 4	0.311064	14.91534	18.39771	0.1437
At most 5 *	0.147927	4.482326	3.841466	0.0342
Trace test indicates 4 cointegrating eqn(s) at the 0.05 level				
* denotes rejection of the hypothesis at the 0.05 level				
**MacKinnon-Haug-Michelis (1999) p-values				

(b)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.860212	55.09357	43.41977	0.0018
At most 1 *	0.747051	38.48793	37.16359	0.0350
At most 2	0.665891	30.69606	30.81507	0.0517
At most 3	0.573390	23.85278	24.25202	0.0564
At most 4	0.311064	10.43301	17.14769	0.3585
At most 5 *	0.147927	4.482326	3.841466	0.0342
Max-eigenvalue test indicates 2 cointegrating eqn(s) at the 0.05 level				
* denotes rejection of the hypothesis at the 0.05 level				
**MacKinnon-Haug-Michelis (1999) p-values				

The presence of co-integration indicated the presence of long-run relationship between the variables. This indicates that there exists an error correction model, which combines the short run effects with the long run. As a result, the vector error correction model is estimated. The "VCM" shown in table-3 indicates the presence of error correction term in the model. The error correction term has the correct negative and significant sign for growth rate of GDP, population growth rate and trade openness. The value of "-0.10" for GDP growth indicates that, about 10% of the previous disequilibrium has been removed in the present period.

Table 3. Vector Error Correction Model

Error Correction:	D(LNGDPGR)	D(LNFDI)	D(LNPG)	D(LNHCEP)	D(LNTOPI)	D(LNEXR)
CointEq1	-0.101583	-0.402661	-1.015735	0.104360	-0.818119	2.373962
	(0.04545)	(0.11955)	(0.01710)	(0.09447)	(0.97116)	(0.64216)
	[-2.23492]	[-3.36812]	[-1.92008]	[1.10465]	[-0.87211]	[3.69684]
D(LNGDPGR(-1))	-0.640425	-0.073893	-0.002147	-0.286902	-0.149949	0.753655
	(0.32371)	(0.05752)	(0.00823)	(0.87285)	(0.46724)	(0.30895)
	[-1.97836]	[-1.28470]	[-0.26089]	[-0.323870]	[-0.32093]	[2.43941]
D(LNFDI(-1))	0.514834	0.893252	-0.026336	0.376271	3.268680	-3.967560
	(1.81174)	(0.32191)	(0.04605)	(0.25439)	(2.61499)	(1.72911)
	[0.28417]	[2.77487]	[-0.57193]	[1.47914]	[1.24998]	[-2.29457]
D(LNPG(-1))	6.973209	1.408695	-0.239515	1.499655	4.334738	-6.254559
	(9.79273)	(1.73996)	(0.24890)	(1.37499)	(14.1344)	(9.34609)
	[0.71208]	[0.80961]	[-0.96231]	[1.09066]	[0.30668]	[-0.66922]
D(LNHCEP(-1))	0.936546	0.342094	0.011506	0.182773	1.794827	-3.882729
	(1.74989)	(0.31092)	(0.04448)	(0.24570)	(2.52571)	(1.67008)
	[0.53520]	[1.10027]	[0.25870]	[0.74388]	[0.71062]	[-2.32488]
D(LNTOPI(-1))	-0.284306	0.026503	-0.006055	-0.045054	-0.514628	-0.106904
	(0.20159)	(0.03582)	(0.00512)	(0.02830)	(0.29096)	(0.19239)
	[-1.41034]	[0.73993]	[-1.18181]	[-1.59176]	[-1.76871]	[-0.55565]
D(LNEXR(-1))	-0.162134	-0.004120	0.000386	-0.024374	0.326736	0.185752

	(0.20690)	(0.03676)	(0.00526)	(0.02905)	(0.29863)	(0.19746)
	[-0.78364]	[-0.11209]	[0.07350]	[-0.83901]	[1.09412]	[0.94070]
C	0.022575	-0.030152	-0.031695	0.033523	-1.271683	2.684815
	(0.68834)	(0.12230)	(0.01750)	(0.09665)	(0.99353)	(0.65695)
	[0.03280]	[-0.24653]	[-1.81162]	[0.34685]	[-1.27997]	[4.08680]

5. Conclusion

This paper determines the nature of relationship between economic growth and globalization in context of Pakistan for the period 1980-2009. Globalization can be helpful for the economic development of LDC's like Pakistan. It can be a useful measure to reduce the income inequalities in the country.

In this study, we use co-integration and error correction model to investigate the long run relationship of economic growth and globalization. The variables include in the study are "total Trade as a percentage of GDP as a measure of trade openness, Net Foreign Direct Investment Inflow, Population Growth Rate, Real Effective Exchange Rate, sum of Public Education and Health expenditure as percentage of GDP used as measure of human capital indicators, and annual GDP growth rate is used as a determinant of economic growth. The results confirm that there is a long run equilibrium relationship between all the variables. Pakistan started the economic liberalization process in the early 80's. Public foreign investment inflows assist the economic growth. However, Trade opens do not have short-run impact on economic development.

Trade openness provides opportunities for a greater access of large trade market, and increase domestic output and employment opportunities. Pakistan should go more for trade openness with improved quality of domestic products. With appropriate policies to promote export and investment lead growth in Pakistan, globalization can be use as efficient sources to control income inequalities, and poverty and it can be a very use full tool to achieve economic development in Pakistan. The research can be extended by introducing other variables such as wage rate, per capita income, unemployment rate, labor force participation rate.

6. References

- Bhanmurthy, N.R. & Mitra, A. (2006). Globalization, Growth and Poverty in India. *UNU-WIDER World Institute for Development Economics Research. Research paper no. 2006/41*.
- Dollar, D. & Kraay, A. (2000). Growth is Good or the Poor. *Working Paper*. World Bank, Washington.
- D.C. Kakwani, N., & Son, H. (2010). *Economics Growth, Inequality, and Poverty: Korea and Thailand*. Greenwood Publishing Group, Inc. for the National University of Singapore.
- Kravis, I. B. (1960). International Differences in the Distribution of Income. *Review of Economics and Statistics*, 42: 408-16.
- Kuznets. S. (1995). Economics Growth and Income Inequality. *American Economic Review XI. V: 1-28*.
- Nasim, S.M. (1998). Globalization, Technology and Asian Economic Growth. *The Pakistan Development Review*, 37(4), 635-659.
- Oshima, H. (1962). International comparison of Size Distribution of Family Incomes with Special Reference to Asia. *Review of Economics and Statistics*, 44:439-45.
- Qurashi, Sarfaraz K. & Arif. G.M. (2001). Profile of Poverty in Pakistan, 1998-99, *MIMAP Technical Paper series*, No. 5, Islamabad, Pakistan: Pakistan Institute of Development Economics.
- Ravallion, M., & Chen, S. H. (1997). What Can New Survey Data Tell Us About Recent Changes in Distribution and Poverty? *World Bank Economic Review*, 11(2): 357-82.
- Streeten, Paul F. (1999). Globalization: Threat or Opportunity, *14th Annual general Meeting and Conference*, 28th-31st January 1999, Islamabad, Pakistan Institute of development Economics.