

## **Business Administration and Business Economic**

### **Western Balkans' Banking Sector Performance in Terms of Macroeconomic and Bank Specific Efficiency Determinants**

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**Abstract:** Main objective of this study is to analyse banking efficiency and productivity considering evidences from Western Balkan countries in pre-crisis, during and the recovery period. Referring to the historic background and the transformations suffered, the WB countries have developed bank based financial structure so the soundness of the banking sector is significantly important for the stability and progress of their economies on the long run. The problems faced by the last recession, the deteriorated macroeconomic indicators and weak, inefficient banking sector translated in slow recovery rates, encouraged this study. Prior studies have been mainly focused in causes and effects of crisis in different sectors of economies while this paper presents relations and dependencies between the macroeconomic and bank specific efficiency determinants. The methodology used is the comparison between countries and as method the financial ratio analysis, intending the presentation of trends and evaluation of changes in efficiency, profitability and performance indicators during 2000-2007 comparing with 2008-2013. Results that the banking sector profitability indicators follow the same negative trend with the economic growth rates and the recovery rates are lower than the forecasted. The results can be used by performance monitors to better identify vulnerabilities and examine uncertainties/risks.

**Keywords:** bank-based structure; efficiency; profitability; macroeconomic indicators; financial ratios

**JEL Classification:** G21; O11; O57

## **1. Introduction**

### **1.1. How is Currently Presented the Macroeconomic Situation of Western Balkan Economies**

The economies of Western Balkan countries are among those that suffered the most the global downturn of 2007 – 2009 and after. Referring to the World Economic Situation and Prospects 2014 (WESP, 2014), the euro zone crisis “remains a

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significant risk factor” for the global economy (p. 27). The euro area is emerging slowly from the last years’ recession. It started recovering during the year 2013.

As published by Monaghan & Warden (2014), the projected economic growth according to ECB (European Central Bank) for the year 2014 was at 1.1% decreased to 0.9% and finalized at almost 0.8%. The same slow growth rates are predicted for the year 2015 and 2016, forecasted before three months from the same institution at 1.6 and 1.9% respectively. Unfortunately, the expectations are falling even more, at 1 and 1.5% according to projections of December, 2014. The recovery will be uneven but still consists in growth. The world gross product has also marked a lower growth than the forecasted, 2.1 instead of 2.4% (WESP, 2013).

Considering the presented economic situation, the main concerns for the “on developing” economies of Western Balkans are the fragility of the banking sector and their sovereign debts. The case of Cypriot banking sector is a reference for the weakness of this industry in the region.

Each Balkan country is aspiring to become EU “full rights” member although their recent mix economic performance and the current different stage of processes. There are considerable differences between the economic indicators of EU countries. Referring to the average GDP per capita (in PPS) between euro areas (18 countries), is averagely at 108 percent as stated by the end of year 2013 while for the same period, that of the newest member Croatia, is at 61 percent. The average of five WB countries candidate for entering EU is at 34.4 percent. Other EU member countries like Spain, Greece and Italy highly affected by crisis, have respectively 95, 98 and 75 percent by the end of the year 2013 (European Statistical System Eurostat, 2014).

Characteristic for the whole period under study (2007 – 2013) is the decreased trend of banking sector performance indicators same as the main macroeconomic ones. The decreased economic growth, the weak banking industry and the problematic fiscal sector are main problems of the WB country members. The problematic situation of euro area “...has hinted the establishment of a regional banking union and this is already agreed in the first round of EU discussions” (WESP, 2014, p. 28).

The study is conducted among the Western Balkan countries due to the below common features: a) all the countries are aiming the EU membership despite the different levels of their economic development b) except Croatia, all the rest countries are listed in “on developing” economies c) the financial structure of their economies is bank-based as the banking sector occupies the major part of total assets of their financial sector and d) the economies considered in this study are small sized e) the financial sector of the economies under consideration have been

directly or indirectly affected by the financial shocks of the last years. The improvements on the banking sector efficiency and performance will accelerate their economic recovery.

The paper consists in the below sections: firstly is analyzed the importance of studying banking sector performance and is emphasized the relationship with the financial system and the economic growth, the second refers to the history of thought, the third refers to the description of methodology used, the source of data for each country and the fourth provides comparative analysis of WB banking sector performance: ROE, ROA, bank capital to assets, bank regulatory capital to risk weighted assets, capital adequacy ratios, bank non performing loans to total loans etc., in line with the macroeconomic efficiency main determinants as GDP, inflation etc.

Conclusions and recommendations refer to the efficiency and profitability trends of individual banking sector of each country under study, their changes and decreasing tendencies same as the macroeconomic indicators during the considered period. The financial and social costs charged because of banking sector inefficiencies enhance the continuous monitoring and evaluation of non-efficient banks. This paper emphasizes the necessity of analyzing the banking sector performance indicators especially of 'on developing' economies. Promoter for analyzing the sector in details is the necessity of identifying vulnerabilities and examining uncertainties, translated in risks due to their fragile financial systems, considerably sensitive to economic stress.

## **1.2 The Importance of Studying Banking Sector Efficiency**

The financial intermediary role of the banking sector in an economy consists in: accumulating capital, providing liquidity, monitoring services offered to depositors and investors, on keeping them informed etc. Any disturbance, interference or irregularities in the banking sector is mainly reflected to the real economy if considering the bank based financial structure of WB countries and the strong relationship between the banking sector and the financial system.

Financial crisis are mainly caused by the prevention of the financial system from its role of enabling financial intermediate process to channel funds from savers to investors. The financial intermediaries are responsible for the efficient allocation of resources to higher return investments. For the above mentioned reasons the continuous monitoring of the banking sector is starting point for measuring financial stability. Banking is the most and well regulated segment of the economy due to its great potential in the financial sector of countries under study. Recently, learning lessons from the liquidity – crisis of the last years, the interest is focused

on the stability of the banking system as a whole. Regarding to this, the global regulatory standards that consist in micro and macro prudential regulations starting from banks' level to system wide risks are constantly reviewed aiming to enable banks to better absorb the financial shocks, for strengthening the sustainability of the banking sector and for avoiding uncertainties.

Considering the regional developments and beyond is suggested the proper assessment of the efficiency and performance of the banking sector, the continuous monitoring by the local supervising authorities and the European authorized institutions, to be closely reviewed the asset quality, the level of capitalization and restructuring.

## **2. Literature Review**

The most widely studied relationship is between the banking sector, the financial system and economic growth.

Analysis of multiple roles performed by banks in the financial system is essential for the economic theory and finance.

Thorsten, Asli & Ross (2003) encouraged by the long discussions about the relationship between the market structure, banking sector and the economic development, studied the concentration of banking sector, the regulatory system and financial institutions during crisis as financial systems' de-stability indicators. Using data from 70 countries, and a time period of seventeen years (1980-1997) with more than 40 episodes of crisis already happened, resulted that the possibilities of occurring economic crisis are lower in countries with high concentration of banking sector, where the competition is less restricted and where institutions encourage it. Economies with sound and developed financial institutions that stimulate competition are less affected by crisis...limitations in banking activities causes instability in the banking sector.

Referring to Drehmann; Borio & Tsatsaronis (2012) the most prudent description of financial cycles is in terms of credit and property prices. The authors, Drehmann & Juselius (2012) explains that the financial cycle (16 years in industrialized countries) has a much lower frequency than the traditional business cycle (8 years) and peaks in the financial cycle are closely related with the systemic banking crisis; recessions that coincide with the concentration phase of the financial cycle are serious.

The main indicators driving to financial crisis are continuous deviations of private sector credit to GDP and asset prices from historical data or norms as pointed by Claudio (2014).

According to David & Paul (2000, p. 20) results that “...*less well – capitalized banks are at greater risk of failure as are banks of high ratios of loans to assets, evidence of poor quality loan portfolios and) banks with low earnings...banks with enhanced freedoms to branch would afford banks greater diversification and reduce their vulnerability to localized economic shocks*”.

Banks as the other financial institutions exist to enhance economic efficiency. Their functions vary depending on economies. Main roles that banks play in the financial system are: as intermediary, to monitor the savers and investors, to manage risks, induces growth and their corporate governance role. Causes of financial crisis are: inappropriate management of financial/economic liberalization including innovation, immediate increase in asset prices and uncertainties of financial markets caused by interruption of financial institution information or their failure.

Initiating factors causing financial crisis as per Frederic & Stanley (2011) are the fiscal debts, financial institutions balance sheets degradation, decrease of asset prices, increase of interest rates and increased uncertainties leading all in moral hazard problems. What follows is a decrease on economic activity and bank crisis.

Due to the historic background and social – economic transformations occurred the countries of WB structured a bank centered financial system. Banks constitute the major part of their financial sectors so their continuous monitoring, the efficiency and performance evaluation require special attention. Other segments are either nonexistent or underdeveloped. The impact will be insignificant in case of their failure due to their low weight in the total financial system.

As backbone of financial system of economies under consideration banks result to be also main source of liquidity. Any interruption of their main function will cause problems for the real economy. The Western Balkans’ economies are small because of their size but due to liberalization are opened up. The problem with emerged and on developing economies is that they abolish restrictions allowing the capital to flow to other nations without strengthening their supervision, government regulators or improving monitoring of credit and other risks. Government fiscal imbalances are also source of weakening the banking system.

Except GDP and inflation other macroeconomic factors are debt level and exchange rates. Banking specific determinants of efficiency are: except size, liquidity, banks’ profitability, operating expenses, the quality of loans, capital adequacy.

In the regulatory determinants of evaluating efficiency are included: ownership, origin, deregulation, reforms, liberalization etc. Considering the small size of the economies under study, a special attention is paid to ‘the contagion theory’.

According to Allen & Gale (2000b) the inter-bank connections reinforce the stability of the system towards the insolvency.

### 3. Data and Method

The main purpose of this paper is to analyze the Western Balkan countries banking sector efficiency and profitability aiming to present an overview of their performance pre crisis, during and one year of starting recovery.

The analysis are focused in seven countries member of Western Balkans, Albania, Bosnia and Herzegovina, Croatia, Macedonia, FYR, Montenegro, Kosovo and Serbia. The methodology used for performance analysis is that of cross-country comparison while as method are compared the financial ratio indicators for the banking sector of all countries under consideration and the macroeconomic efficiency determinants. The data used are from secondary but reliable resources, Euro stat, IMF databases, GFSR and financial statements or annual reports of countries under study. The data used refer to pre and after crisis of the year 2007, starting from 2000 up to 2014.

The hypothesis of this paper is: being bank based structured economies the efficiency of banking sector is significant for the long run stability of financial system and the economic development of these countries. Efficiency monitoring and accurate assessments are significantly important for identifying inefficiencies, taking measures and improving strategies for faster recovery and better achievements.

## 4. How is Presented the WBCs' Banking Sector Performance in Terms of Macroeconomic and Bank Specific Efficiency Determinants

### 4.1. An Overview on the Macroeconomic Determinants of Efficiency

**Table 1. Real GDP growth rate (% change on previous year)**

Geo/time	Pre Crisis (Average)	During crisis (Average)
	2002-2007	2008-2013
EU (28 countries)	2.37	-0.13
EU (27 countries)	2.37	-0.12
Euro area (changing composition)	1.93	-0.25
Euro area (18 countries)	1.97	-0.28
Croatia	4.78	-1.73
Montenegro	5.38	0.73

Former Yugoslav Republic of Macedonia	3.97	2.05
Serbia	5.08	0.65
Greece	4.17	-4.37
Italy	1.17	-1.48
Spain	3.38	-0.97

Source: Authors' calculations based on data extracted from [Eurostat], E. S. (n.d.). *Print-Europa.*, from Eurostat-Tables, Graphs and maps *Interference (TGM)* updated on November 13, 2014 and *Indicators – The World Bank*, Retrieved respectively on November 15, 2014 from: [http://ec.europa.eu/geninfo/legal\\_notices\\_en.htm](http://ec.europa.eu/geninfo/legal_notices_en.htm), and <http://data.worldbank.org/indicator/>.

Referring to the data presented in the above table the average growth rate calculated as percentage change “during crisis” period is lower than “pre crisis”. The average growth rate of Western Balkans is greater than of European countries member before crisis. The same remains during crisis with the difference that the European countries have decreased average growth.

Referring to the data presented in the table above, Greece results with the lower Gross Domestic Product during the period 2008-2013 and Montenegro results with a positive increase during crisis but still at considerable lower average rates than the pre crisis period.

**Table 2. GDP Growth (annual in %)**

Geo/Time	Pre-Crisis (Average)	During Crisis(Average)
	2000-2006	2007-2013
Albania	5.61	3.71
Bosnia and Herzegovina	5.21	1.57
Croatia	4.44	-0.73
Kosovo	7.38	4.56
Macedonia, FYR	2.53	2.66
Montenegro	3.69	2.66
Serbia	5.1	1.33

Source: Authors' calculations based on data extracted [IBRD-IDA], W. B. (n.d.). *Indicators – The World Bank*, Retrieved November 15, 2014 from: <http://data.worldbank.org/indicator/>

Average annual in percentage GDP growth rate of Western Balkan economies is decreased during the crisis period if compared with the pre-crisis. This is an indicator of economic contraction. Kosovo and Albania have the greater average GDP growth rate. The average inflation calculated on annual basis as percentage of GDP deflator is lower than pre – crisis period except Kosovo where is obvious the increase of 2.3 %.

Although the low inflation rates, the decreased interest rates and the stable exchange rate, the demand for credit results not to be increased.

**Table 3. Inflation, GDP deflator (annual %)**

Geo/Time	Pre – Crisis (Average)	During Crisis (Average)
	2000-2006	2007-2013
Albania	3.71	2.43
Bosnia and Herzegovina	7.06	2.94
Croatia	3.91	2.59
Kosovo	0.73	3.03
Macedonia, FYR	3.73	3.1
Montenegro	8.47	3.94
Serbia	34.5	7.8

*Source: Authors' calculations based on [IBRD-IDA], W. B. (n.d.). Indicators - The World Bank, World Bank national accounts and OECD National Accounts Data Files. Retrieved from: <http://data.worldbank.org/indicator/>, Accessed on November 15, 2014*

Based on data presented in table 4, the average domestic credit provided by the financial sector where banks have the greater potential, is almost doubled if comparing with the pre crisis period except Albania where is recorded a decrease at almost 43% compared with the pre crisis period.

**Table 4. Domestic credit provided by financial sector as (% of GDP)**

Geo/Time	Pre – Crisis (Average)	During Crisis (Average)
	2000-2006	2007-2013
Albania	48.2	27.2
Bosnia and Herzegovina	35.6	65.63
Croatia	55.13	86.4
Kosovo	2.27	15.57
Macedonia, FYR	17.56	44.66
Montenegro	18.22	70.57
Serbia	30.83	50.1

*Source: Authors' calculations based on data extracted from International Monetary Fund (IMF), International Financial Statistics and data files, World Bank and OECD GDP estimates. Catalog Sources World Development Indicators, Retrieved From: <http://data.worldbank.org/indicator/> Accessed on November 15, 2014.*

The highest increase on average domestic credit during crisis belongs to Kosovo followed by Montenegro and Macedonia, FYR.



**4.2. Based on Data Presented are Tracked the Financial Sector size Changes and is also Presented an Evidence of the Progress in pre and during Crisis Period**

**Table 5. WBCs’ financial sector size changes, tendencies and developments calculated as % of GDP**

(as % in GDP)	Pre crisis	Crisis Period		
	Average	Average	Max (Average)	Min (Average)
Domestic Credit to private sector	43.91	49.08	69.88 (Montenegro)	34.91 (Kosovo)
Deposits	54.31	46.74	66.32 (Albania)	39.23 (B&H)
Stock Market Capitalization	85.72	33.98	86.48 (Montenegro)	22.81 (Macedonia, FYR)
Insurance Premiums	1.89	1.56	1.99 (Montenegro)	0.62 (Albania)

*Source: Authors’ calculations based on data extracted from USAID, P. f. (2013, November). Financial Sector Benchmarking Studies Database. Retrieved November 15, 2014, from PFS Program : <http://www.pfsprogram.org/> and Indicators - The World Bank ([IBRD-IDA], 2013). Retrieved from: <http://www.pfsprogram.org/> and <http://data.worldbank.org/indicator>*

Analyzing the main ratio indicators of financial sector depth results that except the small size of the financial sector compared with GDP there is a slight increase in the domestic credit to private sector but still remained less than 50% and all the other are decreased. The average of total deposits during the crisis remained at 46.74% of GDP.

Referring to average ratio of total loans to total deposits for all the countries in consideration (except Croatia) it is at 110.47% with the highest indicator of Montenegro at 163.94% and Albania at 59.24%. This growth of gross loans results from the resident deposits having no investment possibilities and because of borrowing from abroad. Taking into consideration that the banking sector of Albania, B&H, Croatia and Macedonia, FYR is dominated by foreign owned banks than the capital inflow is mainly from their parent banks or borrowed from the domestic big banks and the international financial markets. The average annual growth of domestic credit to private sector (as % of GDP) is 3.23% while the average annual growth of deposits is at 2.21%. Albania has the highest ratio of the average deposits to gross domestic product amounting at 66.32%. The average of the total deposits to GDP for the countries sample is at 46.74%. B&H has the

lowest deposited amounts at 39.23%. The stock market capitalization decreased at almost half of pre crisis period.

The Albania and Kosovo have an invalid capital market so the total average market capitalization refers to B&H, Macedonia FYR, Montenegro and Serbia. The historic background and the transformations occurred to the economies of Western Balkan countries are reflected in their financial structure and the progress of financial sector. As often mentioned in the paper the banking sector is the main pillar of the financial sector of these economies while the capital markets and the insurance companies are of a low potential and their effects in the financial development or failure is almost insignificant. There is also a decline in the average insurance premium to GDP ratio. The lowest indicator refers to Albania at 0.62% and the maximum to Montenegro at 1.99 percent of GDP.

#### 4.3. Overview on the Western Balkan Countries Banking Sector Efficiency Determinants during pre and Crisis Period

**Table 6. Evidence on WBCs' banking sectors efficiency determinants in pre and during crisis**

(in %)	Pre crisis		Crisis Period	
	Average	Average	Max (Average)	Min(Average)
Bank Capital to Assets	13	12.24	21.03 (Serbia)	9.03 (Kosovo)
Bank Regulatory Capital to Risk Weighted Assets	18.9	17.43	20.45 (Serbia)	15.76 (Montenegro)
Bank NPL Loans to Total Loans	5.46*	11.78	8.6** (Serbia)	6.85 (Kosovo)

\*The calculations refer to average data for the year 2007; no data available for Montenegro.

*Source: Authors' calculations based on data extracted from Regional Financial Sector Benchmarking System Report, ([FSBS], 2013) and Indicators - The World Bank ([IBRD-IDA], 2013). Retrieved from: <http://www.pfsprogram.org/> and <http://data.worldbank.org/indicator/>*

Main determinants of banking sector efficiency are capital adequacy, profitability, loan quality, liquidity etc. The banking sector of Western Balkan economies results to have remained stable and well capitalized. This is due the implementation of appropriate policies and the low exposure to countries highly affected by crisis. The average capital to total assets is almost in the same levels, indicating a stability of the industry in overall. The average regulatory capital before crisis was at 18.9

% and remained at almost the same levels during the crisis period. The ratio was at higher than the level required. What is worth to be discussed is the increase of non performing loans. The average ratio is almost doubled during crisis period. Montenegro, Serbia and Albania have the worst performance regarding the quality of loans granted. The average NPL to total loans granted are respectively at 17.47, 15.96 and 14.19%. The data are extracted from World Bank statistics but may be even worse than the provided.

If considering the data presented in both tables and the respective analysis it is obvious than the banking sector is related to finance and the macroeconomic indicators. The crisis period was characterized from decreased growth rates, increased inefficiency of the banking sector and the worsening of financial depth indicators. The increased interest of banks to provide credit without simultaneously improving the monitoring and credit risk assessment strategies, the deterioration of economies and macroeconomic indicators led to over risky lending and the worsening of their quality. The “past due” and “bad loans” are provisioned and their increased provisioning has direct impact in the efficiency, ROA and profitability, ROE. The performance ratio indicators for the banking sector of the seven countries are as below presented.

**Table 7. Return on Assets, in % (ROA)**

Geo/Time	Pre Crisis (Average)	During Crisis (Average)
	2005-2007	2008-2014
Albania	1.47	0.57
Bosnia and Herzegovina	0.83	0.25
Croatia	1.57	0.76
Kosovo	n/a	0.70
Macedonia, FYR	1.60	0.50
Montenegro	0.87	-1.02
Serbia	1.50	0.77

*The Sources are retrieved from: IMF, G. F. ([GFSR], October 2009/2014). Global Financial Stability Report. Washington, USA: International Monetary Fund Retrieved from: <http://data.worldbank.org/indicator/FM.AST.CGOV.ZG.M3/countries>; and Bank of Albania, Financial Stability Report 2014H1, ([FSR], 2014H1) [http://www.bankofalbania.org/web/Financial Stability Report 2014 H1 7138 2.php](http://www.bankofalbania.org/web/Financial%20Stability%20Report%202014%20H1_7138_2.php) (p.75)*

The average Return on Assets Ratio for the seven countries under study is obviously decreased in during crisis period if compared with the pre crisis from 1.3 to 0.36 %. The lowest average efficiency ratio is recorded for Montenegro at – 1.02% while the highest refers to Serbia at 0.77%.

**Table 8. Return on Equity, in % (ROE)**

Geo/Time	Pre Crisis (Average)	During Crisis (Average)
	2005-2007	2008-2014
Albania	20.97	5.76
Bosnia and Herzegovina	7.87	2.23
Croatia	12.90	5.11
Kosovo	n/a	7.37
Macedonia, FYR	11.60	4.49
Montenegro	5.73	-10.18
Serbia	8.97	3.77

The Sources are retrieved from: IMF, G. F. ([GFSR], October 2009/2014). *Global Financial Stability Report*. Washington, USA: International Monetary Fund Retrieved from: <http://data.worldbank.org/indicator/FM.AST.CGOV.ZG.M3/countries>; and Bank of Albania, *Financial Stability Report 2014H1*, ([FSR], 2014H1) [http://www.bankofalbania.org/web/Financial\\_Stability\\_Report\\_2014\\_H1\\_7138\\_2.php](http://www.bankofalbania.org/web/Financial_Stability_Report_2014_H1_7138_2.php) (p.75)

The strengthening of the capital, the increase of reserves, the standardization of regulatory requirements were all helpful measures for avoiding contagion effect between banks. As below presented the average lending and deposit interest rates are decreased during crisis if compared with the pre crisis period due to the increased competition, the low demand because of the economic situation. The lack of supervision from the respective authorities encourages unequal competition and obligates banks to decrease lending prices. The continuous decrease of interest rates was not followed by increased demand for lending and other assets in cash although the exchange rates remained stable. The financial situation of Euro zone, the bank risks due to high non performing loans level are not favoring the increase of crediting.

**Table 9: Lending Interest Rate (in %)**

Geo/time	Pre Crisis (Average)	During Crisis (Average)
	2000-2006	2007-2013
Albania	15.6	12.24
Bosnia and Herzegovina	13.67	7.33
Croatia	11.26	9.97
Kosovo	14.47	13.46
Macedonia, FYR	15.5	9.27
Montenegro	11.2	9.43
Serbia	17.84	12.49

Source: International Monetary Fund (IMF), *International Financial Statistics (IFS) and Data Files, Catalog Sources, World Development Indicators*, Retrieved from:

<http://data.worldbank.org/indicator/FM.AST.CGOV.ZG.M3/countries;> Accessed on November 15, 2014

**Table 10. Deposit interest rate (in %)**

Geo/time	Pre Crisis (Average)	During Crisis (Average)
	2000-2006	2007-2012
Albania	7.11	6.17
Bosnia and Herzegovina	5.92	3.27
Croatia	2.23	2.28
Kosovo	3	3.85
Macedonia, FYR	7.89	5.98
Montenegro	4.93	3.8
Serbia	14.36	8.03

Source: International Monetary Fund (IMF), International Financial Statistics (IFS) and Data Files, Catalog Sources, World Development Indicators, Retrieved from: <http://data.worldbank.org/indicator/FM.AST.CGOV.ZG.M3/countries;> Accessed on November 15, 2014

**Table 11. Interest rate spread (lending – deposit rate in %)**

Geo/time	Pre Crisis (Average)	During Crisis (Average)
	2000-2006	2007-2013
Albania	8.47	6.5
Bosnia and Herzegovina	7.73	4.1
Croatia	9.04	7.8
Kosovo	11.47	9.97
Macedonia, FYR	7.61	3.5
Montenegro	6.1	5.62
Serbia	3.47	8.4

Source: Authors' calculations based on data extracted from Indicators – The World Bank, [IBRD-IDA, 2013], Global Financial Stability Report (IMF), October 200 & /November 2013. Retrieved From: <http://data.worldbank.org/indicator/FM.AST.CGOV.ZG.M3/countries;> Accessed on 15/11/2014

The banking sector of the Western Balkan countries is mainly foreign owned. Austrian banks are a potential presence in the banking industry of Western Balkan countries. The same are Greek, Italian, German and French owned banks. Austria and Greece remain main creditors for Western Balkan economies. Their role in providing liquidity, stability and solvency is significant.

A good sample is the panic caused to clients and depositors of the Greek and Italian banks branches in the countries under study when the parent countries were in economic recession and was widely discussed their emergence from EU.

**Table 12. The total loans to total deposits (in %)**

(in %)	During Crisis (Average) 2007-2013
Albania	59.24
Bosnia and Herzegovina	143.71
Kosovo	77.88
Macedonia, FYR	87.53
Montenegro	163.94
Serbia	130.50

*Source: Authors' calculations based on data extracted from Regional Financial Sector Benchmarking System Report, November 2013, ( [FSBS] Report, November 2013).*

*Retrieved from: <http://www.pfsprogram.org/albania>*

Western Balkan countries are attending to become EU members and so they have relations and dependence from the regional developments. Considering that the banking regulations are tightening, the demand for loans decreases because of the economic situation, the non performing loans increases for the same reasons, and the financial position of parent countries is not positive, the problems and the stress of banking sector increase.

## 5. Conclusions

The banking sector of Western Balkans based on evidences presented above results well capitalized during crisis, higher than the required levels, decreased in Serbia and Albania but increased in Croatia, stabilized in the year 2013. The banking sector efficiency of Western Balkan countries has recorded decrease; the same is presented the profitability and the banking performance in overall. The financial depth of the countries studied results worsened compared with the pre crisis period. The efficiency of the banking sector is related to the financial sector and the macroeconomic indicators which results to have a decreased trend. The non performing loans remain the main risk as they tend to increase. The bank -based structure of the countries under consideration, encourages the faster growth but not in the long run as a characteristic of market based financial structured economies. The financial depth analysis of countries under consideration indicated that their capital and the insurance market are of a low potential in the total financial sector so their impact in the recovery, stability and progress of financial development is insignificant. The central source of liquidity for the real economy remains the banking sector. Western Balkans banking sector is in its major part foreign owned so the economic vulnerabilities of parent countries are easily transmitted in the banking industry of these countries. Their role in the stability, in providing liquidity, in maintaining the paying capacity of the banking sector of the WB

countries is crucial. The Western Balkans banking sector in particular and the financial sector as whole are recovering more slowly than anticipated. Influential factor in their progress remain the euro zone problems.

## 6. Recommendations

Considering all the results from the banking sector efficiency analysis, pre and during crisis period the main problem remains the decreased profit and the performance deterioration. The decreased interest rate spread due to the low demand for credit and the high competition, the increase in provisions due to increased non performing/unpaid loans, requires except the strengthen and standardization of the banking regulations in national and international plan important remains the continuous monitoring of the banking industry and the performance evaluation with accuracy, both will be helpful for inefficiencies identification and risks estimation. Monitoring plus efficiency evaluation are inestimable tools for better management.

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**Abbreviations**

ECB	European Central Bank
EU	European Union
EUROSTAT	European Statistical System
FSBS	Financial Sector Benchmarking System
GDP	Gross Domestic Product
GFRS	Global Financial Stability Report
IBRD	International Bank for Reconstruction and Development
IDA	International Development Association
PPS	Purchasing Power Standards
PFS	Partners for Financial Stability
ROE	Return on Equity
ROA	Return on Assets
WESP	World Economic Situation and Prospects
WB	Western Balkan
WBCs	Western Balkan Countries

## **The Model of Integrated Marketing Communication: Who has the Role to Influence Consumer Behaviour**

**Olimpia Elena Mihaela Oancea<sup>1</sup>**

**Abstract:** The purpose of this paper is proposes a theoretical framework to investigate the models of integrated marketing communication that can influence the consumer behaviour, and the development a model of integrated marketing communication. The research goals aim the following aspects: (a) The analyze of the IMC concept; (b) Identifying and analyzing the main models of integrated marketing communication that can influence the consumer behaviour; (c) Identifying the variables that will be included in the conceptual model of integrated marketing communication proposed. A review of the integrated marketing communication literature show the fact that were developed a series models of integrated marketing communication which has the role to influence the consumer buying behavior, but these not capture the correlation between the following factors: sociological variables, external stimuli, integrated marketing communication and consumer behavior. The method used was the secondary research in order to fulfill the research objectives established. The major result of this paper consists in proposing of a new conceptual model of integrated marketing communication that captures the correlation between external stimuli - sociological variables - integrated marketing communication - consumer behavior.

**Keywords:** integrated marketing communication; model; consumer behavior; sociological variables; external stimuli

**JEL Classification:** M30; M31; M37

### **1. Introduction**

With an increase in global competition, technological advances, and fast informed customers, it is important for businesses to make a powerful impact on target audiences and markets. Integrated marketing communication (IMC) is one of the most important communications trends adopted all over (Sisodia & Telrandhe, 2010). IMC is undoubtedly the major communications development of the last decade of the 20th century (Kitchen & Schultz, 1999, 2000); this despite the fact that most of the history of IMC approaches, theory, and contribution is very recent in nature. More organizations consider IMC to be a key competitive advantage associated with marketing (Kitchen & Schultz, 2001; Weilbacher, 2001).

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According to Nowak & Phelps (1994) the success of an integrated marketing communication campaign could be observed in the positive change of brand image and behavior. The goal is to create a holistic campaign - in addition to synergy - which ameliorates brand image and brand awareness whilst induces behavioral changes.

In practice, the impact of integrated marketing communication activity on consumer behavior is mostly sequential, particularly in situations where the buying process involves a several steps. However, the timing of exposure to this sequential communication represents an aspect still unexplored in the context of resource efficiency. Integrated marketing communication offer the possibility of organizations to communicate with their target audiences through multiple channels such as advertising by mass-media, sales promotion, direct marketing, public relations, online marketing etc. Consumer purchasing behavior can be influenced by the communicational messages transmitted through these channels of communication (Gopalakrishna & Chatterjee, 2006).

**The research problem** consisted in identification those gaps that exist in the relevant literature regarding the models of integrated marketing communication developed with the aim to influence consumer buying behavior.

Taking into account that in the relevant literature, although were developed a series models of integrated marketing communication, these not capture the correlation between sociological variables, external stimuli, integrated marketing communication and the consumer behavior.

## 2. Literature Review

The integrated marketing communication is looked upon as the “voice of the company”, this being a means through which may establish a dialogue with consumers and build the relationships with them (Keller, 2009). In the development of an integrated marketing communications program can be taken into consider a number of factors, such as product availability, stage of the product life cycle, market share of product, the positioning (Schultz, Tannenbaum & Lauterborn, 1993).

The American Association of Advertising Agencies (4As) see Integrated Marketing Communication as “*a concept of marketing communication planning that reorganizes the added value of a comprehensive plan that evaluates the strategies roles of a variety of communication disciplines – general advertising, direct response, sales promotion and public relations and combines these disciplines to provide clearly, consistency and maximum communication impact*” (1989). Schultz, Tannenbaum and Lauterborn (1993) defined IMC as “*concept of*

*marketing communication planning that combine and evaluate strategic role of different communication discipline to get the clarity, consistency and greater impact*". "IMC involves proper control over the information available for the company in order to acquire new customers while retaining the loyal ones (Duncan & Caywood, 1996). "IMC is a strategic business process which designs, develops, executes and evaluates coordinated, measurable, and persuasive marketing communications. The following participants are involved in IMC: consumers, customers, potential customers, other market players, relevant external and internal audience (Schultz & Schultz, 1998). According to Gronstedt (2000): "IMC is a strategic management style, which is conducive to the goals of the company and the brand by focusing resources on every point where a contact with the key-consumer or with other stakeholders happens in order to create a profitable relationship". IMC is "the notion and the practice of aligning symbols, messages, procedures and behaviours in order for an organization to communicate with clarity, consistency and continuity within and across formal organizational boundaries" (Thoger, Fuat & Torp, 2008). IMC can be defined as the interactive and systemic process of cross-functional planning and optimization of messages to stakeholders with the aim of communicating with coherence and transparency to achieve synergies and encourage profitable relationships in the short, medium and long-term (Porcu, Del Barrio-Garcia & Kitchen, 2012).

Over time, the specialists in various fields have developed a number of models that describe the process of communication. Harold D. Lasswell proposed, in 1948, next logical demarche that must occur in a communication process (Lasswell, 1948): a) who communicates? b) what communicate? c) by what means? d) to whom communicates? E) with what effects communicate?

H. Lasswell's formulation is considered to be a classical one, present in all theories of communication, mass communication, sociology, psychology and political science. In H. Lasswell's model, the answer to these questions involves identifying the components of a communication process: communicator - message - channel - receiver - effect. Also, the model includes communication sphere consistent with the role proposed through the distribution of participants: the study of management, the study of communication content, selection of the means of communication, the target audience analysis and analyzing the communication process effects.

This model was improved by the American engineer Claude E. Shannon, becoming a benchmark for many specialists. It includes five elements described in a linear fashion (Shannon, 1948): the source of information - the transmitter - channel - receiver - consignee.

Later, C. Shannon's ideas were interpreted by R. Jakobson. Jakobson's model distinguishes six elements, or factors of communication, that are necessary for

communication to occur: (1) context, (2) addresser (sender), (3) addressee (receiver), (4) contact, (5) common code and (6) message. Each factor is the focal point of a relation, or function, that operates between the message and the factor (Jakobson, 1960).

Tom Duncan and Sandra Moriarty (1998) developed a model of communication based on relational marketing. Using three key points where the two disciplines intersect - messages, stakeholders and interactivity - the authors develop a model based on marketing communication. They demonstrate how interactive can be communication at three levels - corporate, marketing and marketing communications (Duncan & Moriarty, 1998). Because stakeholder relationships are strongly influenced by messages from and to a company, a model built on relationship with a brand should consider the brand messages of all internal sources. The model captures the interaction between the different sources of messages in an organization and its various stakeholders (Tom Duncan & Moriarty, 1998).

James W. Peltier, John A. Schibrowsky and Don E. Schultz (2003) proposed a model which conceptualizes the relationship between database management and interactive integrated marketing communication programs. Using a database-driven CRM program provides the mechanism for firms to create and distribute a customized, interactive, integrated marketing communication program. In their model, database management refers to the process of collecting customer data, integrating the data to form customer segments, and then using other data to build predictive models for categorizing other customers and prospects. The IMC portion of their model relates to the development of targeted, personalized and interactive communication programs based on information learned in the data analysis stage. (Peltier, Schibrowsky & Schultz, 2003)

In the opinion of John F. Tanner Jr. and Mary Anne Raymond the communication process illustrates how messages are sent and received. The source (or sender) encodes, or translates, a message so that it's appropriate for the message channel - say, for a print advertisement, TV commercial, or store display - and shows the benefits a value of the offering. The receiver (customer or consumer) then decodes, or interprets, the message. For effective communication to occur, the receiver must interpret the message as the sender intended. Interference, or noise, can distort marketing messages. Interference includes any distractions receivers and senders face during the transmission of a message. Purchasing a product provides the sender with feedback, which often tells the seller that you saw information and wanted to try the product (Tanner Jr. & Raymond, 2011).

### 3. The Model of Integrated Marketing Communication

Communication model proposed, Figure 1, is a holistic model based on the correlation between a series of factors which can influence in a favorable way consumer buying behavior, thus: (a) sociological variables - the family, reference groups, membership groups, social class, culture and subculture; (b) external stimuli - the price, quality, packaging, brand, emotional value, attributes of the product, offers; (c) integrated marketing communication - communicational message, communication tools, such as: Advertising through mass-media, Sales promotion, Direct marketing, Public relations, The sales representatives, Online marketing.

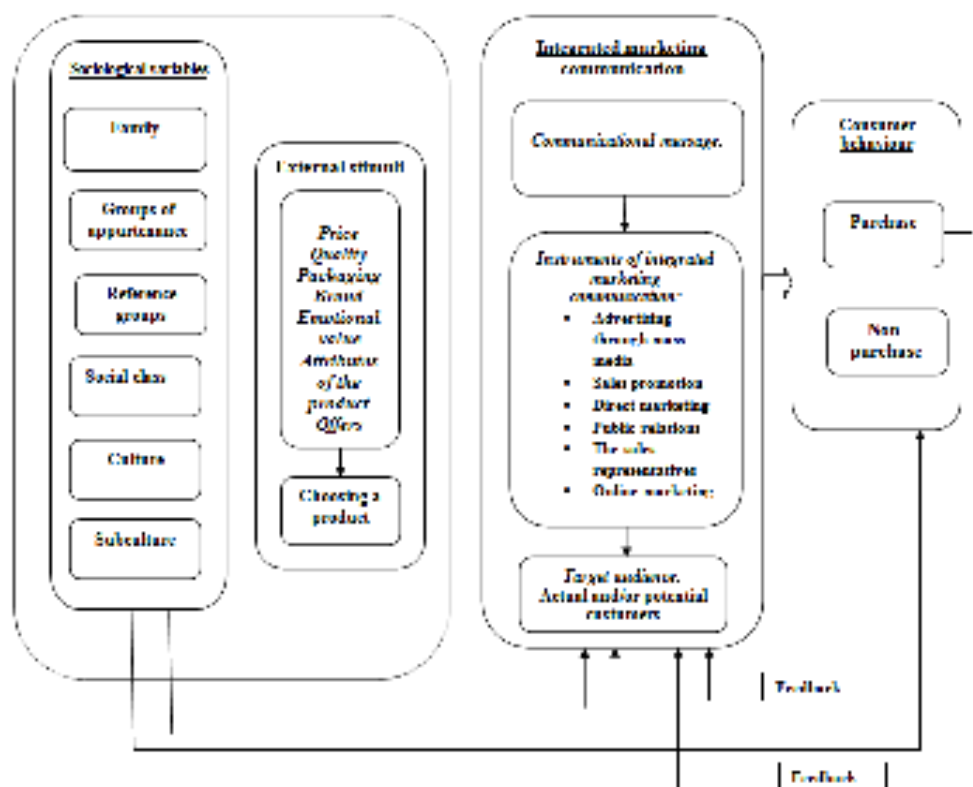


Figure 1. Model of integrated marketing communication

Source: IMC model developed by author

In this model the sociological variables, respectively family, the reference groups, membership groups, the class social, culture and subculture contribute to changing consumer buying behavior. **The family** is composed by two or more people living

together who are related by blood or marriage. It is a part of a household which consists of individuals living singly or together with others in a residential unit. Consumer behavior is influenced by the relationships within families. The family has a strong, most immediate and most pervasive influence on decision-making (Durmaz & Zengin, 2012). Also, system of communication and the interactions between family members, make that this entity have a system of buying and consumption well contoured which must be brought to the attention of researchers in the field of integrated marketing communication (Cătoiu & Teodorescu, 2004).

**The reference groups** can be defined as any person or group of people who significantly influences an individual's behavior (Sakpichaisakul, 2012). The beliefs, values, attitudes, behaviors, and norms of the group are perceived to have relation upon the assessment, behaviors, and aspirations of another individual (Eva & Judit, 2010). Individuals' behavior is strongly influenced through numerous groups. Individuals reference group are those groups that have a direct or indirect influence on the person's attitudes or behavior. Group having a direct influence on a person are called **memberships group**. These are group to which the person belongs and interacts. Some are primary groups. With which there is fairly continuous interaction, such as family, friends, neighbors, and co-workers (Katherine, 2009).

**The social class** refers to grouping of people who are similar in their behavior based upon their economic position in the market place (Engel, Blackwell and Miniard, 1990). Every society stratifies its members into social classes according to their values to the society. The members of social class share common values and ways of thinking, speaking and behaving (Abraham, 2011). Also, knowing the specificities existing between various social categories allows achieving a market segmentation particularly useful for the development of integrated marketing communication programs (Cătoiu & Teodorescu, 2004).

**The culture** is the essential character of a society that distinguishes it from other cultural groups. The underlying elements of every culture are the values, language, myths, customs, rituals, laws, and the artifacts, or products that are transmitted from one generation to the next (Lamb, Hair and Daniel, 2011). Culture is the most fundamental determinant of a person's wants and behavior (Pandey & Dixit, 2011). Courses on culture and behavior of individuals with similar values formed smaller groups are called **sub-culture** (Durmaz, Çelik and Oruç, 2011). The marketer must determine whether the belief, values and customs shared by member of a specific subgroup make them desirable candidates for special marketing attention (Tyagi & Kumar, 2004). Their careful study and consideration of the particularities behavioral, can bring great benefits the activity the integrated marketing communication of an organization (Cătoiu & Teodorescu, 2004).

The external stimuli, respectively the price of a product, the quality, the packaging, brand, the emotional value, attributes of the product, offers can constitute key factors in designing a communicational message, influencing thus the consumer decision to choose and purchase a product. Therefore any organization before you design and implement an integrated marketing communications program should identify and analyze the external stimuli that can be taken into account when designing a communicational message, rational or emotional type, and the sociological factors that contributes significantly to changing in a favorably way to the purchasing behavior of consumers. The tools of integrated marketing communication, respectively advertising through mas-media, sales promotion, direct marketing, public relations, the sales represantives, online marketing represent the means through which the communicational message reaches the target audience which it is addressed, more precisely the potential customers and/or actual of an organization.

In this model, the integrated marketing communication has the role to determine an actual behavior, more precisely the purchase of a product and/or service by the consumers. Where consumers have manifested an purchase behaviour to the product and/or service, the integrated marketing communication model will be maintained or improved, and if this behavior was of the non-purchase, the integrated marketing communication model will be reviewed and modified, so this generates a positive effect on consumer purchasing behavior.

#### **4. Conclusions**

This paper provides a picture of what represent IMC and contribute to a better understanding of the concept. A review of the integrated marketing communication literature show the fact that, over the time, were developed a series models of integrated marketing communication which has the role to influence the consumer buying behavior, but these not capture the correlation between the following factors: sociological variables, external stimuli, integrated marketing communication and consumer behavior.

Also, in the literature seems to be lacking a broad model that includes all these factors, as well as and a specific study regarding the nature and impact of these factors on the consumer behavior. Therefore, the proposed model contributes to the literature in this field, offering a new perspective regarding the marketing communication with actual and prospective clients, in order to determine the actual purchase behavior of their.



## 5. Future Research Directions

**Step 1:** Taking into account the fact that the conceptual model of IMC proposed is very complex, it will be tested empirically on diverse consumer groups, for to identify those variables sociological, external stimuli and integrated marketing communication tools that influence in a greater measure the consumer buying behavior. Also, will be empirically tested the relationships between variables proposed in the model, and its validation.

**Step 2:** The conceptual model of IMC will be improved and it will include only those sociological variables, external stimuli and tools of integrated marketing communication which influencing in a greater measure the consumer buying behavior.

**Step 3:** Will be empirically tested the relationships between variables that will be contained in the new model of IMC, and it will be validated.

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## An Examination and Analysis of Employees' Job Stress in Gas Refinery Company of Ilam

Kobra Roshani<sup>1</sup>, Seidmehdi Veiseh<sup>2</sup>, Ardeshir Shiri<sup>3</sup>  
Kolsoom Roshani<sup>4</sup>

**Abstract:** The aim of the study is to investigate and analyze employees' job stress and also to find out the drivers for job stress, which contribute to create job stress which ultimately reduce efficiency. The study is mainly based on review of the existing literature and collection of data through an adopted questionnaire survey, conducted from the selected sample of employees working in Ilam gas Refinery Company. 140 employees were selected among 230 subjects of population using simple sample way, which are suitable for analysis purpose respecting Kerjcie and Morgan table. In order to examining significance of research questions, t test and explorative factor analysis was applied. The results of research show that personal, group and out-organizational factors have effect on job stress, but outer-organizational factors had no significant effect on employees' job stress and also, the level of employees' stress is medium. The implications of the paper include implementation of the results provided by researcher to decrease the employees' level of stress. The study is conducted first time in the field specifically highlighting the stress factor. It can be a base for the future research in this area.

**Keywords:** Job stress; Job stressor factors; Gas Refinery Company of Ilam province

**JEL Classification:** M1

### 1. Introduction

With gradual inclination of societies towards modern living, one of main concerns during recent decades is stress. Along with industrialization, this phenomenon has gained special importance and has influenced widely and increasingly on employees' health.

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During recent decades, job transition has caused some complications in the area of job, human relations in working environments. Job stress is one of most important outcomes of this phenomenon (Tarshizi, Saadatjou, 2012, P 201).

The main issue of present study is that organizations have no access to lots of information about job stressor factors. This can be true in population of the study; hence researcher is seeking for factors causing job stress and protective skills in employees.

Since increase of efficiency, achieving to high indicators of effectiveness, improving technology and innovation in technology, rising level of competitiveness, respecting human dignity and considering human resources are considered as the main assets of organization. Realizing these objectives requires removing some obstacles, which make achieving to it difficult. One of these obstacles is not to pay attention to stresses and job stresses employees are facing with. Offering some solutions to reduce stressor factors may contribute to reaching for predetermined objectives of the organization (Jazani et al, 2010, p. 129).

Experts and human forces of Gas Refinery Company of Ilam that mostly have high level of acceptable knowledge and capabilities and due to their job nature, must offer desirable job quality and quantity, and in case of facing to job stressor factors, they must be able to overcome by their organization's management. On the other hand, through recognizing these factors, employees must be informed about their professional problems and harmful sources and apply the way of protecting against its unfavorable effects. Due to scientific gap in the field of effect of job stress on employees within workshop environments and companies of gas refinery and also because of legal and application necessity, examination and analysis of employees' job stresses in gas Refinery Company seems inevitable.

Remaining matters of present study are as follows:

First theoretical consideration is addresses, then a review of experimental literature on job stress if offered. Finally in order to finding answer to the research questions, hypotheses are tested, results are examined, and discussion and conclusions are offered.

## **2. Brief Review of Theoretical Consideration on Job Stress**

Fontana (1989) considers the root of the word stress in ancient word of Destress, which means to get in trouble and hard mental and physical situations. This word in middle English is written as "Distress", which after omitting "Di" is transformed into stress (Ranjbar, 2011, p. 24).

Hans Seyle states psychological stress in terms of physical responses in relating to any needs. During this condition, two different and opposite psychological stress is appeared.

1. Eustress: it is helpful or good stress related to happy feeling, exhilaration, and accomplishment (Rowland Croucher, 2002) and stimulate the person for performance and making better opportunities and challenges (Sime, 2002).
2. Distress: it is unhelpful, which is long-term. It causes many psychological stresses and results from high needs and unfavorable tasks (Alvani, 2010, p 295).

Job stress: emotional and physical responses that occurs due to lack of correlation of obligations and job demands with individual's accessible resources and capabilities (Del Valle & Bravo, 2007, p 610)

Job stressors: it is mutual interactions between working situations and personal characteristics, so that it is working environment requirement sand subsequently related stresses, with which employee can cope (Khajeh pour, 1998). In addition, lack of correlation between job requirements with capabilities, resources, or employees' needs is called job stressors (institute of well-being and job safety, 2002), (Ranjbar, 2011, p 21).

Operational definition

Job stress: stress one of states occurring for most of people and it differs for different people during different situations and times. The response, which the respondr makes in gas Refinery Company of Ilam to questions regarding stress within questionnaire, can offer a definition of stress operationally.

## **2.1. Mental Stress Factors**

There are different factors causing stress related to environmental and cultural conditions of organization, some of which are common within most of organization and some others are different. Stressors or mental stress factors are derived from the person, from the group, from inside the organization and out of it (Boroumand, 2001, p 197).

### **2.1.1. Personal factors**

1. Interpersonal conflict

Conflict occurs when two or more conflicted stimulus or tensions compete for appearing (Parsa, 1991, p 235).

2. Role ambiguity

It is a certain job situation in which some required information for implementing job favorably are inadequate or misleading, hence employee does not know how he is expected about his job (Alavi, 1993, p 62).

### 3. Overloaded role

Overloaded role means that if the person is not able to accomplish a task as a part of a certain job, he would get stressed.

### 4. Low role

Low role means a situation in which individual's skills are not used perfectly and completely. Stress resulted from this situation is called low role.

### 5. Role incompatibility

Incompatibility occurs when embracing a body of job obligations is inconsistent or impossible with approving other body of job regulations (Filippo, 2002, p 19).

## **2.1.2. Group Stressors**

### 1. Lack of group integrity and solidarity

The most important factor in proceeding organization and establishing peace among employees is the sense of organizational membership and finally forming an organizational identity for individuals, so that they would consider failure of organization as their own failure and its success as their own success.

### 2. Lack of social support

Human is a societal creature, whose performance is so influenced by others' support. When his performance is not supported by the group due to any reason, the individual gets lost and deprived from the reference evaluating his performance, which is considered as his source of support.

### 3. Intergroup conflict

In some cases, it is possible to occur some conflicts among different units within an organization, so that it may consume all of person's ability and energy. Due to any reason, such conflicts may lead to stress and anxiety with individuals and groups (Baratvand, 2004, p 22).

## **2.1.3. Organizational factors**

### 1. Organizational structure

Organizational structure indicates official relations between different positions. Organizational structure makes clear where obligations and regulations were determined. If the way of legislation is not determined in organizational structure, it may lead to mental stress.

## 2. Organizational atmosphere

By implementing various leadership styles, which subsequently may lead to some practices such as dismissal, separation, and layoff, managers can make organizational atmosphere stressful and offer fears, and concerns for them within the workplace (Sinaee, 2002, p 69).

### **2.1.4. Environmental factors\_ External factors**

Various factors external to the organization cause stress for individuals within the organization, some of which are mentioned below:

1. Social changes: since nowadays people are entrapped in a kind of life full of running, hastening, urbanization, overcrowding, and quick transforming, their welfare is diminished and potential possibilities causing stress within workplace is increased (Lotanz, 2004, p 398).
2. Economic changes: In societies, in which people are forced to have second job to afford their life's needs. This reduces relax and recreation time and accomplishing family duties and rises employees' stress.
3. Family environment: family conditions including a trivial crisis like family conflict or illness of family members or unfriendly relations to spouse or kids for a long time may play a vital role as a significant stressor for employee (Karami, 2011, p 39).
4. Political instability: changes within stable political systems are regular and based on systematic process, which have no significant social effects, subsequently may not lead to mental stress, while within instable countries, these changes cause mental stresses.
5. Technology: Today's advanced equipment such as computer and automatic machines cause stress and anxiety for people who are no so familiar with them, hence it lead to mental stress (Parsaeeyan, A'arabi, 1998, p 116).

## **2.2. The Level of Mental Stress of Life Events**

The effect of various events and the level of their stressfulness differ. During individual's life, some events may occur causing mental stress. A score is attributed to any event, which is compatible to the importance of the event indicating the level of individual's stress. Greater scores show higher mental stress and demand more adaptive behaviors. Underlying logic of this indicator is that accumulating change events during lifetime reduces body's resistance and endangers individual's well-being. Studies show that people who their total score of mental stress is less than 150 per year, they would have better general well-being during next year, while a total score of 150-300 may lead to 50% catching to acute illness during next year and this group of people are listed within precarious mental illness. If



their total score is greater than 300, with probability of 70%, they catch chronic illness during next year (Shiri, 2012, p 135).

### **3. A Review of Experimental Studies**

Lee and Shin (2010) in a study examined the relation between job stress and job satisfaction. They classified stressor factors into three physical, environmental, and psychological categories and examined the effect of these factors on job satisfaction through data mining techniques. Results of their study confirmed a negative significant effect of job stress on job satisfaction.

Catalina (2011) in a study entitled “job stress and organizational commitment in Romania's state organizations” concluded that high level of stress lead to reducing organizational commitment and subsequently causes low overall efficiency of organization.

Shariati et al (2011) examined effective factors on job stress of educates working in Tehran police stations. According to results of this study, there is no relation between personal characteristics of employees and environmental resources and employees' job stress; however there was a significant relation between education, role characteristics, and organizational structure.

Enayati et al (2012) in a research examined the relation of organizational stressor sources with employees' job stress in Gas Company of Mazandaran. Findings showed that there was a significant relation between eight factors causing stress, six factors of organizational expectation, communications, co-workers' support, role characteristics, authorities supporting, work changes and employees' stress. Moreover, there is no significant relation between two factors of authority and physical environment (external factors) and stress.

Kamali Ardakani et al (2013) in a study examined job stress and effective factors upon it for medical students. Findings of their research showed that the level of stress for older individuals is more than who are younger. The stress score is greater for individuals who smoke and also for who get tranquilizer medications and all of mentioned results are statistically significant.

### **4. Methodology**

In terms of research purpose, present study is an applied research, and in terms of nature and research method, it is regarded as a correlative descriptive study. In order to data analyzing, inferential, descriptive statistics was applied. In the part of descriptive statistics, by applying some statistical features such as frequency,

frequency percentage, mean, Standard deviation, minimum and maximum, the sample of study was examined. In the part of inferential statistics, in order to determine normality of data, Kolmogorov – Smirnov test was applied, then for answering research questions, unilateral T tests and explorative factor analysis were used.

#### **4.1. Population of Study and Sampling Method**

Population of study consists of all of 230 employees working in Ilam Gas Refinery Company and in order to choosing research sample, random sampling method based on Morgan table was applied and 140 questionnaires were distributed.

#### **4.2. Data Collecting Tools**

Research questionnaire consisted of three sets of questions. The first set was determined to signifying attributes of sample in terms of gender, age, marital status, education, service experience. The second set of questions were considered to testing 4 first questions of research and third set of questions were intended to testing the last question of research based on an experienced way as follows;

1. Personal characteristic, through which data relating gender, age, education, marital status, and service experience was collected.
2. Job stress factors include questions relating to personal, group, organizational, and external factors.
3. Events occurred during last year to employees.

#### **Research questions**

1. Do personal factors cause job stress to employees of Ilam gas Refinery Company?
2. Do group factors cause job stress to employees of Ilam gas Refinery Company?
3. Do organizational factors job stress to employees of Ilam gas Refinery Company?
4. Do external factors job stress to employees of Ilam gas Refinery Company?
5. What is the level of job stress for employees of Ilam gas Refinery Company?

In this research, items 1-10 of questionnaire answer to first question of study, items 11-16 answer to second question of study, items 17-37 provide answer to third question of study, items 38-46 answer to fourth question within Likert's scale. Items of third set of questionnaire form 1 to 28 provide answer to the last question of study.

### **4.3. Validity and Reliability of Questionnaire**

In order to reaching face validity during present study, a primary questionnaire was provided and was corrected based of experts' opinion and finalized. Before final setting of questionnaire and t evaluate its validity and reliability, it was tested upon 30 subjects within population of study. Calculated alpha for personal, group, inter-organizational, and external factors was respectively, 0.75, 0.77, 0.81, and 0.80 indicating its suitable reliability.

## **5. Research Findings**

### **5.1. Findings of Descriptive Statistics**

among all of statistical population of employees in Ilam refinery company, approximately 13.6 % was at age 30 and below, 62.9% was 30-35, 17,1&% was 35-40,4.3% was 40-45, and 2.1% was over45.the highest percent was devoted to 30 -35. There was 3.6% high school diploma, 14.3% was associate degree, 57.9% was bachelor, 24.3% was master degree and higher. The highest degree was devoted to bachelor degree with approximately 57.9%. About 41.4% of sample was usual staff, 35.7 % was expert,16.4% was responsible, and 6/4% was boss with the highest percent of 41.4 for usual staff. About 15% of respondents had 5 years or less of work experience, 50.7%had 5-10 yrs., 27.9% had 10-15yrs, 3.6% had 15-20yrs, and 2.9% had over 20 years of work experience. 87.1% of respondents stated that no events occurred to them, 10.7% stated that between 1 to 3 years has had no event, and 2.1% stated that no events occurred to them since more than 3 years.

### **5.2. Findings of Inferential Statistics**

In order to examine normality of data distribution, Kolmogorov - Smirnov test was applied to making sure about normality of data. Obtained results from this test indicate that data distribution is normal. Here, for finding answers to the research questions, the questions are raised and required tests are done.

#### **5.2.1. Personal Factors**

In order to finding personal factors affecting on employees' mental stress, one-sample T test and factor analysis were applied. (See appendix A)

According to obtained results, among personal factors, hesitation,, expectation, accomplishment, capability, enough time (opportunities), skill, and ability, have the mean of more than moderate in Likert's scale (3) that indicates that these factors are effective on employees' job stress and other factors (decision, mismatching, and inconsistency), whose mean is less than moderate level in Likert scale(3) has no

significant effect on employees' mental stress. Moreover, respecting statistic of T test for personal factors indicated that factors of hesitation, expectation, accomplishment, capability, enough time (opportunities), skill, and ability have higher t calculated than t of table in significance level of 0.05 and they have upper and lower limits of confidence interval greater than zero (positive), then claim of this hypothesis for these factors is confirmed. Therefore, with confidence interval of 95% it can be stated that these factors have significant effect on employees' job stress and by respecting calculated t for other factors; factors of decision, mismatching, inconsistency have no significant effect on employees' mental stress.

First factor analysis was examined on research samples using Bartlett test sampling adequacy index (KMO).

The value of KMO statistic equals 0.767, which indicates that factor analysis is approved and its results can be generalized to the population. Moreover, value of Bartlett's test is significant at level of 0.01; hence the conditions for accomplishing explorative factor analysis are obtained.

Among ten questions relating to personal factors of job stress, seven variables (hesitation, expectation, accomplishment, capability, enough time (opportunities), skill, and ability), whose mean is greater than 3 were considered as main variables. Varimax rotation method was applied to realize substructure factors of variables and also for determining its simple structure.

According to results, questions 6, 7,8,4,10,2,5 with factor one are so correlated. These seven questions indicate the level of role's expectation and knowledge; hence the term role's ambiguity is appropriate for them.

### **5.2.2. Group Factors**

For finding group factors affecting on employees' mental stress, one sample T test and factor analysis were applied. (See appendix B)

According to obtained results, among group factors, intimacy, job problem, responsible, and respect, have the mean of greater than moderate in Likert's scale (3) that indicates that these factors are effective on employees' job stress and other factors (tension, combining), whose mean is smaller than moderate level in Likert scale (3) has no significant effect on employees' mental stress. Moreover, respecting statistic of T test for group factors indicated that factors of intimacy, job problem, responsible, capability, and respect have higher t calculated than t of table in significance level of 0.05 and they have upper and lower limits of confidence interval greater than zero (positive), then claim of this hypothesis for these factors is confirmed. Therefore, with confidence interval of 95% it can be said that these factors have significant effect on employees' job stress and by respecting calculated

t for other factors; tension and combining have no significant effect on employees' mental stress.

First factor analysis was examined on research samples using Bartlett test sampling adequacy index (KMO).

Among six questions relating to group factors of job stress, 4 questions (intimacy, job problem, responsible, respect), whose mean is greater than 3 were considered as main variables. Varimax rotation method was applied to realize substructure factors of variables and also for determining its simple structure.

According to results, these 4 questions are placed in the same factor that shows that all of variable of group factors such as lack of integrity, lack of support, and contradiction are considered as group factors of mental stress based on one main factor.

### **5.2.3. Organizational Factors**

In order to find organizational factors affecting employees' mental stress, one-sample T test and factor analysis were applied. (See appendix C)

According to obtained results, among organizational factors, vertical surfaces, job title, educational degree, job description, getting guidelines, follow guidelines, cooperation, partnership, freedom, coordination, encouragement, disagreement, working interest, interrelationship, toxics, and appropriate means have the mean of greater than moderate in Likert's scale (3) that indicates that these factors are effective on employees' job stress and other factors (independent, authority, regulations, noise, and lighting), whose mean is smaller than moderate level in Likert scale (3) has no significant effect on employees' mental stress. Moreover, respecting statistic of T test for group factors indicated that factors of vertical surfaces, job titles, educational degree, job description, getting guidelines, follow guidelines, cooperation, partnership, freedom, coordination, encouragement, disagreement, working interest, interrelationships, toxics, and appropriate mean shave higher t calculated than t of table in significance level of 0.05 and they have upper and lower limits of confidence interval greater than zero (positive), then claim of this hypothesis for these factors is confirmed. Therefore, with confidence interval of 95% it can be said that these factors have significant effect on employees' job stress and by respecting calculated t for other factors; decision and inconsistency have no significant effect on employees' mental stress.

First factor analysis was examined on research samples using Bartlett test sampling adequacy index (KMO).

Among 21 questions relating to organizational factors of job stress, 16 questions whose mean is greater than 3 were considered as main variables. Varimax rotation

method was applied to realize substructure factors of variables and also for determining its simple structure.

Respecting the results, it can be said that:

1. Questions 32, 31, 28, 33, 37, 29, 37, 27 are so correlated with factor 1. These 8 questions show organizational culture and environment, therefore the term organizational atmosphere is appropriate to them.
2. Questions 18, 23, 20, 21 have significant correlation to factor 2. Respecting this question indicates task description and guidelines; hence the term officialism is appropriate to them.
3. Questions 36, 24, 25 are so correlated with factor 3. These 3 questions show authority and independence, hence the term centralization is appropriate to them. Questions 24, 25 are the base of centralization, and question 36 indicating unsafe conditions causes decrease of centralization.
4. Questions 17 and 19 are so correlated with factor 4. These questions show organizational levels; hence the term complicacy is appropriate to them.

#### **5.2.4. External Factors**

In order to find external factors affecting on employees mental stress, one-sample T test and factor analysis were applied. (See appendix C)

According to obtained results, among external factors, relatives death and financial status have the mean of greater than moderate in Likert's scale (3) that indicates that these factors are effective on employees' job stress and other factors (education, recreation, residence place, holding party, new member of family, and technology), whose mean is smaller than moderate level in Likert scale (3) has no significant effect on employees' mental stress. Moreover, respecting statistic of T test for external factors indicated that factors of relatives' death and financial status have higher t calculated than t of table in significance level of 0.05 and they have upper and lower limits of confidence interval greater than zero (positive), then claim of this hypothesis for these factors is confirmed. Therefore, with confidence interval of 95% it can be said that these factors have significant effect on employees' job stress and by respecting calculated t for other factors; education place, recreation, political activity, residence place, holding party, new member of family, and technology have no significant effect on employees' mental stress.

First factor analysis was examined on research samples using Bartlett test sampling adequacy index (KMO).

Among 9 questions relating to external factors of job stress, 2 questions whose mean is greater than 3 were considered as main variables. Varimax rotation method was applied to realize substructure factors of variables and also for determining its simple structure.

According to results, these 2 questions are placed in the same factor that shows that factors of family environment and economic changes contradiction are considered as substructure variable of external factors.

#### **5.2.5. The Level of Job Stress**

While the values greater than 300 indicate high job stress of employees and the values smaller than 150 indicate low job stress, finding the level of employees' job stress is done as follows;

Hypotheses H0 and H1 are defined as follows;

Null hypothesis: Low job stress of employees. H0:  $\mu \leq 150$

Alternative hypothesis: Not low job stress of employees: H1:  $\mu > 150$

In order to testing this hypothesis, observed mean and theoretical mean of measurement index were compared using parametric test for comparing one-sample mean. (See appendix E)

As shown, SD equals 59.201 and the mean is 169.25 that is greater than 150. It means that employees have low mental stress. Obtained statistic of t is 3.84 with significance level of 0.00 is greater than t of table. Therefore, H0 is rejected and alternative is accepted. Also it can be concluded by respecting statistical analysis; employees have not low mental stress.

Then in order to find out whether the level of employees' mental stress is high, hypotheses H0 and H1 are defined as follows;

Hypotheses H0 and H1 are defined as follows;

Null hypothesis: not high job stress of employees. H0:  $\mu \leq 300$

Alternative hypothesis: \ high job stress of employees: H1:  $\mu > 300$

In order to testing this hypothesis, observed mean and theoretical mean of measurement index were compared using parametric test for comparing one-sample mean. (See appendix F)

As shown, SD equals 59.201 and the mean is 169.25, which is greater than 300. It means that employees have not high mental stress. Being negative, the value of statistic T is  $-26.13$ , and mean of employees' mental stress smaller than 300. Therefore, there is no reason to reject null hypothesis and it can be said that according to statistical analysis, the hypothesis of high mental stress of employees and managers is rejected.

Considering two mentioned hypotheses and their tests, it can be concluded employees' mental stress is between 150 and 300 that means employees will be faced to moderate mental stress during next year.

## 6. Discussion and Conclusions

During present study, first demographic data relating to considered sample, frequency indices, and variables scattering were explained and in the inferential part through applying research indices and SPSS and one-sample T tests, research questions were analyzed. Then in order to determine important factors in making job stress after experiencing, factor analysis was done and required explanations were accomplished.

Respecting results of factor analysis, only one substructure factor of role ambiguity (the level of knowledge and role expectations) have effective role in job stress. Results of present study are consistent with findings of Muhamadkhani (2007) and Rezaeeyan (2012).

All of variables in group factors such as lack of integrity, lack of support, and inconsistency are considered as group factors of mental stress based on a main factor. Results of this part are consistent with results of Rezaeeyan (2012).

Factors of organizational atmosphere (organizational culture and environment), officialism (task description and guidelines), centralization (authority and independence), and complicacy (organizational levels) have effective role on job stress. Results of present study are consistent with findings of Sinaee (2004).

According to obtained results, it was observed that hypothesis of low and high job stress has been rejected and employees' job stress during next year would be between 150 and 300 (moderate). Results of this part are consistent with findings of Giurian et al (2010) and Kabirzadeh et al (2008).

- Employees under 30 years old are more facing to low mental stress and would have good well-being during next month. Employees with 30 years and over are more facing to moderate mental stress and would develop severe diseases with probability of more than 50%. Considering these results, the company must pay more attention to employees over 30. The results of this part are consistent with findings of Kamaliardakani et al (2013), however they have no consistency to findings of Rezaeeyan (2012).
- Difference in educational degree was no effective on job stress, so that in all of educational levels, employees are more facing to moderate mental stress and by changing in the level of education, there was no change in frequency distribution.



The results of this part have no consistent to findings of Givarian et al (2010) and Rezaeeyan (2012).

- Single individuals are more facing to low mental stress and married one are more facing to moderate mental stress. With probability of more than 50%, they would develop severe diseases. The company must pay more attention to married individuals. The results of this part are consistent to findings of Al-Omar (2003), but opposed to Rezaeeyan findings (2012).
- Employees at level of usual staff are more facing to low mental stress, but employees in other organizational levels are more facing to moderate mental stress. With probability of more than 50%, they would develop severe diseases during next year. Results of this part are consistent to findings of Rezaeeyan (2012).
- Employees with fewer than 5 years of work experience are more facing to low mental stress, but those with more than 5 years are more facing to moderate mental stress. With probability of 50% they would severe diseases during next year. Results of this part are consistent with findings of Rezaeeyan (2012). However they have no consistency to Gvorian et al (2010).
- Employees at all of event levels are more facing to moderate mental stress and with probability of 50% they would severe diseases during next year.

## **7. Suggestions Based on Research's Findings**

1. Conflicts resulted from lack of balance between professional life and family life causes mental stress. Therefore, it is suggested to managers to provide flexible agenda and various advantages of working for establishing a kind of balance between work life and family.
2. Role ambiguity is accompanied with increase of stress. Most of employees are trained for certain tasks, but some other tasks and activities may be referred to them, which are inconsistent to their expectations and job description, for example some tasks may be delegated from senior managers to employees that are contrary to the standards and regulations of organization and in case of resist it may lead to dissatisfaction of senior managers. Moreover, it is possible to refer some task to employees, which are beyond their skills and trainings. Here, it is probable that the tasks are accomplished with low quality and lead to employee's stress. Therefore, it is suggested that employees get informed of guidelines and regulations through studying and participating in training courses on the arrival to Gas Company.
3. Since task description and guidelines can be considered as one of factors making stress for employees, it is probable that some of regulations are incomplete and not appropriate to current conditions of company, hence managers who are responsible and have authority must review and determine these regulations according to

culture, equipment, needs, and capability of society and reduce current stresses through reasonable adjustments and making them more flexible.

4. Applying personality tests to get information from personality dimensions appropriate to professional activities is suggested, based on which personality dimensions every individual is served according to his profession and the level of stress.

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### Appendix A

Hypotheses H0 and H1 are defined as follows;

Null hypothesis: personal factors cause no mental stress for employees.  $H_0: \mu \leq 3$

Alternative hypothesis: personal factors cause mental stress for employees.  $H_1: \mu > 3$

If statistic of calculated T test is more than the value of T in the table with significance level of 0.05 (1.645), null hypothesis is rejected indicating ineffectiveness of factors on job stress.

**Results of T test, mean, and SD**

Personalfactors questions		Value of test-3						
		Mean	SD	T	Freedom degree	Significance level	Confidence limits	
							High	Low
1	Decision	2/14	0/69	-14/59	139	0/000	-0/74	-0/97
2	Hesitation	3/22	0/94	2/77	139	0/006	0/38	0/06
3	Mismatch	2/81	0/82	-2/65	139	0/009	-0/05	-0/32
4	Expectation	3/96	0/84	13/50	139	0/000	1/10	0/82
5	Accomplishment	4/2	0/68	20/86	139	0/000	1/31	1/09
6	Capability	3/66	0/84	9/3	139	0/000	0/81	0/52
7	Enough time	3/61	0/94	7/72	139	0/000	0/77	0/46
8	Skill	3/41	0/75	6/35	139	0/000	0/53	0/28
9	Inconsistency	2/4	0/91	-7/78	139	0/000	-0/45	-0/75
10	Ability	4/28	0/6	25/19	139	0/000	1/38	1/18
Mean		3/37	0/44	9/833	139	0/000	0/44	0/295

**Values of KMO and Bartlett**

Sampling adequacy index (KMO)	Bartlett's test		
	Chi-square statistic	Freedom degree	Significance level
0.767	130.09	21	0.000

**Factor loads**

Factors	1
5	0.66
2	0.66
10	0.64
4	0.59
8	0.59
7	0.53
6	0.48

**Appendix B**

Hypotheses H0 and H1 are defined as follows;

Null hypothesis: group factors cause no mental stress on employees. H0:  $\mu \leq 3$

Alternative hypothesis: group factors cause mental stress on employees. H1:  $\mu > 3$

If statistic of calculated T test is greater than the value of T of table with significance level of 0.05 (1.645), null hypothesis is rejected indicating ineffectiveness of factors on job stress.

**Results of T test, mean and SD**

Group factors questions		Value of test-3						
		Mean	SD	T	Freedom degree	Significance level	Confidence limits	
							High	Low
11	Tensions	2/61	0/82	-5/67	139	0/000	-0/26	-0/53
12	Intimacy	3/76	0/81	11/16	139	0/000	0/9	0/63
13	Job problem	3/62	0/8	9/09	139	0/000	0/76	0/49
14	Combining	2/8	0/78	-3/00	139	0/003	0/07	-0/33
15	Responsible	3/54	0/94	6/74	139	0/000	0/69	0/38
16	Respect	4/19	0/69	20/21	139	0/000	1/31	1/08
Mean		3/42	0/56	8/74	139	0/000	0/515	0/325

**Values of KMO and Bartlett**

Sampling adequacy index (KMO)	Bartlett's test		
	Chi-square statistic	Freedom degree	Significance level
0.716	133.04	6	0.000

**Factor loads**

Factors	1
16	0.86
12	0.76
13	0.65
15	0.60

### Appendix C

Hypotheses H0 and H1 are defined as follows;

Null hypothesis: organizational factors cause no mental stress on employees.

H0:  $\mu \leq 3$

Alternative hypothesis: organizational factors cause mental stress on employees.

H1:  $\mu > 3$

If statistic of calculated T test is more than the value of T in the table with significance level of 0.05 (1.645), null hypothesis is rejected indicating ineffectiveness of factors on job stress.

#### Results of T test, mean and SD

Organizational factors questions		Value of test-3						
		Mean	SD	T	Freedom degree	Significance level	Confidence limits	
							High	Low
17	Vertical surfaces	3/21	1	2/45	139	0/000	0/37	0/04
18	Job titles	3/58	1/15	5/94	139	0/000	0/77	0/39
19	Educational degree	3/51	0/98	6/08	139	0/000	0/67	0/34
20	Job description	3/79	0/94	9/86	139	0/000	0/94	0/63
21	Get guidelines	3/65	0/96	7/95	139	0/000	0/81	0/49
22	Independent	2/63	1/06	-4/13	139	0/000	-0/19	-0/55
23	Follow guidelines	4/25	0/84	17/41	139	0/000	1/39	1/11
24	Cooperation	3/63	0/85	8/73	139	0/000	0/77	0/49
25	Partnership	3/47	1/05	5/28	139	0/000	0/65	0/3
26	Authority	2/49	1/15	-5/2	139	0/000	-0/31	-0/7
27	Freedom	3/5	0/97	6/04	139	0/000	0/66	0/34
28	Coordination	3/63	0/93	7/91	139	0/000	0/79	0/47
29	Encouragement	3/37	1/1	3/96	139	0/000	0/56	0/19
30	Regulations	2/87	0/96	-1/57	139	0/118	0/03	-0/29
31	Disagreement	3/29	1/07	3/14	139	0/002	0/47	0/11
32	Working interest	3/75	1/16	7/62	139	0/000	0/94	0/56
33	Interrelationships	3/76	0/91	9/81	139	0/000	0/91	0/6
34	Noise	2/86	1/03	-1/55	139	0/122	0/04	-0/31
35	Lighting	2/62	1/11	-4/01	139	0/000	-0/19	-0/56
36	Toxics	3/31	1/26	2/86	139	0/005	0/52	0/1
37	Appropriate means	3/62	1/04	7/05	139	0/000	0/8	0/54
Mean		3/37	0/45	9/72	139	0/000	0/445	0/295

#### Values of KMO and Bartlett

Sampling adequacy index (KMO)	Bartlett's test		
	Chi-square statistic	Freedom degree	Significance level
0.839	744.146	120	0.000

Rotated factor loads

Factors	1	2	3	4
32	0/76			
31	0/72			
28	0/72			
33	0/71			
29	0/68			
37	0/65			
37	0/65			
18		0/73		
23		0/72		
20		0/57		
21		0/53		
24			0/68	
36			-0/62	
25			0/55	
17				0/72
19				0/48

Appendix D

Hypotheses H0 and H1 are defined as follows;

Null hypothesis: External factors cause no mental stress on employees.  $H_0: \mu \leq 3$

Alternative hypothesis: External factors cause mental stress on employees.  $H_1: \mu > 3$

If statistic of calculated T test is more than the value of T in the table with significance level of 0.05 (1.645), null hypothesis is rejected indicating ineffectiveness of factors on job stress.

Results of T test, mean and SD

Organizational factors questions		Value of test-3						
		Mean	SD	T	Freedom degree	Significance level	Confidence limits	
							High	Low
38	Education place	2/79	1/06	-2/38	139	0/019	-0/04	-0/39
39	Recreation	4/42	1/04	-6/54	139	0/000	-0/4	-0/75
40	Political activity	2/43	0/99	-6/77	139	0/000	-0/4	-0/74
41	Residence place	2/39	1/13	-6/40	139	0/000	-0/42	-0/8
42	Holding party	2/98	1/01	-0/25	139	0/803	0/15	-0/19
43	Relatives' death	3/69	0/99	8/13	139	0/000	0/85	0/52
44	Financial status	3/74	1/02	8/61	139	0/000	0/91	0/57
45	New member of family	2/55	1/09	-4/86	139	0/000	-0/27	-0/63
46	Technology	2/38	0/98	-7/46	139	0/000	-0/46	-0/79
Mean		2/81	0/61	-3/48	139	0/000	-0/07	-0/28

**Values of KMO and Bartlett**

Sampling adequacy index (KMO)	Bartlett's test		
	Chi-square statistic	Freedom degree	Significance level
0.5	21.39	1	0.000

**Factor loads**

Factors	1
32	0.83
31	0.83

**Appendix E****Results of T test- mean and SD**

Low job stress of employees	Value of test-150						
	Mean	SD	T	Freedom degree	Significance level	Confidence limits	
						High	Low
	169.25	59.201	3.84	139	0.00	29.14	9.36

**Appendix F****Results of T test- mean and SD**

High level of job stress of employees	Value of test-150						
	Mean	SD	T	Freedom degree	Significance level	Confidence limits	
						High	Low
	169.25	59.201	_26.13	139	0.00	_120.86	_140.64



## **Predictors of Facebook Shopping Intentions among South African Generation Y Students**

**Hilda Bongazana Mahlangu<sup>1</sup>**

**Abstract:** The purpose of this paper was to investigate predictors of Facebook shopping intentions. The sample of this study was students registered at two higher education institutions in the Gauteng province of South Africa. The author selected students because the majority of Facebook users are college students. This segment is also active in the marketplace and seeks value in their purchases. Participants were selected randomly and 300 questionnaires were distributed to the participants. Out of 300 questionnaires, 31 were discarded because of missing data resulting in a final sample of 269 participants. The findings of this study showed self-efficacy had a positive effect on both perceived ease of use and perceived usefulness on Facebook shopping intentions. Perceived usefulness in turn influences intention. Contrary to the findings of previous research, perceived ease of use does not have an effect on intention to use Facebook as a shopping channel. The study has important implications to marketers, as it will help in developing marketing strategies of organisations. Customers who are confident about Facebook shopping and who believe that this medium will provide useful information and enable quicker shopping are likely to use the medium for purchasing a product or a service of their choice.

**Keywords:** self-efficacy; perceived usefulness; perceived ease of use; Facebook shopping

**JEL Classification:** I23

### **1. Introduction**

Facebook, a medium that enables marketers to deliver information about their offerings, is quickly becoming the driver of electronic commerce. Facebook is increasingly outpacing other electronic commerce channels with \$3.203 billion in advertising revenues generated in the third quarter of 2014 (eMarketer, 2014). This makes it a value-laden medium, effective as a shopping channel, which offers exceptional opportunities to tailor individualised product offerings (eMarketer, 2014), and thereby allowing individuals to have customised offerings delivered to their Facebook sites. One important benefit associated with Facebook is its ability to turn potential prospects into customers (eMarketer, 2013). Although there is an

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observed growth in global use of Facebook as a communication medium, little research has been conducted on shopping opportunities this medium offers. The commercial success of Facebook depends, as noted in the context of online shopping, in part on consumers' intention to use it and the perceptions of its functional and utilitarian dimensions- particularly the ease of use and usefulness (Cha, 2011). Davis (1985) explains the dimensions in his study. He conceptualises the drivers of behaviour in technology related environments as an all-encompassing phenomenon that manifests itself in a myriad of factors. Accordingly, he developed a model to study such behaviours, hereinafter referred to as the technology acceptance model (TAM). His model illuminates how intentions and attitudes determine an individual's behaviours. It also pinpoints how perceived usefulness and perceived ease of use influence an individual's attitudes and intentions.

In 1996, Venkatesh and Davis extended TAM to incorporate external variables such as self-efficacy. They found that self-efficacy is an important variable in information technology usage behaviours as it relates to a person's subjective judgement of his or her own abilities. Subjective judgement of abilities is described best by the self-efficacy construct of Bandura (1977) who posits that the "*strength of people's convictions in their own effectiveness is likely to affect whether they will even try to cope with given situations*" (p. 193).

In the context of electronic commerce, Kim and Kim (2005) observe that those customers who have high beliefs about their ability to purchase online are likely to attempt risky online transactions. This suggests that individuals tend to use information technology when they believe they have the necessary abilities for using such technology. Furthermore, self-efficacy influences both the constructs of TAM, perceived ease of use and perceived usefulness (Davis, 1989; Agarwal & Karahanna, 2000; Venkatesh, 2000; Venkatesh & Bala, 2008; Chen, Chen & Yen, 2011). In an online shopping environment, Dash and Saji (2000) noted that TAM explains the effects of self-efficacy on perceived usefulness. Thus, they found that self-efficacy has a positive effect on perceived usefulness. Self-efficacy and usefulness in turn influence intentions to make a purchase (Vijayasathy, 2004; Hernández, Jiménez, & Martín, 2010). Against this backdrop, and in consideration of the lack of studies addressing Facebook shopping behaviours in developing countries like South Africa, the purpose of this study is to examine predictors of Facebook shopping intentions among South African Generation Y students. Intentions may assist in driving prospects into actual purchases (eMarketer, 2014). Generation Y age cohort, born between 1986 and 2005 (Markert, 2004), is active in the marketplace and are fashion conscious, and show high levels of brand awareness and seek value in purchases (Noble, Haytko & Phillips, 2009). Hence, the focus of this study is on this cohort.

## **2. Theoretical Background and Hypotheses**

### **2.1 Self-efficacy in Information Technology Environments**

Bandura (1977) observed that people who believe they can successfully perform a particular behaviour would persevere even in the presence of difficulties. He maintained that performance of behaviour is linked to an outcome. Thus, individuals perform an activity only if they believe they are capable of performing it and that the activity will produce a desired outcome (Hill & Beatty, 2011). Generally, beliefs about outcomes are important in understanding human-technology interactions. These beliefs are determined by individual characteristics such as self-efficacy. Ordinarily, self-efficacy entails assessment of behaviours that influence the decision-making process and effort put into the behaviour under study (Hsu, Ju, Yen & Chang, 2007). The effect of self-efficacy on the information technology environment such as the Web, Internet, computer use and mobile devices has been studied.

In the context of computers, self-efficacy describes “an individual's perceptions of his or her ability to use computers in the accomplishment of a task rather than reflecting simple component skills”. As a result, computer self-efficacy has been found to predict users' behaviours (Agarwal, Sambamurthy & Stair, 2000; Hsu, et al., 2007). When conducting a study of this nature, it is important to draw on the computer self-efficacy model derived from Compeau and Higgins (1995); and building on existing literature on self-efficacy in information technology environments, this study predicts that self-efficacy will play a significant role in Facebook shopping behaviours. In this study, self-efficacy refers to the evaluation of one's ability to use Facebook for shopping purposes.

### **2.2. Perceived Ease of Use**

Perceived ease of use is defined as “the degree to which a person believes that using a particular system would be free of effort” (Davis, 1989:320). Information technology users often use technology only if they believe they have the ability and skills necessary to use such technology. Such perceptions are derived from the self-efficacy theory. Self-efficacy is regarded as a stronger predictor of perceived ease of use of information technology and users who believe that using a particular information technology would assist in accomplishing a certain task are more likely to have high self-efficacy (Hsu, et al., 2007). In the Facebook context, perceived ease of use will refer to ease of browsing an organisation's Facebook pages or ease of browsing the advertisements on user's Facebook pages.

### 2.3. Perceived usefulness

Perceived usefulness is defined as the “degree to which a person believes that using a particular system would enhance his or her job performance” (Davis, 1989:320). Wang, Wang, Lin and Tang (2003) identified a relationship between perceived usefulness and self-efficacy. They assert that individuals with high self-efficacy will find computer technology useful. Several studies (Agarwal & Karahanna, 2000; Luarn & Lin, 2005; Guriting, & Ndubisi, 2006; Wu, Wang & Lin, 2007) showed that self-efficacy was positively related to perceived usefulness. In the Facebook context, self-efficacy had a significant and positive effect on perceived usefulness (Wang, Xu & Chan, 2008). In this study, perceived usefulness refers to the extent to which individuals believe Facebook will provide useful information and enable quicker shopping.

### 2.4. Intention

Usually, an individual’s behaviour determines their intention to perform the behaviour (Venkatesh, Brown, Maruping & Bala, 2008). According to Ajzen (1991:181) intentions “are indications of how hard people are willing to try, of how much of an effort they are planning to exert, in order to perform the behavior”. Thus, intention refers to a person’s decision to act and is assumed to reflect the effort a person is likely to exert in order to achieve a goal or perform a behaviour (Abraham & Sheeran, 2003) and could be increased through self-efficacy (Venkatesh, Morris, Davis & Davis, 2003; Hsu & Chiu, 2004; Luarn & Lin, 2005).

## 3. Research Model and Hypotheses

Figure 1 depicts the research model of this study. As shown below, the model proposes that self-efficacy would influence selected constructs in the TAM, namely, perceived ease of use and perceived usefulness. These constructs are expected to play a major role in shaping intention to shop on Facebook. Therefore, this study hypothesises that:

H<sub>1</sub>: Facebook shopping self-efficacy will have a positive effect on perceived ease of use

H<sub>2</sub>: Facebook shopping self-efficacy will have a positive effect on perceived usefulness

H<sub>3</sub>: Perceived ease of use significantly influences respondents’ intention to use Facebook sites to purchase products

H<sub>4</sub>: Perceived usefulness significantly influences respondents' intention to use Facebook sites to purchase products

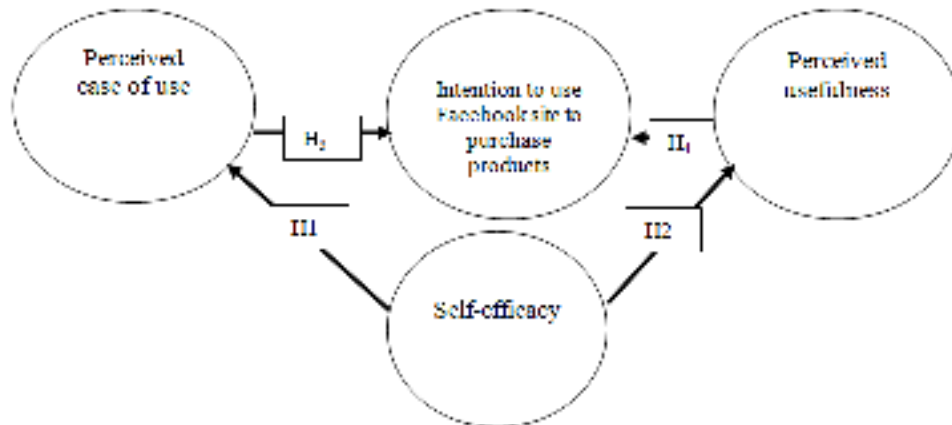


Figure 1. Proposed research model

Source: A research model based on previous TAM literature

## 4. Methodology

### 4.1 Sample and Data Collection

The sample of this study was students registered at two higher education institutions in the Gauteng province of South Africa. The author selected students because the majority of Facebook users are college students (Roblyer, McDaniel, Webb, Herman, & Witty, 2010). Importantly, Wolburg and Pokrywczynski (2001) maintain that university students are an essential segment for marketers. Participants were selected randomly and 300 questionnaires were distributed to the participants. Out of 300 questionnaires, 31 were discarded because of missing data resulting in a final sample of 269 participants. Approximately 55 percent of the respondents were female and 45 percent male.

### 4.2 Measurement Instrument

The questions were adapted from the original instrument developed by Davis (1985). The key TAM constructs measured were perceived ease of use, perceived usefulness and intention. Modifications were made to fit the current research context and purpose. Items from self-efficacy were adapted from the Compeau and Higgins (1995) instrument. Participants were asked to indicate their level of self-efficacy, perceived of use, perceived usefulness and intention for each scenario

using a six-point scale ranging from strongly disagree (1) to strongly agree (6) as end points.

## 5. Data Analysis and Results

To analyse data of this study, structural equation modelling (SEM) was applied using SPSS Amos 22. A two-step analytic process was followed by assessing the measurement and the structural model (McDonald & Ho, 2002). Descriptive statistics were performed using SPSS. Furthermore, tests on confirmatory factor analysis, model fit, reliability and validity were performed.

### 5.1 Construct Reliability and Validity

Reliabilities of constructs were assessed using composite reliability (CR) and variance extracted (AVE). Recommended values for CR and AVE are 0.70 and 0.50 respectively. The composite reliability for all the factors were above the 0.70 recommended value suggested by Malhotra (2010). Furthermore, the average variances extracted are greater than the 0.50 value recommended by Bagozzi and Yi (1988). Table 1 presents factor loadings, and composite and average variance extracted (AVE) values.

Validity of the constructs were assessed using convergent and discriminant validity. In order for the items of the scale to exhibit convergent validity, they should highly correlate and the values should be greater than zero (Churchill Jr, 1979). Factor loadings exceeded the recommended value of 0.50 suggested by Bagozzi and Yi (1988), indicating the existence of convergent validity. Factor loadings ranged from 0.670 to 0.904.

**Table 1. Standardised factor loadings, CR and AVE**

Construct	Factor loadings	CR	AVE
Self-efficacy			
I feel capable of using Facebook for making purchases	0.794	0.97	0.92
I feel comfortable searching for information about a product on Facebook	0.725 0.788		
Learning to use Facebook for searching products was easy for me			
Perceived ease of use			
Using Facebook to purchase a product would not require a lot of mental effort	0.638 0.755	0.91	0.84
Using Facebook to acquire a product would permit me to purchase more efficiently			

Perceived usefulness			
Using Facebook to acquire a product would permit me to purchase more quickly	0.670 0.729	0.91	0.84
Using Facebook to acquire a product would be useful to make my purchases			
Intention			
I intend to purchase through Facebook site in the near future	0.878	0.98	0.94
It is likely that I will purchase through Facebook site in the near future	0.904 0.830		
I expect to purchase through Facebook site in the near future			

Source: Author's computation based on field survey 2014

To examine discriminant validity, guidelines suggested by Malhotra (2010) stating that the square root AVE should exceed the values of the correlation of the related construct were followed. Table 2 lists the inter-correlation matrix and shows that the square root of average variance extracted illustrated in diagonal, exceeded the values of the construct correlations confirming discriminant validity. Overall, the measurement model shows reliability and validity.

**Table 2. Inter-item construct correlation matrix and discriminant validity**

	Self- efficacy	Perceived usefulness	Perceived ease of use	Intention
Self-efficacy	<b>0.96</b>			
Perceived usefulness	0.602	<b>0.92</b>		
Perceived ease of use	0.799	0.685	<b>0.92</b>	
Intention	0.574	0.520	0.513	<b>0.97</b>

\*\*p<0.01 (two-tailed)

Source: Author's computation

## 5.2 Predicting Shopping on Facebook

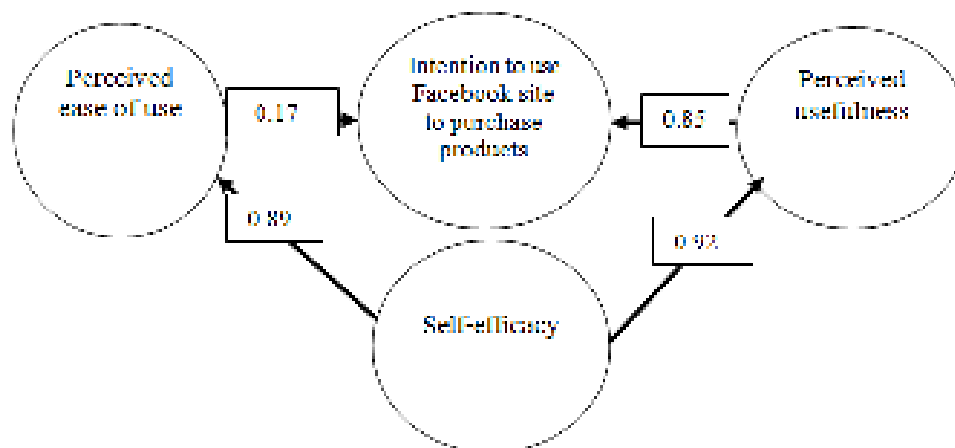
To predict shopping on the Facebook medium, a structural model was formulated. The structural model was evaluated using the maximum likelihood method. To assess the fit indices of the hypothesised model, a comparison of fit indices with their corresponding values was provided. The fit indices of this study are within accepted recommended values. The fit indices for the constructs are listed in Table 3.

**Table 3. Fit indices**

Fit indices	Recommended value	Score for this study
GFI	$\geq 0.90$	0.912
AGFI	$\geq 0.80$	0.843
NFI	$\geq 0.90$	0.914
CFI	$\geq 0.90$	0.933
RMSR	$< 0.08$	0.0552

*Source: Author's computation*

The results of the proposed model indicate strong support for the proposed hypotheses with the exception of a proposed relationship between perceived ease of use and intention to use Facebook.



**Figure 2. Results of structural equation model**

*Source: Computation based on results of author's field survey 2014*

The standardised parameter estimates indicate that the path ( $\beta=0.89$ ) between self-efficacy and perceived ease of use of Facebook are positive and significant ( $p=0.000<0.05$ ). There is a positive significant ( $p=0.000<0.05$ ) relationship between self-efficacy and perceived usefulness of Facebook ( $\beta=0.92$ ). Perceived usefulness significantly ( $p=0.000<0.05$ ) influences intention to use Facebook ( $\beta=0.85$ ). Therefore, the results of this study supported hypothesis 1, 2 and 4 respectively. The hypothesis that perceived ease of use significantly influences intention to use Facebook, has not been confirmed by the results of this study. The findings indicate that perceived ease of use had a negative and insignificant



( $p=0.406>0.05$ ) effect on intention to use Facebook ( $\beta=-0.17$ ). Thus, hypothesis 3 was not supported. Table 4 shows a summary of the hypotheses.

**Table 4. Summary of hypotheses results**

Hypothesis	Results
H <sub>1</sub> : Self-efficacy will have a positive effect on perceived ease of use of Facebook	Supported
H <sub>2</sub> : Self-efficacy will have a positive effect on perceived usefulness of Facebook	Supported
H <sub>3</sub> : Perceived ease of use significantly influences intention to use Facebook	Rejected
H <sub>4</sub> : Perceived usefulness significantly influences intention to use Facebook	Supported

*Source: Author's field survey results 2014*

## 6. Discussion of Results and Conclusion

An individual's intention to use a communication medium depends upon their confidence in their own capabilities. Furthermore, other important aspects are the extent to which such medium is easy to use and whether it is useful in assisting individuals to perform their tasks. The purpose of this study was to investigate predictors of shopping on the Facebook medium. The study applied TAM as a research framework to predict behavioural intentions of Facebook shopping. The findings of this study showed that of the TAM constructs, self-efficacy influence intentions to use Facebook as a shopping medium and the influence occurs through perceived usefulness and perceived ease of use. This is consistent with other studies (Venkatesh, 2000; Yi & Hwang, 2003; Wang & Lin, 2007; Zhang & Mao, 2008), which indicated that self-efficacy is a dominant factor in predicting both perceived ease of use and perceived usefulness. Even in their study, Dash and Saji (2000) point out that self-efficacy played a major role in online shopping behaviours. Hence, they found a significant relationship between self-efficacy and perceived usefulness. In this study, perceived usefulness is another factor influencing respondents' intention to use Facebook as a shopping medium. It seems that respondents' intention to use Facebook intensifies when they believe it is useful for shopping purposes. This finding is consistent with the results of Cha (2011) who reported, in the context of the Internet, that perceived usefulness is more prominent when the respondents believe the Internet will help them purchase the items they require.

Contradictory to the findings of previous TAM research, perceived ease of use does not have an effect on intention to use Facebook as a shopping medium. These findings are unexpected and surprising. The underlying reason for this finding may be attributed partly to the nature of Facebook. By its invention, Facebook social networking site is designed in such a way that it is easy to perform tasks such as updating, analysing and sharing of information (Mazman & Usluel, 2010). Hence, even the inexperienced user may find it easy to navigate and browse through the medium quickly. However, consistent to studies of perceived ease of use in the online shopping context, the findings of this study are similar to the results of Domina, Lee and MacGillivray (2012), who reported that perceived ease of use had no influence on shopping intentions in virtual environments. Overall, results of this study suggest that self-efficacy and perceived usefulness better predicts Facebook shopping intentions than perceived ease of use.

## **7. Implications, Limitation and Future Research**

The findings of this study provide useful information to practitioners. As the number of organisations adopting Facebook as a preferred electronic commerce medium increase, it is important that people have a strong belief in their own abilities in using this medium for shopping purposes. By designing ease to use and a useful site, this will enhance users' confidence in their capabilities in browsing and navigating the site. As such, customers who are confident about Facebook shopping and who believe that this medium will provide useful information and enable quicker shopping are likely to use the medium for purchasing a product or a service.

The findings of this study have theoretical implications. From a theoretical point of review, this study introduced self-efficacy, perceived ease of use, perceived usefulness and intentions to the research of Facebook. While these constructs have been used in numerous research studies and have been accepted as valid constructs to predict online shopping behaviours, little is known on the influence of self-efficacy, perceived ease of use and perceived usefulness in the context of Facebook shopping intentions especially in a developing country like South Africa. As such, this study demonstrated that perceived usefulness and self-efficacy are the main predictors of Facebook shopping intentions among Generation Y students in a developing country like South Africa.

Despite a strong support of the use TAM to study Facebook shopping behaviours, this study is fraught with limitations. This study used selected constructs of the TAM. Based on previous research on TAM, attitudes play an important role in determining online shopping behaviours. Additionally, researchers using TAM

often incorporate trust as an important variable in studying online shopping behaviours. Perhaps one of the observed limitations of this study is the omission of this important construct. Future research should investigate trust-based construct in the context of Facebook shopping behaviours, as this construct is believed to inhibit successful shopping behaviours in the online context. Furthermore, other variables such as actual purchase experience of the products advertised on Facebook sites and benefits derived from such purchase, should be examined. Finally, another major limitation is the use of a student cohort. In order to see the benefits of Facebook commerce, the non-student segment should be studied.

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## Innovative Approaches to Management Health Needed to Make Ecosanogenesis

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**Abstract:** In this study we analysed the field of ecosanogenesis in correlation the health workplace, the health of consumers, and the health of the environment. Concerns of this vast area are primarily related to the implications of technology and various health products, because their effects are far more extensive and more generalized. This concept involves man to major issues, human actions and their results, nature and society, material and spiritual culture in a global relational context. If health depends on the relation human - product - nature, and promote better health will depend on optimal functionality of this relationship. Innovation as a result of research and inventions, in the service ecosanogenesis, offers a balance and sustainable development progress for sure, we could call it positive progress. Health classic resources financing, general taxes or health insurance contributions, are becoming increasingly inadequate, regardless of the type of health system, national health system or health insurance system. From increasingly many factors adversely affect the health of the population impersonal, that are not the result of human action or inaction in terms of his daily activities eg: environmental pollution, deforestation, use of non-biodegradable products etc. In conclusion we describe the innovative approaches to management health in context of ecosanogenesis.

**Keywords:** information technology; innovation; medical research

**JEL Classification:** H15

### 1. Introduction

Health systems are resource intensive for the past 30 years recorded a continuous increase in the level of resources required, increase mainly due to: aging; drug discovery more efficient and more advanced technology, but also more expensive; increasing the number of persons requiring and receiving healthcare.

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Allocation of financial resources within the health care providers (clinics, hospitals, laboratories, etc.) is done on the “money follows the patient”, although desired. Financing is one of the main factors determining the sustainable functioning of the health system and create favourable conditions to meet the needs and demand of the population in health care quality and adequate volume. The correct and timely regulatory levers allow the system funding increase equitable access to medical services, ensure the rational and efficient use of resources and promotes positive motivation of health care providers.

Although with the introduction of mandatory health insurance has increased the financial stability of the system and improved public access to basic health services, a number of issues have not been resolved. The remains substantial share of the population outside the system of mandatory health insurance, especially among self-employed persons, low-income and predominantly rural. Inequities persist, both vertically and horizontally, in financial contributions to health arising from reduced flexibility mechanisms collection and training of healthcare insurance funds and the lack of objective criteria for assessing the degree of social vulnerability population categories for allocating budgetary resources.

The relatively high burden of contributions from the state budget mandatory health insurance fund for a wide range of categories, without taking into account their real income and capacity to pay insurance premiums. The lack of competitiveness in the procurement of services and providers rigidity of payment of mandatory health insurance funds, which does not take into account regional differences and socio-demographic structure encourages equalizer artificial disproportionate distribution of providers with duplication of functions, which increases the financial burden on budgets and public funds for health. No real mechanisms were implemented to increase efficiency, both technical and allocate as the health system or provider leverages motivation for developing performance. Medical institutions remains limited rights to manage their own resources and means obtained for sustainable development. There remain a number of discrepancies between the functions and responsibilities as delegated their decision.

Lack of resource use rules, imperfect mechanism for calculating the cost of care, inefficient negotiation process because of rigid tariffs undermine the financial security providers and medical staff motivation to turn honest. It is significant participation of local government health infrastructure development in the territory. Persist in inertia investments in disease and not health, but it is increasingly obvious and justified the need to invest in prevention and health promotion, both externally funded programs, as well as vice tax applying to market products with increased risk for health, the orientation of these funds to the Ministry of Health. Ongoing national programs do not receive adequate funding, relative to their real needs and costs.

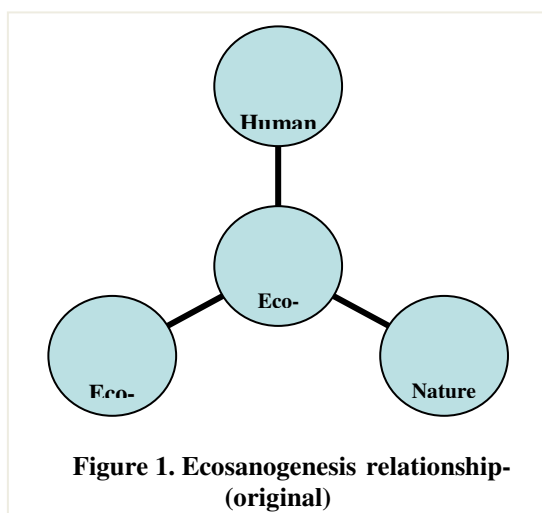
European Commission presented a multiannual program for action on health 2014 - 2020. This program addresses the need to support Member States in an effort to improve the health of citizens and to ensure sustainable healthcare systems in line with the Europe 2020 Strategy .

## 2. Results and Discussions

Ecosanogenesis concept requires that an objective requirement, because man can only survive in a healthy environment. Field of ecosanogenesis covers both workplace health, the health of consumers, and the environment. Concerns of this vast area are primarily related to the implications of technology and various health products, because their effects are far more extensive and more generalized. This concept involves man to major issues, human actions and their results, nature and society, material and spiritual culture in a global relational context.

If health depends on the human - product - nature, and promote better health will depend on optimal functionality of this relationship (fig.1). Concept aims and objectives ecosystem management quality production correlated with environmental requirements.

These objectives are related to the interests of consumers and producers and environmental issues raised by advanced biotechnology principles and eco-economy.



European society is characterized by high costs for the health system and a workforce increasingly smaller due to medical reasons and aging. These issues put



enormous pressure on economic and social system. Personal lifestyle and environmental factors are the most significant risk factors that influence health. e-Health is a relatively recent term used for the practice of using electronic processes and communication. In recent years, focusing on investment in e-health was made as part of e-health.

Currently, the medical system in Romania is still dominated by the public health system is financed through a combination of employer and employee contributions to the National Health Fund (FNS) and direct allocations from the state budget.

Romania has a single integrated system (SUI), a solution aimed at more efficient management of the National Unique Fund for Health Insurance by collecting online and real-time processing of all medical information of patients insured.

By June 2013, out of SUI, Romania has implemented the following medical sector-specific IT systems:

- classification system diagnostic groups (Diagnosis Related Groups - GDI); the electronic prescription (RE);
- health Insurance Card System (SCAS) - under implementation;
- the electronic management of patient records - during the auction.

In Romania, telemedicine has generated increased interest in the last nine years, the first such centre is located in Targu Mures. Through telemedicine means the electronic transfer of medical data (high definition images, sounds, live video recordings on the patient) from one place to another, from a distance. This transfer of medical data may use different types of technology, including but not limited to the technologies listed below - telephone lines, Internet, Intranet and satellite. Telemedicine is used by suppliers in increasingly more medical specialties including dermatology, oncology, radiology, surgery, cardiology and psychiatry.

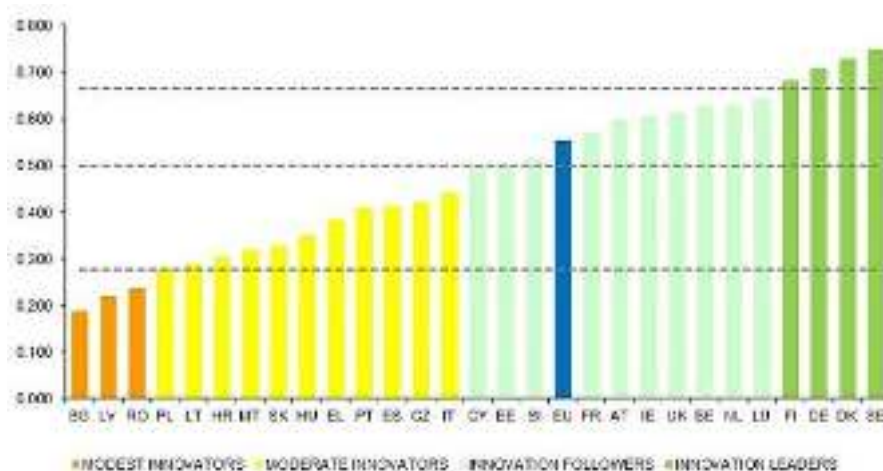
Romania currently has no express authority appointed to coordinate e-Health policy and technical partner for the European Commission common objectives such as, for example, interoperability, Ministry of Health is the only institution that covers all medical activities carried out country.

Manual used to assess innovative activities and innovative performance measurement in OECD countries and the EU: *The Measurement of Scientific and Technological Activities* (Third edition, 2005). According to the Manual, innovation refers to all stages of the innovation process: scientific, technological, organizational, financial and trade leading to the implementation of innovations.

Components of innovative performance, according to “*The Measurement of Scientific and Technological Activities* (third edition, 2005)”, are: research activity, development activities, with some degree of novelty, support activities such as

training and market research, implementing activities dedicated to innovation developments, such as new marketing methods or new organizational methods, different product or process innovation.

According to the latest sources of innovation synthetic index is quite modest to us, the modest innovators in the cluster. In 2014 Romania has climbed two places in the ranking (Fig.2). Synthetic Innovation Index has a direct correlation with the percentage of GDP for research, development and innovation.



**Figure 2. Source- Union Innovation Scoreboard, 2014**

World Health Organization (WHO) launched in 2013 “Report on Global Health - Research for ensuring universal access to health care”, making a call to all countries to ask them to invest and help achieve such research that aim to provide universal access to health, according to the needs of individual countries. Through research we can put a number of questions about how to achieve universal access to health services and find answers to improve human health and increase welfare. All countries should be both producers and consumers of such research. Creativity and skills of experts should be used for the investigations not only in academic centres, but also in public health programs, close to those who ask and offer medical services.

Since 2010, all hospitals (427) and all family doctors in Romania (over 11,500) have minimal facilities, IT. A 2008 study indicated that 411 public hospitals under the Ministry of Health had an average of about 8 hospital beds to a computer, half of which is used in the clinical and pre-clinical.

Cloud Computing (latest technology ICT) refers to the storage, processing and use of data on computers and remote computers that can be accessed through an

Internet connection. Implementation of Cloud technologies in the public institutions can help increase service efficiency, reduce management costs and increase the availability of services. In Romania there are multiple initiatives for Cloud Computing in several public institutions. In the absence of harmonized legislation on the operation's cloud, these initiatives remain only “internal” or private Cloud sites, cannot really benefit from the latest technology.

The main benefits of the introduction of IT technology in the health system are reduced costs, improved service and a more rapid delivery of new services: model that allows sustainable cost reduction for IT services in the public sector, including hardware, software and operations; reduce costs for migration to new platforms services; reduce the number of applications and services (redundant) in the public sector; reduce time and cost for the acquisition of new services; switch IT investments to more efficient computing platforms; promote the use of green IT by reducing the total energy and the surface properties of the government data center. Implementation the IT Infrastructure in the Cloud is faster, safer and constantly updated with the latest application versions and updates.

It is important to reduce carbon emissions by optimizing the use of data center resources, removing redundant services and purchase of ecological systems to reduce energy consumption and increase security in the data center by implementing standard security solutions, updated and tested. Data will be much less susceptible to loss because the backups for data in a Cloud environment will cause the data to be safer. The data will be inaccessible by assailants in a Government Cloud-based cyber infrastructure of public institutions as cyber security specialists will be able to offer non-stop monitoring and protection. Cloud government must be formed on a Romanian state-owned infrastructure.

### **3. Conclusion**

Solutions to solve the widening gap between health expenditure (which are in continuous growth) and financial resources allocated to health is to identify new sources of funding, through the realization ecosanogenesis.

Identifying new sources of funding for health cannot be achieved only by means of research, innovation and ICT.

In a first step, it is necessary to disseminate information about ecosanogenesis, educating people in the spirit of the requirement of quality goods that are not produced at the expense of the environment and human health. ICT can support this process through the Internet, dedicated software (e-health, e-learning, newsletters, social networks etc.).

Also be aware and informed properly authorized state institutions in policy environmental, economic and health. In this respect, sociological research, comparative studies and statistical modelling can provide a serious support decision making at national level.

Only in this way can fulfill ecosanogenesis one of its most important to require proper management and to have a preventive role, to prevent spasms and irreversible developments on human health and the environment.

It is important to balance the allocation of financial resources (territorial and between categories of services) and cost control; reducing inequities in access to services offer; improving satisfaction of care providers and users; reducing inappropriate use of modern technologies.

It is necessary to introduce controlled competition (between the public and / or private insurance organizations etc.); establishment of contractual relations between suppliers and buyers of services; pay doctors and institutions based on performance criteria; introduction of mechanisms for ensuring the quality of health care; introducing modern management methods in health services management.

Creation software, like DRG System (System Diagnostic major groups) CNAS used to finance public hospitals, which should be fairly finance all health care providers (clinics, hospitals, laboratories, rehabilitation centers etc. ).

It is necessary to restructure the funding sources of the health sector by giving greater importance to obtain new sources of funding to come directly from those individuals and companies who, through their work or lack of action, disturbances of the state of population health (food producers with a strong detrimental impact on health, the companies that pollute the soil, water and atmosphere etc.).

In this context, research, innovation and ICT specialists can provide support by: use mathematical modelling to identify interdependencies between home use of products harmful to health and health status indicators.

#### **4. Acknowledgment**

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**Microeconomics****The Complete Theory of Cobb-Douglas Production Function**Cătălin Angelo Ioan<sup>1</sup>, Gina Ioan<sup>2</sup>

**Abstract.** The paper treats various aspects concerning the Cobb-Douglas production function. On the one hand were highlighted conditions for the existence of the Cobb-Douglas function. Also were calculated the main indicators of it and short and long-term costs. It has also been studied the dependence of long-term cost of the parameters of the production function. The determination of profit was made both for perfect competition market and maximizes its conditions. Also we have studied the effects of Hicks and Slutsky and the production efficiency problem.

**Keywords:** production function; Cobb-Douglas; Hicks; Slutsky

**1. Introduction**

To conduct any economic activity is absolutely indispensable the existence of inputs, in other words of any number of resources required for a good deployment of the production process. We will assume that all resources are indefinitely divisible.

We define on  $\mathbf{R}^n$  the production space for  $n$  fixed resources as  $SP = \{(x_1, \dots, x_n) \mid x_i \geq 0, i = \overline{1, n}\}$  where  $x \in SP$ ,  $x = (x_1, \dots, x_n)$  is an ordered set of resources and, because inside a production process, depending on the nature of applied technology, not any amount of resources is possible, we will restrict production space to a convex subset  $D_p \subset SP$  – called the domain of production.

We will call a production function an application:

$$Q: D_p \rightarrow \mathbf{R}_+, (x_1, \dots, x_n) \rightarrow Q(x_1, \dots, x_n) \in \mathbf{R}_+ \quad \forall (x_1, \dots, x_n) \in D_p$$

which satisfies the following axioms:

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A1.  $Q(0, \dots, 0) = 0$ ;

A2. The production function is of class  $C^2$  on  $D_p$  that is it admits partial derivatives of order 2 and they are continuous on  $D_p$ ;

A3. The production function is monotonically increasing in each variable, that is:

$$\frac{\partial Q}{\partial x_i} \geq 0, i = \overline{1, n};$$

A4. The production function is quasi-concave (*see Appendix*).

Considering a production function  $Q: D_p \rightarrow \mathbf{R}_+$  and  $Q_0 \in \mathbf{R}_+$  - fixed, the set of inputs which generate the production  $Q_0$  called isoquant. An isoquant is therefore characterized by:  $\{(x_1, \dots, x_n) \in D_p \mid Q(x_1, \dots, x_n) = Q_0\}$  or, in other words, it is the inverse image  $Q^{-1}(Q_0)$ .

We will say that a production function  $Q: D_p \rightarrow \mathbf{R}_+$  is constant return to scale if  $Q(\lambda x_1, \dots, \lambda x_n) = \lambda Q(x_1, \dots, x_n)$ , with increasing return to scale if  $Q(\lambda x_1, \dots, \lambda x_n) > \lambda Q(x_1, \dots, x_n)$  and decreasing return to scale if  $Q(\lambda x_1, \dots, \lambda x_n) < \lambda Q(x_1, \dots, x_n) \forall \lambda \in (1, \infty) \forall (x_1, \dots, x_n) \in D_p$ .

## 2. The Cobb-Douglas Production Function

The Cobb-Douglas function has the following expression:

$$Q: D \subset \mathbf{R}_+^n - \{0\} \rightarrow \mathbf{R}_+, (x_1, \dots, x_n) \rightarrow Q(x_1, \dots, x_n) = A x_1^{\alpha_1} \dots x_n^{\alpha_n} \in \mathbf{R}_+ \forall (x_1, \dots, x_n) \in D, \\ A \in \mathbf{R}_+^*, \alpha_1, \dots, \alpha_n \in \mathbf{R}^*$$

Computing the partial derivatives of first and second order, we get:

$$Q'_{x_i} = \alpha_i A x_1^{\alpha_1} \dots x_i^{\alpha_i - 1} \dots x_n^{\alpha_n} = \frac{\alpha_i Q}{x_i} \quad \forall i = \overline{1, n}$$

$$Q''_{x_i x_j} = \alpha_i \alpha_j A x_1^{\alpha_1} \dots x_i^{\alpha_i - 1} \dots x_j^{\alpha_j - 1} \dots x_n^{\alpha_n} = \frac{\alpha_i \alpha_j Q}{x_i x_j} \quad \forall i \neq j = \overline{1, n}$$

$$Q''_{x_i x_i} = \alpha_i (\alpha_i - 1) A x_1^{\alpha_1} \dots x_i^{\alpha_i - 2} \dots x_n^{\alpha_n} = \frac{\alpha_i (\alpha_i - 1) Q}{x_i^2} \quad \forall i = \overline{1, n}$$

Let the bordered Hessian matrix:

$$H^B(Q) = \begin{pmatrix} 0 & \frac{\alpha_1 Q}{x_1} & \frac{\alpha_2 Q}{x_2} & \dots & \frac{\alpha_n Q}{x_n} \\ \frac{\alpha_1 Q}{x_1} & \frac{\alpha_1(\alpha_1 - 1)Q}{x_1^2} & \frac{\alpha_1 \alpha_2 Q}{x_1 x_2} & \dots & \frac{\alpha_1 \alpha_n Q}{x_1 x_n} \\ \frac{\alpha_2 Q}{x_2} & \frac{\alpha_1 \alpha_2 Q}{x_1 x_2} & \frac{\alpha_2(\alpha_2 - 1)Q}{x_2^2} & \dots & \frac{\alpha_2 \alpha_n Q}{x_2 x_n} \\ \dots & \dots & \dots & \dots & \dots \\ \frac{\alpha_n Q}{x_n} & \frac{\alpha_1 \alpha_n Q}{x_1 x_n} & \frac{\alpha_2 \alpha_n Q}{x_2 x_n} & \dots & \frac{\alpha_n(\alpha_n - 1)Q}{x_n^2} \end{pmatrix}$$

We find (not so easy):  $\Delta_k^B = (-1)^k Q^{k+1} \frac{\prod_{i=1}^k \alpha_i \sum_{i=1}^k \alpha_i}{\left(\prod_{i=1}^k x_i\right)^2}$ ,  $k = \overline{1, n}$ .

Because  $(-1)^k \Delta_k^B = Q^{k+1} \frac{\prod_{i=1}^k \alpha_i \sum_{i=1}^k \alpha_i}{\left(\prod_{i=1}^k x_i\right)^2}$ , if  $\prod_{i=1}^k \alpha_i \sum_{i=1}^k \alpha_i > 0$ ,  $k = \overline{1, n}$  it follows that the

function is strictly quasi-concave. Also, if the function is quasi-concave we have that  $\prod_{i=1}^k \alpha_i \sum_{i=1}^k \alpha_i \geq 0$ .

But from the axiom A3 we must have that  $Q'_{x_i} = \frac{\alpha_i Q}{x_i} \geq 0$  that is  $\alpha_i > 0$ . After these considerations we have that if  $\alpha_i > 0$ ,  $i = \overline{1, n}$  the Cobb-Douglas function is strictly quasi-concave.

We have now:  $q(\chi_1, \dots, \chi_{n-1}) = Q(\chi_1, \dots, \chi_{n-1}, 1) = A \chi_1^{\alpha_1} \dots \chi_{n-1}^{\alpha_{n-1}}$  and  $r = \sum_{k=1}^n \alpha_k$ .

The main indicators are:

- $\eta_{x_i} = A \alpha_i x_1^{\alpha_1} \dots x_i^{\alpha_i - 1} \dots x_n^{\alpha_n} = \frac{\alpha_i Q}{x_i}$ ,  $i = \overline{1, n}$
- $w_{x_i} = A x_1^{\alpha_1} \dots x_i^{\alpha_i - 1} \dots x_n^{\alpha_n} = \frac{Q}{x_i}$ ,  $i = \overline{1, n}$



- $RMS(i,j) = \frac{\alpha_i x_j}{\alpha_j x_i}$ ,  $i, j = \overline{1, n}$
- $RMS(i) = \frac{\alpha_i}{x_i \sqrt{\sum_{\substack{j=1 \\ j \neq i}}^{n-1} \frac{\alpha_j^2}{x_j^2}}}$ ,  $i = \overline{1, n}$
- $\varepsilon_{x_i} = \alpha_i$ ,  $i = \overline{1, n}$
- $\sigma_{ij} = -1$ ,  $i, j = \overline{1, n}$

Reciprocally, if for a homogenous production function of degree  $r$ :  $\varepsilon_{x_i} = \alpha_i$ ,  $i = \overline{1, n}$

we have that:  $\frac{\frac{\partial q}{\partial \chi_i}}{\frac{q}{\chi_i}} = \alpha_i$ ,  $i = \overline{1, n-1}$  and  $\frac{rq - \sum_{i=1}^{n-1} \frac{\partial q}{\partial \chi_i} \chi_i}{q} = \alpha_n$ .

But now, we have:

$$\frac{\partial q}{\partial \chi_i} = \alpha_i \frac{q}{\chi_i} = q \frac{\partial \ln \chi_i^{\alpha_i}}{\partial \chi_i} \Leftrightarrow \frac{\frac{\partial q}{\partial \chi_i}}{q} = \frac{\partial \ln \chi_i^{\alpha_i}}{\partial \chi_i} \Leftrightarrow \frac{\partial \ln q}{\partial \chi_i} = \frac{\partial \ln \chi_i^{\alpha_i}}{\partial \chi_i} \Leftrightarrow \frac{\partial \ln \frac{q}{\chi_i^{\alpha_i}}}{\partial \chi_i} = 0 \Rightarrow$$

$$\ln \frac{q}{\chi_i^{\alpha_i}} = F_i(\chi_1, \dots, \hat{\chi}_i, \dots, \chi_{n-1}) \text{ (where } \hat{\ } \text{ means that the variable is missing).}$$

We have now:  $q = \chi_i^{\alpha_i} e^{F_i(\chi_1, \dots, \hat{\chi}_i, \dots, \chi_{n-1})}$ . For  $j \neq i$  we obtain now:  $\alpha_j = \chi_j \frac{\partial F_i}{\partial \chi_j}$  therefore:

$$\frac{\partial F_i}{\partial \chi_j} = \frac{\alpha_j}{\chi_j}. \text{ Integrating with respect to } \chi_j :$$

$$F_i = \alpha_j \ln \chi_j + g_i(\chi_1, \dots, \hat{\chi}_i, \dots, \hat{\chi}_j, \dots, \chi_{n-1}) \text{ therefore: } q = \chi_i^{\alpha_i} \chi_j^{\alpha_j} e^{g_i(\chi_1, \dots, \hat{\chi}_i, \dots, \hat{\chi}_j, \dots, \chi_{n-1})}.$$

Analogously, by recurrence:  $q = A \chi_1^{\alpha_1} \dots \chi_{n-1}^{\alpha_{n-1}}$  with  $A = \text{constant}$  with respect to

$$\chi_1, \dots, \chi_{n-1}. \text{ But: } \alpha_n = \frac{rq - \sum_{i=1}^{n-1} \frac{\partial q}{\partial \chi_i} \chi_i}{q} = r - \sum_{i=1}^{n-1} \alpha_i \Leftrightarrow r = \sum_{i=1}^n \alpha_i. \text{ After these}$$

considerations it follows that if it is homogenous of degree  $r$ ,  $r$  must be  $\sum_{i=1}^n \alpha_i$ .

Finally:  $q = Ax_1^{\alpha_1} \dots x_{n-1}^{\alpha_{n-1}}$  implies that:

$$Q(x_1, \dots, x_n) = x_n^r q(x_1, \dots, x_{n-1}) = Ax_n^r \frac{x_1^{\alpha_1}}{x_n^{\alpha_1}} \dots \frac{x_{n-1}^{\alpha_{n-1}}}{x_n^{\alpha_{n-1}}} = Ax_n^{r - \sum_{k=1}^{n-1} \alpha_k} x_1^{\alpha_1} \dots x_{n-1}^{\alpha_{n-1}} =$$

$Ax_1^{\alpha_1} \dots x_{n-1}^{\alpha_{n-1}} x_n^{\alpha_n}$  - the Cobb-Douglas production function.

Considering now again the Cobb-Douglas production:  $Q(x_1, \dots, x_n) = Ax_1^{\alpha_1} \dots x_n^{\alpha_n}$  let search the dependence of the parameters  $\alpha_1, \dots, \alpha_n$ .

We have:  $\frac{\partial Q}{\partial \alpha_i} = Ax_1^{\alpha_1} \dots x_n^{\alpha_n} \ln x_i = Q \ln x_i \geq 0 \quad \forall x_i \geq 1, i = \overline{1, n}$ . From this relation we

have that at an increasing of a parameter  $\alpha_i$  the production  $Q$  will increase also.

In particular, for the Cobb-Douglas function related to capital  $K$  and labor  $L$ :  $Q = AK^\alpha L^\beta$  we have that the main indicators are:

- $\eta_K = A\alpha K^{\alpha-1} L^\beta$
- $\eta_L = A\beta K^\alpha L^{\beta-1}$
- $w_K = AK^{\alpha-1} L^\beta$
- $w_L = AK^\alpha L^{\beta-1}$
- $RMS(K, L) = RMS(K) = \frac{\alpha L}{\beta K}$
- $RMS(L, K) = RMS(L) = \frac{\beta K}{\alpha L}$
- $\varepsilon_K = \alpha$
- $\varepsilon_L = \beta$
- $\sigma = \sigma_{KL} = -1$

### 3. The Costs of the Cobb-Douglas Production Function

Considering now the problem of minimizing costs for a given production  $Q_0$ , where the prices of inputs are  $p_i, i = \overline{1, n}$ , we have:

$$\begin{cases} \min \sum_{k=1}^n p_k x_k \\ Ax_1^{\alpha_1} \dots x_n^{\alpha_n} \geq Q_0 \\ x_1, \dots, x_n \geq 0 \end{cases}$$

From the obvious relations:  $\begin{cases} \frac{\alpha_1}{p_1 x_1} = \dots = \frac{\alpha_n}{p_n x_n} \\ Ax_1^{\alpha_1} \dots x_n^{\alpha_n} = Q_0 \end{cases}$  we obtain:

$$\begin{cases} x_k = \frac{\alpha_k p_n}{\alpha_n p_k} x_n, k = \overline{1, n-1} \\ Ax_1^{\alpha_1} \dots x_n^{\alpha_n} = Q_0 \end{cases} \text{ and from the second equation:}$$

$$A \frac{p_n^{\sum_{k=1}^{n-1} \alpha_k} \prod_{k=1}^{n-1} \alpha_k^{\alpha_k}}{\alpha_n^{\sum_{k=1}^{n-1} \alpha_k} \prod_{k=1}^{n-1} p_k^{\alpha_k}} x_n^{\sum_{k=1}^n \alpha_k} = Q_0. \text{ Noting } r = \sum_{k=1}^n \alpha_k \text{ we finally obtain:}$$

$$\bar{x}_k = \frac{\left( \prod_{k=1}^n p_k^{\alpha_k} \right)^{1/r}}{\left( \prod_{k=1}^n \alpha_k^{\alpha_k} \right)^{1/r}} \frac{\alpha_k}{p_k} \frac{Q_0^{1/r}}{A^{1/r}}, k = \overline{1, n}$$

The total cost is:

$$TC = \sum_{k=1}^n p_k \bar{x}_k = \frac{\left( \prod_{i=1}^n p_i^{\alpha_i} \right)^{1/r}}{\left( \prod_{i=1}^n \alpha_i^{\alpha_i} \right)^{1/r}} \frac{r Q_0^{1/r}}{A^{1/r}}.$$

At a price change of one factor, i.e.  $x_k$ , from the value  $p_k$  to  $\bar{p}_k$  we have:

$$\bar{TC} = \frac{\left( \prod_{\substack{i=1 \\ i \neq k}}^n p_i^{\alpha_i} \right)^{1/r} \bar{p}_k^{\alpha_k/r}}{\left( \prod_{i=1}^n \alpha_i^{\alpha_i} \right)^{1/r}} \frac{rQ_0^{1/r}}{A^{1/r}}$$

where the relative variation of the total cost is:  $\frac{\Delta TC}{TC} = \frac{\bar{TC} - TC}{TC} = \left( \frac{\bar{p}_k}{p_k} \right)^{\alpha_k/r} - 1$ .

Let us now consider the behavior of the total cost of production function at a parameters variation. We have:

$$\frac{\partial TC}{\partial \alpha_k} = \frac{\left( \frac{\prod_{i=1}^n p_i^{\alpha_i} Q_0}{\prod_{i=1}^n \alpha_i^{\alpha_i} A} \right)^{\frac{1}{r}}}{\left( \sum_{j=1}^n \alpha_j \right)^2} \ln \left( \frac{A p_k^{\sum_{j=1}^n \alpha_j} \prod_{\substack{i=1 \\ i \neq k}}^n \alpha_i^{\alpha_i}}{Q_0 \alpha_k^{\sum_{j=1}^n \alpha_j} \prod_{\substack{i=1 \\ i \neq k}}^n p_i^{\alpha_i}} - r \right)$$

Therefore:  $\frac{\partial TC}{\partial \alpha_k} \geq 0 \Leftrightarrow e^{\sum_{j=1}^n \alpha_j} \alpha_k^{\sum_{j=1}^n \alpha_j} \leq \frac{A p_k^{\sum_{j=1}^n \alpha_j} \prod_{\substack{i=1 \\ i \neq k}}^n \alpha_i^{\alpha_i}}{Q_0 \prod_{\substack{i=1 \\ i \neq k}}^n p_i^{\alpha_i}}$ . If we note:  $\Gamma = \sum_{\substack{j=1 \\ j \neq k}}^n \alpha_j > 0$  and

$$M = \frac{A p_k^{\sum_{j=1}^n \alpha_j} \prod_{\substack{i=1 \\ i \neq k}}^n \alpha_i^{\alpha_i}}{Q_0 \prod_{\substack{i=1 \\ i \neq k}}^n p_i^{\alpha_i}} > 0 \text{ we obtain that: } \frac{\partial TC}{\partial \alpha_k} \geq 0 \Leftrightarrow e^{\alpha_k} \alpha_k^\Gamma \leq \frac{M}{e^\Gamma}.$$

Considering now the function  $f: (0, \infty) \rightarrow \mathbf{R}$ ,  $f(x) = x^\Gamma e^x$  we have  $f'(x) = x^{\Gamma-1} e^x (x + \Gamma) > 0$  therefore  $f$  is strictly increasing. Because  $\lim_{x \rightarrow 0} x^\Gamma e^x = 0$

and  $\lim_{x \rightarrow \infty} x^\Gamma e^x = \infty$  the equation:  $x^\Gamma e^x = \frac{M}{e^\Gamma}$  has a unique solution  $\alpha_k^0$  called cost threshold with respect to the k-th parameter. After these considerations we have that for  $\alpha_k \leq \alpha_k^0$  the total cost will increase at an increasing of  $\alpha_k$  and after it will decrease.

The situation may seem paradoxical that at the growth of the elasticity of one input, total cost increases. Fortunately, due to the sharp rise of  $f$ , the values of  $\alpha_k^0$  are very small so it does not significantly affect processes.

Like an example, considering the production function  $Q(K,L)=K^\alpha L^\beta$ ,  $\alpha, \beta > 0$  we have that the behavior of  $\beta^0$  related to  $\alpha$  is (for  $Q=5$ ):

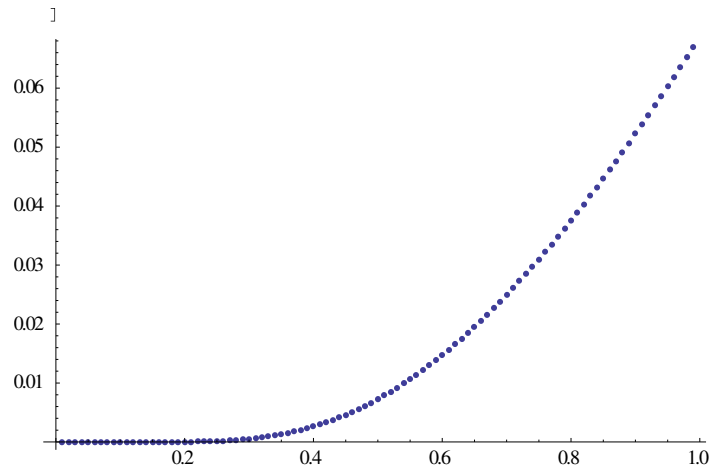


Figure 1

The long-term total cost for  $\alpha=0.7$  and variable  $\beta$  is shown in figure 2:

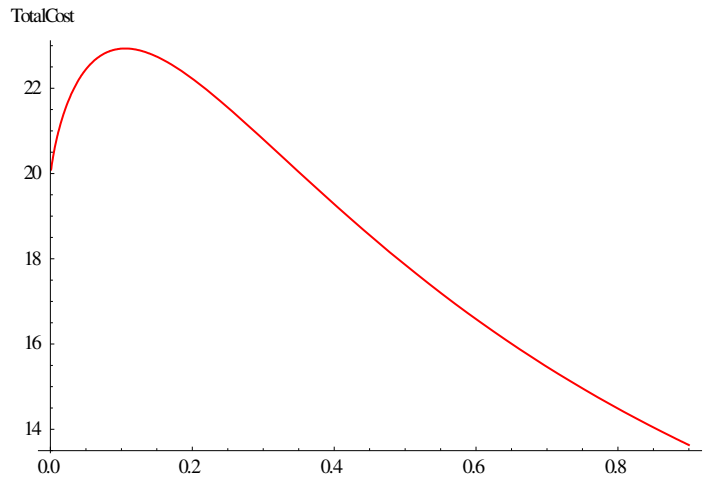


Figure 2

where the maximum value is reached for  $\beta=0.025$ .

If we consider for a given output  $Q_0$ , the inputs  $x_1, \dots, x_n$  such that:  $Ax_1^{\alpha_1} \dots x_n^{\alpha_n} = Q_0$

let  $x_k = \frac{Q_0^{\frac{1}{\alpha_k}}}{A^{\frac{1}{\alpha_k}} x_1^{\alpha_1} \dots \hat{x}_k \dots x_n^{\alpha_n}}$  where  $\hat{\phantom{x}}$  means that the term is missing.

We have  $STC_k = \sum_{i=1}^n p_i x_i = \sum_{\substack{i=1 \\ i \neq k}}^n p_i x_i + p_k \frac{Q_0^{\frac{1}{\alpha_k}}}{A^{\frac{1}{\alpha_k}} x_1^{\alpha_1} \dots \hat{x}_k \dots x_n^{\alpha_n}}$  representing the short-

term total cost when factors  $x_1, \dots, \hat{x}_k, \dots, x_n$  remain constant.

We put now the question of determining the envelope of the family of hypersurfaces:

$$f(Q_0, x_1, \dots, \hat{x}_k, \dots, x_n) = \sum_{\substack{i=1 \\ i \neq k}}^n p_i x_i + p_k \frac{Q_0^{\frac{1}{\alpha_k}}}{A^{\frac{1}{\alpha_k}} x_1^{\alpha_1} \dots \hat{x}_k \dots x_n^{\alpha_n}}$$

Conditions to be met are:

$$\begin{cases} \text{TC} = f(Q_0, x_1, \dots, \hat{x}_k, \dots, x_n) \\ \frac{\partial f}{\partial x_i} = 0, i = \overline{1, n}, i \neq k \end{cases}$$

After the elimination of parameters  $x_1, \dots, \hat{x}_k, \dots, x_n$  we have either the locus of singular points of hypersurfaces (which is not the case for the present issue) or envelope sought.

We have therefore:

$$\begin{cases} \text{TC} = \sum_{\substack{i=1 \\ i \neq k}}^n p_i x_i + p_k \frac{Q_0^{\frac{1}{\alpha_k}}}{A^{\frac{1}{\alpha_k}} x_1^{\frac{\alpha_1}{\alpha_k}} \dots \hat{x}_k^{\frac{\alpha_k}{\alpha_k}} \dots x_n^{\frac{\alpha_n}{\alpha_k}}} \\ p_i - \frac{\alpha_i}{\alpha_k} \frac{p_k Q_0^{\frac{1}{\alpha_k}}}{A^{\frac{1}{\alpha_k}} x_1^{\frac{\alpha_1}{\alpha_k}} \dots x_i^{\frac{\alpha_i}{\alpha_k}} \dots \hat{x}_k^{\frac{\alpha_k}{\alpha_k}} \dots x_n^{\frac{\alpha_n}{\alpha_k}}} = 0, i = \overline{1, n}, i \neq k \end{cases}$$

Noting:  $\Psi = \frac{Q_0^{\frac{1}{\alpha_k}}}{A^{\frac{1}{\alpha_k}} x_1^{\frac{\alpha_1}{\alpha_k}} \dots \hat{x}_k^{\frac{\alpha_k}{\alpha_k}} \dots x_n^{\frac{\alpha_n}{\alpha_k}}}$  it follows:

$$\begin{cases} \text{TC} = \sum_{\substack{i=1 \\ i \neq k}}^n p_i x_i + p_k \Psi \\ p_i - \frac{\alpha_i}{\alpha_k} \frac{p_k \Psi}{x_i} = 0, i = \overline{1, n}, i \neq k \end{cases}$$

from where:  $x_i = \frac{\alpha_i}{\alpha_k} \frac{p_k \Psi}{p_i}, i = \overline{1, n}, i \neq k$ . Finally:  $\Psi = \frac{\alpha_k (p_1^{\alpha_1} \dots p_n^{\alpha_n})^{1/r} Q_0^{1/r}}{A^{1/r} p_k (\alpha_1^{\alpha_1} \dots \alpha_n^{\alpha_n})^{1/r}}$  and

replacing:

$$x_i = \frac{\alpha_i \left( \prod_{i=1}^n p_i^{\alpha_i} \right)^{1/r} Q_0^{1/r}}{A^{1/r} p_i \left( \prod_{i=1}^n \alpha_i^{\alpha_i} \right)^{1/r}}, i = \overline{1, n}, i \neq k$$

$$TC = \frac{\left(\prod_{i=1}^n p_i^{\alpha_i}\right)^{1/r}}{\left(\prod_{i=1}^n \alpha_i^{\alpha_i}\right)^{1/r}} \frac{rQ_0^{1/r}}{A^{1/r}}$$

We obtained so that the envelope of the family of hypersurfaces of the short-term total cost when all inputs are constant except one is just the long-term cost obtained from nonlinear optimization problem with respect to the minimizing of the cost for a given production.

Calculating the costs derived from the (long-term or short-term) total cost now, we have:

$$ATC = \frac{TC}{Q_0} = \frac{\left(\prod_{i=1}^n p_i^{\alpha_i}\right)^{1/r}}{\left(\prod_{i=1}^n \alpha_i^{\alpha_i}\right)^{1/r}} \frac{rQ_0^{1/r-1}}{A^{1/r}} \quad (\text{average long-term total cost})$$

$$MTC = \frac{\partial TC}{\partial Q_0} = \frac{\left(\prod_{i=1}^n p_i^{\alpha_i}\right)^{1/r}}{\left(\prod_{i=1}^n \alpha_i^{\alpha_i}\right)^{1/r}} \frac{Q_0^{1/r-1}}{A^{1/r}} = \frac{ATC}{r} \quad (\text{marginal long-term total cost})$$

$$ASTC_k = \frac{STC_k}{Q_0} = \frac{\sum_{i=1, i \neq k}^n p_i x_i}{Q_0} + p_k \frac{Q_0^{\frac{1}{r}-1}}{A^{\frac{1}{r}} x_1^{\alpha_1} \dots \hat{x}_k \dots x_n^{\alpha_n}} \quad (\text{average short-term total cost})$$

$$MC_k = \frac{\partial STC_k}{\partial Q_0} = p_k \frac{Q_0^{\frac{1}{r}-1}}{\alpha_k A^{\frac{1}{r}} x_1^{\alpha_1} \dots \hat{x}_k \dots x_n^{\alpha_n}} \quad (\text{marginal short-term total cost})$$

$$VTC_k = p_k \frac{Q_0^{\frac{1}{r}}}{A^{\frac{1}{r}} x_1^{\alpha_1} \dots \hat{x}_k \dots x_n^{\alpha_n}} \quad (\text{variable short-term total cost})$$



$$AVTC_k = \frac{VTC_k}{Q_0} = p_k \frac{Q_0^{\frac{1}{\alpha_k}-1}}{A^{\frac{1}{\alpha_k}} x_1^{\frac{\alpha_1}{\alpha_k}} \dots \hat{x}_k^{\frac{\alpha_k}{\alpha_k}} \dots x_n^{\frac{\alpha_n}{\alpha_k}}} \text{ (average variable short-term total cost)}$$

$$FTC_k = \sum_{\substack{i=1 \\ i \neq k}}^n p_i x_i \text{ (fixed short-term total cost)}$$

$$AFTC_k = \frac{FTC_k}{Q_0} = \frac{\sum_{\substack{i=1 \\ i \neq k}}^n p_i x_i}{Q_0} \text{ (average fixed short-term total cost)}$$

The extreme of the function  $ASTC_k(Q_0) = \frac{\sum_{\substack{i=1 \\ i \neq k}}^n p_i x_i}{Q_0} + p_k \frac{Q_0^{\frac{1}{\alpha_k}-1}}{A^{\frac{1}{\alpha_k}} x_1^{\frac{\alpha_1}{\alpha_k}} \dots \hat{x}_k^{\frac{\alpha_k}{\alpha_k}} \dots x_n^{\frac{\alpha_n}{\alpha_k}}}$  are given

by:

$$ASTC_k'(Q_0) = \frac{p_k \left( \frac{1}{\alpha_k} - 1 \right) Q_0^{\frac{1}{\alpha_k}} - A^{\frac{1}{\alpha_k}} x_1^{\frac{\alpha_1}{\alpha_k}} \dots \hat{x}_k^{\frac{\alpha_k}{\alpha_k}} \dots x_n^{\frac{\alpha_n}{\alpha_k}} \sum_{\substack{i=1 \\ i \neq k}}^n p_i x_i}{Q_0^2 A^{\frac{1}{\alpha_k}} x_1^{\frac{\alpha_1}{\alpha_k}} \dots \hat{x}_k^{\frac{\alpha_k}{\alpha_k}} \dots x_n^{\frac{\alpha_n}{\alpha_k}}} = 0 \text{ from where:}$$

$$Q_{0,d-root} = \frac{A x_1^{\alpha_1} \dots \hat{x}_k^{\alpha_k} \dots x_n^{\alpha_n} \left( \sum_{\substack{i=1 \\ i \neq k}}^n p_i x_i \right)^{\alpha_k}}{p_k^{\alpha_k} \left( \frac{1}{\alpha_k} - 1 \right)^{\alpha_k}} \text{ when } \alpha_k < 1 \text{ and the minimum value is:}$$

$$ASTC_k(Q_{0-d-root}) = \frac{p_k^{\alpha_k} \left( \frac{1}{\alpha_k} - 1 \right)^{\alpha_k-1}}{A \alpha_k x_1^{\alpha_1} \dots \hat{x}_k^{\alpha_k} \dots x_n^{\alpha_n} \left( \sum_{\substack{i=1 \\ i \neq k}}^n p_i x_i \right)^{\alpha_k-1}}.$$

If  $\alpha_k \geq 1$  it follows that  $ASTC_k'(Q_0) < 0$  therefore the average short-term total cost will decrease.

Finally we have:

$$\varepsilon_{p_k} = \frac{\frac{\partial CT}{\partial p_k}}{\frac{CT}{p_k}} = \frac{\alpha_k}{r} - \text{the coefficient of elasticity of long-term total cost with respect to the price factor } i$$

$$\varepsilon_Q = \frac{\frac{\partial CT}{\partial Q_0}}{\frac{CT}{Q_0}} = \frac{1}{r} - \text{the coefficient of elasticity of long-term total cost with respect to the production } Q_0$$

$$\varepsilon_{av,p_k} = \frac{\frac{\partial ATC}{\partial p_k}}{\frac{ATC}{p_k}} = \frac{\alpha_k}{r} - \text{the coefficient of elasticity of average long-term total cost with respect to the price factor } i$$

$$\varepsilon_{marg,p_k} = \frac{\frac{\partial MTC}{\partial p_k}}{\frac{MTC}{p_k}} = \frac{\alpha_k}{r} - \text{the coefficient of elasticity of marginal long-term total cost with respect to the price factor } i$$

In particular, for the Cobb-Douglas function related to capital K and labor L:  $Q = AK^\alpha L^\beta$  we have:

$$\bar{K} = \frac{\left(\frac{\alpha}{p_K p_L}\right)^{1/(\alpha+\beta)} \alpha Q_0^{1/(\alpha+\beta)}}{\left(\alpha \beta\right)^{1/(\alpha+\beta)} p_K A^{1/(\alpha+\beta)}}$$

$$\bar{L} = \frac{\left(\frac{\alpha}{p_K p_L}\right)^{1/(\alpha+\beta)} \beta Q_0^{1/(\alpha+\beta)}}{\left(\alpha \beta\right)^{1/(\alpha+\beta)} p_L A^{1/(\alpha+\beta)}}$$

$$TC = \frac{(p_K^\alpha p_L^\beta)^{1/(\alpha+\beta)} (\alpha + \beta) Q_0^{1/(\alpha+\beta)}}{(\alpha^\alpha \beta^\beta)^{1/(\alpha+\beta)} A^{1/(\alpha+\beta)}}$$

On the short-term, we have for constancy of K:  $STC_L = p_K K + p_L \frac{Q_0^{\frac{1}{\alpha}}}{A^{\frac{1}{\beta}} K^{\frac{\alpha}{\beta}}}$  and

$$ATC = \frac{(p_K^\alpha p_L^\beta)^{1/(\alpha+\beta)} (\alpha + \beta) Q_0^{1/(\alpha+\beta)-1}}{(\alpha^\alpha \beta^\beta)^{1/(\alpha+\beta)} A^{1/(\alpha+\beta)}}$$

$$MTC = \frac{(p_K^\alpha p_L^\beta)^{1/(\alpha+\beta)} Q_0^{1/(\alpha+\beta)-1}}{(\alpha^\alpha \beta^\beta)^{1/(\alpha+\beta)} A^{1/(\alpha+\beta)}}$$

$$ASTC_L = \frac{p_K K}{Q_0} + p_L \frac{Q_0^{\frac{1}{\alpha}-1}}{A^{\frac{1}{\beta}} K^{\frac{\alpha}{\beta}}}$$

$$MC_L = \frac{p_L Q_0^{\frac{1}{\alpha}-1}}{\beta A^{\frac{1}{\beta}} K^{\frac{\alpha}{\beta}}}$$

$$VTC_L = p_K \frac{Q_0^{\frac{1}{\alpha}}}{A^{\frac{1}{\beta}} L^{\frac{\alpha}{\beta}}}$$

$$AVTC_L = p_K \frac{Q_0^{\frac{1}{\alpha}-1}}{A^{\frac{1}{\beta}} L^{\frac{\alpha}{\beta}}}$$

$$FTC_L = p_L L$$

$$AFTC_L = \frac{p_L L}{Q_0}$$

The extreme of the function:  $ASTC_L(Q_0)$  is given by:  $Q_{0,d-root} = \frac{Ap_K^\beta K^{\alpha+\beta}}{p_L^\alpha \left(\frac{1}{\beta} - 1\right)^\beta}$  when

$\beta < 1$  and the minimum value is:  $ASTC_L(Q_{0-d-root}) = \frac{p_L^\beta (1-\beta)^{\beta-1}}{Ap_K^{\beta-1} \beta^\beta K^{\alpha+\beta-1}}$ .

$$\varepsilon_{p_K} = \frac{\alpha}{\alpha + \beta}, \varepsilon_{p_L} = \frac{\beta}{\alpha + \beta}, \varepsilon_Q = \frac{1}{\alpha + \beta}, \varepsilon_{av,p_K} = \frac{\alpha}{\alpha + \beta}, \varepsilon_{av,p_L} = \frac{\beta}{\alpha + \beta},$$

$$\varepsilon_{marg,p_K} = \frac{\alpha}{\alpha + \beta}, \varepsilon_{marg,p_L} = \frac{\beta}{\alpha + \beta}.$$

#### 4. The Profit

Now consider a sale price of output  $Q_0$ :  $p(Q_0)$ . The profit is therefore:

$$\Pi(Q_0) = p(Q_0) \cdot Q_0 - TC(Q_0)$$

It is known that in a market with perfect competition, the price is given and equals marginal cost. The profit on long-term becomes:

$$\Pi(Q_0) = p(Q_0) \cdot Q_0 - TC(Q_0) = MTC(Q_0) \cdot Q_0 -$$

$$TC(Q_0) = ATC(Q_0) Q_0^2 = \frac{\left(\prod_{i=1}^n p_i^{\alpha_i}\right)^{1/r}}{\left(\prod_{i=1}^n \alpha_i^{\alpha_i}\right)^{1/r}} \frac{r Q_0^{1/r+1}}{A^{1/r}}$$

In particular, for the Cobb-Douglas function related to capital  $K$  and labor  $L$ :

$$Q = AK^\alpha L^\beta \text{ we have: } \Pi(Q_0) = \frac{(p_K^\alpha p_L^\beta)^{1/(\alpha+\beta)}}{(\alpha^\alpha \beta^\beta)^{1/(\alpha+\beta)}} \frac{(\alpha + \beta) Q_0^{1/r+1}}{A^{1/r}}.$$

On short-term, when factors  $x_1, \dots, \hat{x}_k, \dots, x_n$  remain constant, we have:

$$\Pi(Q_0) = p(Q_0) \cdot Q_0 - STC_k(Q_0) = MTC(Q_0) \cdot Q_0 - STC_k(Q_0) = AVTC_k'(Q_0) Q_0^2 - FTC_k$$

therefore:

$$\Pi(Q_0) = \left( \frac{1}{\alpha_k} - 1 \right) p_k \frac{Q_0^{\frac{1}{\alpha_k}}}{A^{\frac{1}{\alpha_k}} x_1^{\frac{\alpha_1}{\alpha_k}} \dots \hat{x}_k \dots x_n^{\frac{\alpha_n}{\alpha_k}}} - \sum_{\substack{i=1 \\ i \neq k}}^n p_i x_i = \left( \frac{1}{\alpha_k} - 1 \right) p_k x_k - \sum_{\substack{i=1 \\ i \neq k}}^n p_i x_i$$

Like a conclusion, the company will make a profit in the short-term if, under constancy factors  $x_1, \dots, \hat{x}_k, \dots, x_n$ , the amount of factor  $x_k$  will be higher than

$\frac{\sum_{\substack{i=1 \\ i \neq k}}^n p_i x_i}{\left( \frac{1}{\alpha_k} - 1 \right) p_k}$  if  $\alpha_k < 1$  and less than  $\frac{\sum_{\substack{i=1 \\ i \neq k}}^n p_i x_i}{\left( \frac{1}{\alpha_k} - 1 \right) p_k}$  if  $\alpha_k > 1$ . If  $\alpha_k = 1$  then the firm will incur losses.

For  $Q = AK^{\alpha}L^{\beta}$  we have that if  $K = \text{constant}$ , the company will make a profit in the short-term in the case  $\beta < 1$  if  $L > \frac{\beta p_K K}{(1 - \beta)p_L}$  and in the case  $\beta > 1$  if  $L < \frac{\beta p_K K}{(1 - \beta)p_L}$ . If  $\beta = 1$  the firm will incur losses.

The condition of profit maximization for an arbitrarily price  $p$ , depending on the factors of production, is:  $\max \Pi(x_1, \dots, x_n) = \max \left( pQ(x_1, \dots, x_n) - \sum_{i=1}^n p_i x_i \right)$  from where  $\frac{\partial Q}{\partial x_i} = \frac{p_i}{p}$ ,  $i = \overline{1, n}$  or otherwise:  $\alpha_i \frac{Q}{x_i} = \frac{p_i}{p}$  and finally:  $\bar{x}_i = \frac{\alpha_i p Q}{p_i}$ . Because  $Q$  is quasi-concave the solution of the characteristic system is the unique point of maximum. How  $Q = Ax_1^{\alpha_1} \dots x_n^{\alpha_n}$  we obtain that the appropriate production is:

$$\bar{Q} = A \bar{x}_1^{\alpha_1} \dots \bar{x}_n^{\alpha_n} = \frac{A p^r \prod_{k=1}^n \alpha_k^{\alpha_k}}{\prod_{k=1}^n p_k^{\alpha_k}} \bar{Q}^r \text{ therefore, if } r \neq 1: \bar{Q} = \left( \frac{\prod_{k=1}^n p_k^{\alpha_k}}{A p^r \prod_{k=1}^n \alpha_k^{\alpha_k}} \right)^{\frac{1}{r-1}} \text{ and the}$$

$$\text{factors: } \bar{x}_i = \left( \frac{\alpha_i^{r-1-\alpha_i} \prod_{\substack{k=1 \\ k \neq i}}^n p_k^{\alpha_k}}{A p_i^{r-1-\alpha_i} \prod_{\substack{k=1 \\ k \neq i}}^n \alpha_k^{\alpha_k}} \right)^{\frac{1}{r-1}}, i = \overline{1, n}.$$

The maximum profit is: 
$$\Pi(\bar{x}_1, \dots, \bar{x}_n) = (p - r) \left( \frac{\prod_{k=1}^n p_k^{\alpha_k}}{A p^r \prod_{k=1}^n \alpha_k^{\alpha_k}} \right)^{\frac{1}{r-1}}$$

If  $r=1$  the necessary condition for profit maximization is:  $\prod_{k=1}^n p_k^{\alpha_k} = A p \prod_{k=1}^n \alpha_k^{\alpha_k}$  or  $p$

must be:  $p = \frac{\prod_{k=1}^n p_k^{\alpha_k}}{A \prod_{k=1}^n \alpha_k^{\alpha_k}}$  therefore the amount of factors are not independent, that is,

for a fixed factor, let say  $x_s$ :  $\bar{Q} = \frac{A p_s \prod_{k=1}^n \alpha_k^{\alpha_k}}{\alpha_s \prod_{k=1}^n p_k^{\alpha_k}} \bar{x}_s$  and:  $\bar{x}_i = \frac{\alpha_i p_s}{\alpha_s p_i} \bar{x}_s, i=1, n, i \neq s$ . The

profit is:  $\Pi(\bar{x}_1, \dots, \bar{x}_n) = 0$  for any amount of  $x_s$ .

For  $Q = AK^\alpha L^\beta$  we have that, if  $\alpha + \beta \neq 1$ :

$$\bar{Q} = \left( \frac{p_K^\alpha p_L^\beta}{A p^r \alpha^\alpha \beta^\beta} \right)^{\frac{1}{\alpha + \beta - 1}}, \bar{K} = \left( \frac{\alpha^{\beta-1} p_L^\beta}{A p p_K^{\beta-1} \beta^\beta} \right)^{\frac{1}{\alpha + \beta - 1}}, \bar{L} = \left( \frac{\beta^{\alpha-1} p_K^\alpha}{A p p_L^{\alpha-1} \alpha^\alpha} \right)^{\frac{1}{\alpha + \beta - 1}},$$

$$\Pi(\bar{K}, \bar{L}) = (p - \alpha - \beta) \left( \frac{p_K^\alpha p_L^\beta}{A p^{\alpha + \beta} \alpha^\alpha \beta^\beta} \right)^{\frac{1}{\alpha + \beta - 1}}$$

and if  $\alpha + \beta = 1$  the necessary condition for profit maximization is:

$$p = \frac{p_K^{1-\beta} p_L^\beta}{A (1-\beta)^{1-\beta} \beta^\beta}, \bar{L} = \frac{\beta p_K}{(1-\beta) p_L} \bar{K}, \bar{Q} = \frac{A \beta^\beta p_K^\beta}{(1-\beta)^\beta p_L^\beta} \bar{K}, \Pi(\bar{K}, \bar{L}) = 0.$$

At a variable price  $p(Q)$  we have now:  $\Pi(Q) = p(Q) \cdot Q - CT(Q)$  therefore the necessary condition for profit maximization is  $\Pi'(Q) = 0$  therefore:  $p'(Q)Q + p(Q) - MTQ = 0$ .

Substituting the expression of MTC we obtain:  $p'(Q)Q + p(Q) - \Gamma Q^{1/r-1} = 0$  where

we noted:  $\Gamma = \frac{\left(\prod_{i=1}^n p_i^{\alpha_i}\right)^{1/r}}{A^{1/r} \left(\prod_{i=1}^n \alpha_i^{\alpha_i}\right)^{1/r}}$ . But this differential equation gives us:

$p(Q) = r\Gamma Q^{1/r-1} + \frac{C}{Q}$ ,  $C \in \mathbf{R}_+$  and the profit  $\Pi(Q) = p(Q) \cdot Q - CT(Q) = C$ . Therefore, for

the maximum of the profit  $\Pi(Q) = C$  we must have the price  $p(Q) = r\Gamma Q^{1/r-1} + \frac{C}{Q}$ ,

$C \in \mathbf{R}_+$ . Because:  $p'(Q) = \frac{(1-r)\Gamma Q^{1/r} - C}{Q^2}$  we have that if  $r > 1$ , the price will

decrease with production and if  $r < 1$  for  $Q \leq \left(\frac{C}{(1-r)\Gamma}\right)^r$  the price will decrease and

for  $Q \geq \left(\frac{C}{(1-r)\Gamma}\right)^r$  the price will increase. If  $r = 1$  we have that  $p(Q) = \Gamma + \frac{C}{Q}$  and the price will decrease with production.

For  $Q = AK^\alpha L^\beta$  we have  $\Gamma = \left(\frac{P_K^\alpha P_L^\beta}{A\alpha^\alpha \beta^\beta}\right)^{1/(\alpha+\beta)}$  and for the profit  $\Pi(Q) = C$  we must

have the price  $p(Q) = (\alpha + \beta)\Gamma Q^{1/(\alpha+\beta)-1} + \frac{C}{Q}$ . If  $\alpha + \beta > 1$ , the price will decrease

with production and if  $\alpha + \beta < 1$  for  $Q \leq \left(\frac{C}{(1-\alpha-\beta)\Gamma}\right)^{\alpha+\beta}$  the price will decrease and

for  $Q \geq \left(\frac{C}{(1-\alpha-\beta)\Gamma}\right)^{\alpha+\beta}$  the price will increase. If  $\alpha + \beta = 1$  we have that

$p(Q) = \Gamma + \frac{C}{Q}$  and the price will decrease with production.

## 5. The Hicks and Slutsky Effects for the Cobb-Douglas Production Function

Now consider the production function  $Q(x_1, \dots, x_n) = Ax_1^{\alpha_1} \dots x_n^{\alpha_n}$  and factor prices  $(p_i)_{i=\overline{1, n}}$ . The non-linear programming problem relative to maximize production at a given total cost ( $CT_0$ ) is:

$$\begin{cases} \max Ax_1^{\alpha_1} \dots x_n^{\alpha_n} \\ \sum_{k=1}^n p_k x_k = CT_0 \\ x_1, \dots, x_n \geq 0 \end{cases}$$

Because the objective function is quasi-concave and also the restriction (being affine) and the partial derivatives are all positive we find that the Karush-Kuhn-Tucker conditions are also sufficient. Therefore, we have:

$$\begin{cases} \frac{\frac{\partial Q}{\partial x_1}(\bar{x}_1, \dots, \bar{x}_n)}{p_1} = \dots = \frac{\frac{\partial Q}{\partial x_n}(\bar{x}_1, \dots, \bar{x}_n)}{p_n} \\ \sum_{k=1}^n p_k x_k = CT_0 \end{cases}$$

From the first equations we obtain:

$$\begin{cases} \frac{\alpha_1}{p_1 x_1} = \dots = \frac{\alpha_n}{p_n x_n} \\ \sum_{k=1}^n p_k x_k = CT_0 \end{cases}$$

therefore:

$$\begin{cases} x_k = \frac{\alpha_k p_n}{\alpha_n p_k} x_n, k = \overline{1, n-1} \\ \sum_{k=1}^n p_k x_k = CT_0 \end{cases}$$



Substituting the first  $n-1$  relations into the last we finally find that:

$$x_{0,k} = \frac{\alpha_k CT_0}{r p_k}, k = \overline{1, n} \quad \text{and} \quad \text{the} \quad \text{appropriate} \quad \text{production:}$$

$$Q_0(x_1, \dots, x_n) = A \frac{\prod_{i=1}^n \alpha_i^{\alpha_i}}{r^r \prod_{i=1}^n p_i^{\alpha_i}} CT_0^r.$$

Suppose now that some of the prices of factors of production (possibly after renumbering, we may assume that they are:  $x_1, \dots, x_s$ ) is modified to values  $\bar{p}_1, \dots, \bar{p}_s$ , the rest remain constant.

From the above, it results:

$$\left\{ \begin{array}{l} x_{f,k} = \frac{\alpha_k CT_0}{r p_k}, k = \overline{1, s} \\ \\ x_{f,k} = \frac{\alpha_k CT_0}{r p_k}, k = \overline{s+1, n} \\ \\ Q_f = A \frac{\prod_{i=1}^n \alpha_i^{\alpha_i}}{r^r \prod_{i=1}^s \bar{p}_i^{\alpha_i} \prod_{j=s+1}^n p_j^{\alpha_j}} CT_0^r \end{array} \right.$$

We will apply in the following, the method of Hicks. To an input price change, let consider that it remains unchanged, leading thus to a change of the total cost. We therefore have:

$$A \frac{\prod_{i=1}^n \alpha_i^{\alpha_i}}{r^r \prod_{i=1}^s \bar{p}_i^{\alpha_i} \prod_{j=s+1}^n p_j^{\alpha_j}} \overline{CT_0^r} = A \frac{\prod_{i=1}^n \alpha_i^{\alpha_i}}{r^r \prod_{j=1}^n p_j^{\alpha_j}} CT_0^r$$

from where:

$$\overline{CT}_0^r = \frac{\prod_{i=1}^s \overline{p}_i^{\alpha_i}}{\prod_{j=1}^s p_j^{\alpha_j}} CT_0^r$$

With the new total cost, the optimal amounts of inputs become:

$$\left\{ \begin{array}{l} x_{int,k} = \frac{\alpha_k \left( \prod_{i=1}^s \overline{p}_i^{\alpha_i} \right)^{1/r} CT_0}{r \overline{p}_k \left( \prod_{j=1}^s p_j^{\alpha_j} \right)^{1/r}}, k = \overline{1, s} \\ x_{int,d} = \frac{\alpha_d \left( \prod_{i=1}^s \overline{p}_i^{\alpha_i} \right)^{1/r} CT_0}{r p_d \left( \prod_{j=1}^s p_j^{\alpha_j} \right)^{1/r}}, d = \overline{s+1, n} \end{array} \right.$$

The Hicks substitution effect which preserves the production is therefore:

$$\left\{ \begin{array}{l} \Delta_{IH} x_k = x_{int,k} - x_{0,k} = \left[ \frac{p_k}{\overline{p}_k} \prod_{i=1}^s \left( \frac{\overline{p}_i}{p_i} \right)^{\alpha_i/r} - 1 \right] \frac{\alpha_k CT_0}{r p_k}, k = \overline{1, s} \\ \Delta_{IH} x_d = x_{int,d} - x_{0,d} = \left[ \prod_{i=1}^s \left( \frac{\overline{p}_i}{p_i} \right)^{\alpha_i/r} - 1 \right] \frac{\alpha_d CT_0}{r p_d}, d = \overline{s+1, n} \end{array} \right.$$

The difference caused by the old cost instead the new total cost one is therefore:

$$\left\{ \begin{array}{l} \Delta_{2H} x_k = x_{f,k} - x_{int,k} = \left[ 1 - \prod_{i=1}^s \left( \frac{\overline{p}_i}{p_i} \right)^{\alpha_i/r} \right] \frac{\alpha_k CT_0}{r \overline{p}_k}, k = \overline{1, s} \\ \Delta_{2H} x_d = x_{f,d} - x_{int,d} = \left[ 1 - \prod_{i=1}^s \left( \frac{\overline{p}_i}{p_i} \right)^{\alpha_i/r} \right] \frac{\alpha_d CT_0}{r p_d}, d = \overline{s+1, n} \end{array} \right.$$

Let now calculate the new prices influence to the effects of substitution and of new cost in the Hicks effect.

We have:

$$\frac{\partial \Delta_{1H} x_k}{\partial \bar{p}_k} = -\frac{p_k}{\bar{p}_k^2} \prod_{i=1}^s \left( \frac{\bar{p}_i}{p_i} \right)^{\alpha_i/r} \frac{\alpha_k CT_0}{r^2 p_k} \sum_{\substack{i=1 \\ i \neq k}}^n \alpha_i, k = \overline{1, s}$$

$$\frac{\partial \Delta_{2H} x_k}{\partial \bar{p}_k} = -\frac{\alpha_k CT_0}{r^2 \bar{p}_k^2} \left[ \prod_{i=1}^s \left( \frac{\bar{p}_i}{p_i} \right)^{\alpha_i/r} \sum_{\substack{j=1 \\ j \neq k}}^n \alpha_j - r \right], k = \overline{1, s}$$

$$\frac{\partial \Delta_{1H} x_k}{\partial \bar{p}_t} = \prod_{i=1}^s \left( \frac{\bar{p}_i}{p_i} \right)^{\alpha_i/r} \frac{\alpha_k \alpha_t CT_0}{r^2 \bar{p}_k \bar{p}_t}, k = \overline{1, s}, t = \overline{1, s}, t \neq k$$

$$\frac{\partial \Delta_{2H} x_k}{\partial \bar{p}_t} = -\prod_{i=1}^s \left( \frac{\bar{p}_i}{p_i} \right)^{\alpha_i/r} \frac{\alpha_k \alpha_t CT_0}{r^2 \bar{p}_k \bar{p}_t}, k = \overline{1, s}, t = \overline{1, s}, t \neq k$$

$$\frac{\partial \Delta_{1H} x_d}{\partial \bar{p}_k} = \prod_{i=1}^s \left( \frac{\bar{p}_i}{p_i} \right)^{\alpha_i/r} \frac{\alpha_k \alpha_d CT_0}{r^2 p_d \bar{p}_k}, d = \overline{s+1, n}, k = \overline{1, s}$$

$$\frac{\partial \Delta_{2H} x_d}{\partial \bar{p}_k} = -\prod_{i=1}^s \left( \frac{\bar{p}_i}{p_i} \right)^{\alpha_i/r} \frac{\alpha_d \alpha_k CT_0}{r^2 p_d \bar{p}_k}, d = \overline{s+1, n}, k = \overline{1, s}$$

After these relations, it follows that the effect of substitution at the increase of the price  $x_k$ ,  $k = \overline{1, s}$  is reduced, while the effect of new cost is reduced if

$$\left[ \prod_{i=1}^s \left( \frac{\bar{p}_i}{p_i} \right)^{\alpha_i/r} \sum_{\substack{j=1 \\ j \neq k}}^n \alpha_j - r \right] > 0 \text{ or it increase if } \left[ \prod_{i=1}^s \left( \frac{\bar{p}_i}{p_i} \right)^{\alpha_i/r} \sum_{\substack{j=1 \\ j \neq k}}^n \alpha_j - r \right] < 0.$$

We shall apply now the Slutsky method for our analysis.

At the modify of the price of the factors  $x_1, \dots, x_s$ , the total cost for the same optimal combination of factors is:

$$CT_{int} = \frac{CT_0}{r} \left( \sum_{j=s+1}^n \alpha_j + \sum_{i=1}^s \alpha_i \frac{\bar{p}_i}{p_i} \right)$$

therefore:

$$\begin{cases} x_{\text{int},k} = \frac{\alpha_k \text{CT}_0}{r^2 \bar{p}_k} \left( \sum_{j=s+1}^n \alpha_j + \sum_{i=1}^s \alpha_i \frac{\bar{p}_i}{p_i} \right), k = \overline{1, s} \\ x_{\text{int},d} = \frac{\alpha_d \text{CT}_0}{r^2 p_d} \left( \sum_{j=s+1}^n \alpha_j + \sum_{i=1}^s \alpha_i \frac{\bar{p}_i}{p_i} \right), d = \overline{s+1, n} \end{cases}$$

The appropriate production is:

$$Q_{\text{int}}(x_{\text{int},1}, \dots, x_{\text{int},n}) = A \frac{\prod_{i=1}^n \alpha_i^{\alpha_i} \text{CT}_0^r}{r^{2r} \prod_{k=1}^s \bar{p}_k^{\alpha_k} \prod_{d=s+1}^n p_d^{\alpha_d}} \left( \sum_{j=s+1}^n \alpha_j + \sum_{i=1}^s \alpha_i \frac{\bar{p}_i}{p_i} \right)^r$$

The Slutsky substitution effect which not preserves the production is therefore:

$$\begin{cases} \Delta_{1S} x_k = x_{\text{int},k} - x_{0,k} = \frac{\alpha_k \text{CT}_0}{r^2 \bar{p}_k} \left( \sum_{j=s+1}^n \alpha_j + \sum_{i=1}^s \alpha_i \frac{\bar{p}_i}{p_i} - r \frac{\bar{p}_k}{p_k} \right), k = \overline{1, s} \\ \Delta_{1S} x_d = x_{\text{int},d} - x_{0,d} = \frac{\alpha_d \text{CT}_0}{r^2 p_d} \left( \sum_{j=s+1}^n \alpha_j + \sum_{i=1}^s \alpha_i \frac{\bar{p}_i}{p_i} - r \right), d = \overline{s+1, n} \end{cases}$$

and the difference caused by the old production instead the new production one is therefore:

$$\begin{cases} \Delta_{2S} x_k = x_{f,k} - x_{\text{int},k} = \frac{\alpha_k \text{CT}_0}{r^2 \bar{p}_k} \sum_{i=1}^s \alpha_i \left( 1 - \frac{\bar{p}_i}{p_i} \right), k = \overline{1, s} \\ \Delta_{2S} x_d = x_{f,d} - x_{\text{int},d} = \frac{\alpha_d \text{CT}_0}{r^2 p_d} \sum_{i=1}^s \alpha_i \left( 1 - \frac{\bar{p}_i}{p_i} \right), d = \overline{s+1, n} \end{cases}$$

Let us now calculate the influence of the new prices on the effects of substitution and new cost in Slutsky effect.

We have:

$$\frac{\partial \Delta_{1S} x_k}{\partial \bar{p}_k} = - \frac{\alpha_k \text{CT}_0}{r^2 \bar{p}_k^2} \left( \sum_{j=s+1}^n \alpha_j + \sum_{\substack{i=1 \\ i \neq k}}^s \alpha_i \frac{\bar{p}_i}{p_i} \right), k = \overline{1, s}$$

$$\frac{\partial \Delta_{2S} x_k}{\partial \bar{p}_k} = - \frac{\alpha_k \text{CT}_0}{r^2 \bar{p}_k^2} \left( \alpha_k + \sum_{\substack{i=1 \\ i \neq k}}^s \alpha_i \left( 1 - \frac{\bar{p}_i}{p_i} \right) \right), k = \overline{1, s}$$

$$\frac{\partial \Delta_{1S} x_k}{\partial \bar{p}_t} = \frac{\alpha_k \alpha_t C T_0}{r^2 \bar{p}_k p_t}, k = \overline{1, s}, t = \overline{1, s}, t \neq k$$

$$\frac{\partial \Delta_{2S} x_k}{\partial \bar{p}_t} = -\frac{\alpha_k \alpha_t C T_0}{r^2 \bar{p}_k p_t}, k = \overline{1, s}, t = \overline{1, s}, t \neq k$$

$$\frac{\partial \Delta_{1S} x_d}{\partial \bar{p}_k} = \frac{\alpha_d \alpha_k C T_0}{r^2 p_d p_k}, d = \overline{s+1, n}, k = \overline{1, s}$$

$$\frac{\partial \Delta_{2S} x_d}{\partial \bar{p}_k} = -\frac{\alpha_d \alpha_k C T_0}{r^2 p_d p_k}, d = \overline{s+1, n}, k = \overline{1, s}$$

Therefore the effect of substitution at a price increase of the factor  $x_k, k = \overline{1, s}$  is reduced, while the effect due to production decrease in case  $\alpha_k + \sum_{\substack{i=1 \\ i \neq k}}^s \alpha_i \left(1 - \frac{\bar{p}_i}{p_i}\right) > 0$

or it increase if  $\alpha_k + \sum_{\substack{i=1 \\ i \neq k}}^s \alpha_i \left(1 - \frac{\bar{p}_i}{p_i}\right) < 0$ .

## 6. Production Efficiency of Cobb-Douglas Production Function

Let now two Cobb-Douglas production functions for two goods  $\Phi, \Psi$  and a number of  $n$  inputs  $F_1, \dots, F_n$  available in quantities  $\bar{x}_1, \dots, \bar{x}_n$ . The production functions of  $\Phi$  or  $\Psi$  are:

$$Q_\Phi(x_1, \dots, x_n) = A x_1^{\alpha_1} \dots x_n^{\alpha_n}, Q_\Psi(x_1, \dots, x_n) = B x_1^{\beta_1} \dots x_n^{\beta_n}$$

appropriate to the consumption of  $x_k$  units of factor  $F_k, k = \overline{1, n}$ .

We have seen that:  $\eta_{\Phi, x_i} = A \alpha_i x_1^{\alpha_1} \dots x_i^{\alpha_i - 1} \dots x_n^{\alpha_n}, \eta_{\Psi, x_i} = B \beta_i x_1^{\beta_1} \dots x_i^{\beta_i - 1} \dots x_n^{\beta_n}, i = \overline{1, n}$ .

The production contract curve satisfies:

$$\frac{\eta_{\Phi, x_i}}{\eta_{\Psi, x_i}} = \frac{A \alpha_i \bar{x}_1^{\alpha_1} \dots \bar{x}_i^{\alpha_i - 1} \dots \bar{x}_n^{\alpha_n}}{B \beta_i (\bar{x}_1 - x_1)^{\beta_1} \dots (\bar{x}_i - x_i)^{\beta_i - 1} \dots (\bar{x}_n - x_n)^{\beta_n}} = \mu, i = \overline{1, n}$$

Dividing for  $i \neq j$ :  $x_j = \frac{\alpha_j \beta_1 \bar{x}_j x_i}{(\alpha_j \beta_1 - \alpha_i \beta_j) x_i + \alpha_i \beta_j \bar{x}_i}$  and for  $i=1$ :

$x_j = \frac{\alpha_j \beta_1 \bar{x}_j x_1}{(\alpha_j \beta_1 - \alpha_1 \beta_j) x_1 + \alpha_1 \beta_j \bar{x}_1}$ ,  $j = \overline{2, n}$ . Finally, for  $x_1 = \lambda$  we have the equation of

production contract curve:

$$\begin{cases} x_1 = \lambda \\ x_j = \frac{\alpha_j \beta_1 \bar{x}_j \lambda}{(\alpha_j \beta_1 - \alpha_1 \beta_j) \lambda + \alpha_1 \beta_j \bar{x}_1} \end{cases}, \lambda \in \mathbf{R}$$

If we consider now the input prices:  $p_1, \dots, p_n$  we have that for the production contract curve:  $x_1 = g_1(\lambda), \dots, x_n = g_n(\lambda)$ ,  $\lambda \in \mathbf{R}$ :

$$\begin{cases} x_1 = g_1(\lambda) = \lambda \\ x_j = g_j(\lambda) = \frac{\alpha_j \beta_1 \bar{x}_j \lambda}{(\alpha_j \beta_1 - \alpha_1 \beta_j) \lambda + \alpha_1 \beta_j \bar{x}_1} \end{cases}, \lambda \in \mathbf{R}$$

and:

$$p_j = \frac{\eta_{\phi, x_j}(g_1(\lambda), \dots, g_n(\lambda))}{\eta_{\phi, x_1}(g_1(\lambda), \dots, g_n(\lambda))} v = \frac{A \alpha_j g_1^{\alpha_1}(\lambda) \dots g_j^{\alpha_j-1}(\lambda) \dots g_n^{\alpha_n}(\lambda)}{A \alpha_1 g_1^{\alpha_1-1}(\lambda) g_2^{\alpha_2}(\lambda) \dots g_n^{\alpha_n}(\lambda)} v = \frac{\alpha_j g_1(\lambda)}{\alpha_1 g_j(\lambda)} v = \frac{(\alpha_j \beta_1 - \alpha_1 \beta_j) \lambda + \alpha_1 \beta_j \bar{x}_1}{\alpha_1 \beta_1 \bar{x}_j} v$$

,  $j = \overline{1, n}$ .

For  $v=1$  we then obtain:  $p_1=1$ ,  $p_j = \frac{(\alpha_j \beta_1 - \alpha_1 \beta_j) \lambda + \alpha_1 \beta_j \bar{x}_1}{\alpha_1 \beta_1 \bar{x}_j}$ ,  $j = \overline{2, n}$ .

If the initial allocation of factors of production was  $x_\phi = (a_1, \dots, a_n)$  we have that

$\sum_{j=1}^n p_j (a_j - x_j) = 0$  therefore:

$$\lambda^* = \frac{a_1 \alpha_1 \beta_1 + \alpha_1 \bar{x}_1 \sum_{j=2}^n \frac{a_j \beta_j}{\bar{x}_j}}{\alpha_1 \beta_1 - \sum_{j=2}^n \frac{a_j \alpha_j \beta_1 - a_j \alpha_1 \beta_j - \alpha_j \beta_1 \bar{x}_j}{\bar{x}_j}} = \frac{a_1 \alpha_1 \beta_1 + \alpha_1 \bar{x}_1 \sum_{j=2}^n \frac{a_j \beta_j}{\bar{x}_j}}{r_1 \beta_1 - \sum_{j=2}^n \frac{a_j (\alpha_j \beta_1 - \alpha_1 \beta_j)}{\bar{x}_j}} \text{ where } r_1 = \sum_{k=1}^n \alpha_k.$$

For this value we find now the final allocation:  $\begin{cases} x_1 = \lambda^* \\ x_j = \frac{\alpha_j \beta_1 \bar{x}_j \lambda^*}{(\alpha_j \beta_1 - \alpha_1 \beta_j) \lambda^* + \alpha_1 \beta_j \bar{x}_1} \end{cases}$

If now the two production functions are:  $Q_\Phi(K,L)=AK^{\alpha_1}L^{\alpha_2}$ ,  $Q_\Psi(K,L)=AK^{\beta_1}L^{\beta_2}$  we have that for limited quantities of capital ( $\bar{K}$ ) and labor ( $\bar{L}$ ) the equation of production contract curve is:

$$\begin{cases} K = \lambda \\ L = \frac{\alpha_2\beta_1\bar{L}\lambda}{(\alpha_2\beta_1 - \alpha_1\beta_2)\lambda + \alpha_1\beta_2\bar{K}}, \lambda \in \mathbf{R} \end{cases}$$

and the final allocation for an initial one:  $x_\Phi = (K_1, L_1)$ :

$$\begin{cases} K = \lambda^* \\ L = \frac{\alpha_2\beta_1\bar{L}\lambda^*}{(\alpha_2\beta_1 - \alpha_1\beta_2)\lambda^* + \alpha_1\beta_2\bar{K}} \end{cases} \text{ where } \lambda^* = \frac{K_1\bar{L}\alpha_1\beta_1 + \alpha_1\bar{K}L_1\beta_2}{(\alpha_1 + \alpha_2)\beta_1\bar{L} - L_1(\alpha_2\beta_1 - \alpha_1\beta_2)}$$

### 7. The Concrete Determination of the Cobb-Douglas Production Function

Considering an affine function:  $f:\mathbf{R}^n \rightarrow \mathbf{R}$ ,  $f(x_1, \dots, x_n) = \beta_1x_1 + \dots + \beta_nx_n + \beta_{n+1}$  and a set of  $m > n+1$  data:  $(x_1^k, \dots, x_n^k, f^k)$ ,  $k = \overline{1, m}$  the problem of determining  $\beta_i$ ,  $i = \overline{1, n+1}$  using the least square method is to minimize the expression:

$\sum_{k=1}^m (\beta_1x_1^k + \dots + \beta_nx_n^k + \beta_{n+1} - f^k)^2$  that is to solve the system:

$$\begin{cases} \beta_1 \sum_{k=1}^m x_1^k x_1^k + \dots + \beta_n \sum_{k=1}^m x_n^k x_n^k + \beta_{n+1} \sum_{k=1}^m x_i^k = \sum_{k=1}^m f^k x_i^k, i = \overline{1, n} \\ \beta_1 \sum_{k=1}^m x_1^k + \dots + \beta_n \sum_{k=1}^m x_n^k + m\beta_{n+1} = \sum_{k=1}^m f^k \end{cases}$$

Considering the matrix:

$$\Theta = \begin{pmatrix} \sum_{k=1}^m (x_1^k)^2 & \sum_{k=1}^m x_1^k x_2^k & \dots & \sum_{k=1}^m x_1^k \\ \sum_{k=1}^m x_1^k x_2^k & \sum_{k=1}^m (x_2^k)^2 & \dots & \sum_{k=1}^m x_2^k \\ \dots & \dots & \dots & \dots \\ \sum_{k=1}^m x_1^k & \sum_{k=1}^m x_2^k & \dots & m \end{pmatrix}$$

and  $\Theta_{ij}$  the cofactor of the (i,j)-element in  $\Theta$  we will obtain:

$$\beta_i = \frac{\Theta_{1i} \sum_{k=1}^m f^k x_1^k + \dots + \Theta_{ni} \sum_{k=1}^m f^k x_n^k + \Theta_{n+1,i} \sum_{k=1}^m f^k}{\det \Theta}, i = \overline{1, n+1}$$

Considering now a production function  $Q(x_1, \dots, x_n) = Ax_1^{\alpha_1} \dots x_n^{\alpha_n}$  we put the problem of concrete determination of the parameters  $A, \alpha_i, i = \overline{1, n}$ .

Let therefore a set of  $m > n+1$  data:  $(x_1^k, \dots, x_n^k, Q^k), k = \overline{1, m}$ .

Considering the logarithm of  $Q$ , we have:  $\ln Q = \alpha_1 \ln x_1 + \dots + \alpha_n \ln x_n + \ln A$  therefore we will modify the data set to the new one:  $(\ln x_1^k, \dots, \ln x_n^k, \ln Q^k), k = \overline{1, m}$ .

From above:

$$\alpha_i = \frac{\Theta_{1i} \sum_{k=1}^m \ln Q^k \ln x_1^k + \dots + \Theta_{ni} \sum_{k=1}^m \ln Q^k \ln x_n^k + \Theta_{n+1,i} \sum_{k=1}^m \ln Q^k}{\det \Theta}, i = \overline{1, n}$$

$$\ln A = \frac{\Theta_{1,n+1} \sum_{k=1}^m \ln Q^k \ln x_1^k + \dots + \Theta_{n,n+1} \sum_{k=1}^m \ln Q^k \ln x_n^k + \Theta_{n+1,n+1} \sum_{k=1}^m \ln Q^k}{\det \Theta}$$

where  $\Theta = \begin{pmatrix} \sum_{k=1}^m (\ln x_1^k)^2 & \sum_{k=1}^m \ln x_1^k \ln x_2^k & \dots & \sum_{k=1}^m \ln x_1^k \\ \sum_{k=1}^m \ln x_1^k \ln x_2^k & \sum_{k=1}^m (\ln x_2^k)^2 & \dots & \sum_{k=1}^m \ln x_2^k \\ \dots & \dots & \dots & \dots \\ \sum_{k=1}^m \ln x_1^k & \sum_{k=1}^m \ln x_2^k & \dots & m \end{pmatrix}$ .

For the Cobb-Douglas  $Q=Q(K,L)=AK^\alpha L^\beta$  we have therefore, for the set:  $(K_i, L_i, Q_i)_{i=\overline{1, m}}$ :



$$\alpha = \frac{\begin{vmatrix} \sum_{i=1}^m \ln Q_i \ln K_i & \sum_{i=1}^m \ln K_i \ln L_i & \sum_{i=1}^m \ln K_i \\ \sum_{i=1}^m \ln Q_i \ln L_i & \sum_{i=1}^m \ln^2 L_i & \sum_{i=1}^m \ln L_i \\ \sum_{i=1}^m \ln Q_i & \sum_{i=1}^m \ln L_i & m \end{vmatrix}}{\begin{vmatrix} \sum_{i=1}^m \ln^2 K_i & \sum_{i=1}^m \ln K_i \ln L_i & \sum_{i=1}^m \ln K_i \\ \sum_{i=1}^m \ln K_i \ln L_i & \sum_{i=1}^m \ln^2 L_i & \sum_{i=1}^m \ln L_i \\ \sum_{i=1}^m \ln K_i & \sum_{i=1}^m \ln L_i & m \end{vmatrix}},$$

$$\beta = \frac{\begin{vmatrix} \sum_{i=1}^m \ln^2 K_i & \sum_{i=1}^m \ln Q_i \ln K_i & \sum_{i=1}^m \ln K_i \\ \sum_{i=1}^m \ln K_i \ln L_i & \sum_{i=1}^m \ln Q_i \ln L_i & \sum_{i=1}^m \ln L_i \\ \sum_{i=1}^m \ln K_i & \sum_{i=1}^m \ln Q_i & m \end{vmatrix}}{\begin{vmatrix} \sum_{i=1}^m \ln^2 K_i & \sum_{i=1}^m \ln K_i \ln L_i & \sum_{i=1}^m \ln K_i \\ \sum_{i=1}^m \ln K_i \ln L_i & \sum_{i=1}^m \ln^2 L_i & \sum_{i=1}^m \ln L_i \\ \sum_{i=1}^m \ln K_i & \sum_{i=1}^m \ln L_i & m \end{vmatrix}},$$

$$\ln A = \frac{\begin{vmatrix} \sum_{i=1}^m \ln^2 K_i & \sum_{i=1}^m \ln K_i \ln L_i & \sum_{i=1}^m \ln Q_i \ln K_i \\ \sum_{i=1}^m \ln K_i \ln L_i & \sum_{i=1}^m \ln^2 L_i & \sum_{i=1}^m \ln Q_i \ln L_i \\ \sum_{i=1}^m \ln K_i & \sum_{i=1}^m \ln L_i & \sum_{i=1}^m \ln Q_i \end{vmatrix}}{\begin{vmatrix} \sum_{i=1}^m \ln^2 K_i & \sum_{i=1}^m \ln K_i \ln L_i & \sum_{i=1}^m \ln K_i \\ \sum_{i=1}^m \ln K_i \ln L_i & \sum_{i=1}^m \ln^2 L_i & \sum_{i=1}^m \ln L_i \\ \sum_{i=1}^m \ln K_i & \sum_{i=1}^m \ln L_i & m \end{vmatrix}}$$

**8. Conclusions**

The above analysis reveals several aspects. On the one hand were highlighted conditions for the existence of the Cobb-Douglas function. Also were calculated the main indicators of its and short and long-term costs. It has also been studied the dependence of long-term cost of the parameters of the production function. The determination of profit was made both for perfect competition market and maximize its conditions. Also we have studied the effects of Hicks and Slutsky and the production efficiency problem.

**9. Appendix**

A.1. Mathematical concepts

A function  $Q: D \subset \mathbf{R}^n \rightarrow \mathbf{R}$ ,  $D$  – convex set, is quasi-concave if:

$$Q(\lambda x + (1-\lambda)y) \geq \min(Q(x), Q(y)) \quad \forall \lambda \in [0, 1] \quad \forall x, y \in D$$

and is strictly quasi-concave if:

$$Q(\lambda x + (1-\lambda)y) > \min(Q(x), Q(y)) \quad \forall \lambda \in (0, 1) \quad \forall x, y \in D$$

A function  $Q: D \subset \mathbf{R}^n \rightarrow \mathbf{R}$ ,  $D$  – convex set, is quasi-convex if:

$$Q(\lambda x + (1-\lambda)y) \leq \max(Q(x), Q(y)) \quad \forall \lambda \in [0, 1] \quad \forall x, y \in D$$

and is strictly quasi-convex if:

$$Q(\lambda x + (1-\lambda)y) < \max(Q(x), Q(y)) \quad \forall \lambda \in (0, 1) \quad \forall x, y \in D$$

Geometrically speaking, a quasi-concave function has the property to be above the lowest values recorded at the ends of some segment. This property is equivalent with the convexity of the set  $Q^{-1}[a, \infty) = \{x \in D \mid Q(x) \geq a\} \forall a \in \mathbf{R}$ .

Note also that if  $f$  and  $g$  are arbitrary functions:

- $f$  – quasi-concave (quasi-convex) implies that  $-f$  is quasi-convex (quasi-concave);
- $f$  – strictly quasi-concave (quasi-convex) implies that  $f$  is quasi-concave (quasi-convex);
- $f$  - quasi-concave (quasi-convex) implies that  $\alpha f$  is quasi-concave (quasi-convex) for any  $\alpha \geq 0$ ;
- $f, g$  - quasi-concave (quasi-convex) imply that  $\min(\alpha f, \beta g)$  ( $\max(\alpha f, \beta g)$ ) is quasi-concave (quasi-convex) for any  $\alpha, \beta \geq 0$ ;
- $f$  - quasi-concave (quasi-convex) and  $g: \mathbf{R} \rightarrow \mathbf{R}$  is increasing imply that  $g \circ f: D \rightarrow \mathbf{R}$  is quasi-concave (quasi-convex);
- $f \in C^1(D)$  is (strictly) quasi-concave if and only if:  $f(x) \geq f(y) \Rightarrow \sum_{i=1}^n \frac{\partial f}{\partial x_i}(y)(x_i - y_i) \geq (>) 0 \forall x, y \in D$ ;
- $f \in C^1(D)$  is (strictly) quasi-convex if and only if:  $f(x) \geq f(y) \Rightarrow \sum_{i=1}^n \frac{\partial f}{\partial x_i}(x)(x_i - y_i) \geq (>) 0 \forall x, y \in D$ ;
- A monotonically function  $f: D \subset \mathbf{R} \rightarrow \mathbf{R}$  is quasi-concave and quasi-convex;
- Any affine function is quasi-concave and quasi-convex.

Considering now the bordered hessian matrix:

$$H^B(f) = \begin{pmatrix} 0 & f'_{x_1} & f'_{x_2} & \dots & f'_{x_n} \\ f'_{x_1} & f''_{x_1x_1} & f''_{x_1x_2} & \dots & f''_{x_1x_n} \\ f'_{x_2} & f''_{x_1x_2} & f''_{x_2x_2} & \dots & f''_{x_2x_n} \\ \dots & \dots & \dots & \dots & \dots \\ f'_{x_n} & f''_{x_1x_n} & f''_{x_2x_n} & \dots & f''_{x_nx_n} \end{pmatrix}$$

and the bordered principal diagonal determinants:

$$\Delta_k^B = \begin{vmatrix} 0 & f'_{x_1} & f'_{x_2} & \dots & f'_{x_k} \\ f'_{x_1} & f''_{x_1x_1} & f''_{x_1x_2} & \dots & f''_{x_1x_k} \\ f'_{x_2} & f''_{x_1x_2} & f''_{x_2x_2} & \dots & f''_{x_2x_k} \\ \dots & \dots & \dots & \dots & \dots \\ f'_{x_k} & f''_{x_1x_k} & f''_{x_2x_k} & \dots & f''_{x_kx_k} \end{vmatrix}, k = \overline{1, n}$$

we have the following theorems:

**Theorem** If the function  $f: D \subset \mathbf{R}_+^n \rightarrow \mathbf{R}$ ,  $D$  – convex,  $f \in C^2(D)$  is quasi-concave then  $(-1)^k \Delta_k^B \geq 0, k = \overline{1, n}$ .

**Theorem** In order that the function  $f: D \subset \mathbf{R}_+^n \rightarrow \mathbf{R}$ ,  $D$  – convex,  $f \in C^2(D)$  be quasi-concave is sufficient that  $(-1)^k \Delta_k^B > 0, k = \overline{1, n}$ .

A.2. The main indicators of production functions

Let a production function:

$$Q: D_p \rightarrow \mathbf{R}_+, (x_1, \dots, x_n) \rightarrow Q(x_1, \dots, x_n) \in \mathbf{R}_+, \forall (x_1, \dots, x_n) \in D_p$$

We will call the marginal productivity relative to a production factor  $x_i$ :  $\eta_{x_i} = \frac{\partial Q}{\partial x_i}$  representing the trend of variation of production at the variation of the factor  $x_i$ . In particular, for a production function of the form:  $Q=Q(K,L)$  we have  $\eta_K = \frac{\partial Q}{\partial K}$  - called the marginal productivity of capital and  $\eta_L = \frac{\partial Q}{\partial L}$  - called the marginal productivity of labor.

We call the average productivity relative to a production factor  $x_i$ :  $w_{x_i} = \frac{Q}{x_i}$  representing the value of production at the consumption of a unit of factor  $x_i$ . In particular, for a production function of the form:  $Q=Q(K,L)$  we have:  $w_K = \frac{Q}{K}$  - called the productivity of capital, and  $w_L = \frac{Q}{L}$  - the productivity of labor.

Considering the factors  $i$  and  $j$  with  $i \neq j$ , we define the restriction of production area:  $P_{ij} = \{(x_1, \dots, x_n) \mid x_k = a_k = \text{const}, k = \overline{1, n}, k \neq i, j, x_i, x_j \in D_p\}$  relative to the two factors

when the others have fixed values. Also, let:  $D_{ij} = \{(x_i, x_j) \mid (x_1, \dots, x_n) \in P_{ij}\}$  - the domain of production relative to factors i and j.

We define:  $Q_{ij}: D_{ij} \rightarrow \mathbf{R}_+$  - the restriction of the production function to the factors i and j, i.e.:  $Q_{ij}(x_i, x_j) = Q(a_1, \dots, a_{i-1}, x_i, a_{i+1}, \dots, a_{j-1}, x_j, a_{j+1}, \dots, a_n)$ . The functions  $Q_{ij}$  define a surface in  $\mathbf{R}^3$  for every pair of factors (i,j).

We will call partial marginal rate of technical substitution of the factors i and j, relative to  $D_{ij}$  (caeteris paribus), the opposite change in the amount of factor j to substitute a variation of the quantity of factor i in the situation of conservation production level.

We will note:  $RMS(i,j) = -\frac{dx_j}{dx_i}$  and we have, since  $Q_{ij}(x_i, x_j) = Q_0 = \text{constant}$ :

$$RMS(i,j) = \frac{\eta_{x_i} \Big|_{D_{ij}}}{\eta_{x_j} \Big|_{D_{ij}}}. \text{ Obviously } RMS(i,j) = \frac{1}{RMS(j,i)}. \text{ We also define the global}$$

marginal rate of substitution between the i-th factor and the others:

$$RMS(i) = \frac{\eta_{x_i}}{\sqrt{\sum_{\substack{j=1 \\ j \neq i}}^n \eta_{x_j}^2}}. \text{ The global marginal rate of technical substitution is the}$$

minimum (in the meaning of norm) of changes in consumption of factors so that the total production remain unchanged.

In particular, for a production function of the form:  $Q = Q(K, L)$  we have:

$$RMS(K,L) = RMS(K) = \frac{\eta_K}{\eta_L}, \quad RMS(L,K) = RMS(L) = \frac{\eta_L}{\eta_K}.$$

It is called elasticity of production in relation to a production factor  $x_i$ :

$$\varepsilon_{x_i} = \frac{\frac{\partial Q}{\partial x_i}}{\frac{Q}{x_i}} = \frac{\eta_{x_i}}{w_{x_i}} - \text{the relative variation of production at the relative variation of}$$

factor  $x_i$ .

In particular, for a production function of the form:  $Q=Q(K,L)$  we have  $\varepsilon_K = \frac{\eta_K}{w_K}$  - called the elasticity of production in relation to the capital and  $\varepsilon_L = \frac{\eta_L}{w_L}$  - the elasticity factor of production in relation to the labor.

Let note now for arbitrary factors  $x_i, x_j$ :  $\xi_{ij} = \frac{x_i}{x_j}$ ,  $i, j = \overline{1, n}$ ,  $i \neq j$  and we call the factor endowment ratio with the factor  $i$  relative to factor  $j$ .

It is called the elasticity of marginal rate of technical substitution for a production

function relative to inputs  $i$  and  $j$ :  $\sigma_{ij} = \frac{\frac{\partial \text{RMS}(i, j)}{\partial \xi_{ij}}}{\text{RMS}(i, j) \cdot \xi_{ij}}$ ,  $i, j = \overline{1, n}$ ,  $i \neq j$  and represents the

relative variation of marginal rate of technical substitution relative to factors  $i$  and  $j$  at the relative variation of the factor endowment ratio with factor  $i$  relative to factor  $j$ .

We have therefore: 
$$\sigma_{ij} = \frac{x_i \frac{\partial \text{RMS}(i, j)}{\partial x_i}}{\text{RMS}(i, j)} = x_i \frac{\partial \ln \text{RMS}(i, j)}{\partial x_i}.$$

Considering now a production function  $Q: D_p \rightarrow \mathbf{R}_+$ ,  $(x_1, \dots, x_n) \rightarrow Q(x_1, \dots, x_n) \in \mathbf{R}_+$   $\forall (x_1, \dots, x_n) \in D_p$ , homogenous of degree  $r$ , let note for an arbitrary factor (for example  $x_n$ ):  $\chi_i = \frac{x_i}{x_n}$ ,  $i = \overline{1, n-1}$ . Of course:  $\xi_{ij} = \frac{\chi_i}{\chi_j}$ .

We obviously have:

$$Q(x_1, \dots, x_n) = x_n^r Q\left(\frac{x_1}{x_n}, \dots, \frac{x_{n-1}}{x_n}, \frac{x_n}{x_n}\right) = x_n^r Q(\chi_1, \dots, \chi_{n-1}, 1)$$

Considering the restriction of the production function at  $D_p \cap \mathbf{R}_+^{n-1} \times \{1\}$ :  $q(\chi_1, \dots, \chi_{n-1}) = Q(\chi_1, \dots, \chi_{n-1}, 1)$  we can write:

$$Q(x_1, \dots, x_n) = x_n^r q(\chi_1, \dots, \chi_{n-1})$$

With the new function introduced, the above indicators are:

- $\eta_{x_i} = x_n^{r-1} \frac{\partial q}{\partial \chi_i}, i = \overline{1, n-1}$
- $\eta_{x_n} = x_n^{r-1} \left( r q - \sum_{i=1}^{n-1} \frac{\partial q}{\partial \chi_i} \chi_i \right)$
- $w_{x_i} = x_n^{r-1} \frac{q}{\chi_i}, i = \overline{1, n-1}$
- $w_{x_n} = x_n^{r-1} q$
- $RMS(i,j) = \frac{\frac{\partial q}{\partial \chi_i}}{\frac{\partial q}{\partial \chi_j}}, i, j = \overline{1, n-1}$
- $RMS(i,n) = \frac{\frac{\partial q}{\partial \chi_i}}{r q - \sum_{i=1}^{n-1} \frac{\partial q}{\partial \chi_i} \chi_i}, i = \overline{1, n-1}$
- $RMS(i) = \frac{\frac{\partial q}{\partial \chi_i}}{\sqrt{\left( r q - \sum_{j=1}^{n-1} \frac{\partial q}{\partial \chi_j} \chi_j \right)^2 + \sum_{\substack{j=1 \\ j \neq i}}^{n-1} \left( \frac{\partial q}{\partial \chi_j} \right)^2}}, i = \overline{1, n-1}$
- $RMS(n) = \frac{r q - \sum_{j=1}^{n-1} \frac{\partial q}{\partial \chi_j} \chi_j}{\sqrt{\sum_{j=1}^{n-1} \left( \frac{\partial q}{\partial \chi_j} \right)^2}}$
- $\varepsilon_{x_i} = \frac{\frac{\partial q}{\partial \chi_i}}{\frac{q}{\chi_i}}, i = \overline{1, n-1}$
- $\varepsilon_{x_n} = \frac{r q - \sum_{i=1}^{n-1} \frac{\partial q}{\partial \chi_i} \chi_i}{q}$

$$\begin{aligned} & \frac{\partial^2 q}{\partial \chi_i^2} \frac{\partial q}{\partial \chi_j} - \frac{\partial q}{\partial \chi_i} \frac{\partial^2 q}{\partial \chi_i \partial \chi_j} \\ \bullet \quad \sigma_{ij} &= \chi_i \frac{\frac{\partial^2 q}{\partial \chi_i^2} \frac{\partial q}{\partial \chi_j} - \frac{\partial q}{\partial \chi_i} \frac{\partial^2 q}{\partial \chi_i \partial \chi_j}}{\frac{\partial q}{\partial \chi_i} \frac{\partial q}{\partial \chi_j}} \\ & r q \frac{\partial^2 q}{\partial \chi_i^2} + (1-r) \left( \frac{\partial q}{\partial \chi_i} \right)^2 + \frac{\partial q}{\partial \chi_i} \sum_{k=1, k \neq i}^{n-1} \frac{\partial^2 q}{\partial \chi_k \partial \chi_i} \chi_k - \frac{\partial^2 q}{\partial \chi_i^2} \sum_{k=1, k \neq i}^{n-1} \frac{\partial q}{\partial \chi_k} \chi_k \\ \bullet \quad \sigma_{in} &= \chi_i \frac{r q \frac{\partial^2 q}{\partial \chi_i^2} + (1-r) \left( \frac{\partial q}{\partial \chi_i} \right)^2 + \frac{\partial q}{\partial \chi_i} \sum_{k=1, k \neq i}^{n-1} \frac{\partial^2 q}{\partial \chi_k \partial \chi_i} \chi_k - \frac{\partial^2 q}{\partial \chi_i^2} \sum_{k=1, k \neq i}^{n-1} \frac{\partial q}{\partial \chi_k} \chi_k}{\frac{\partial q}{\partial \chi_i} \left( r q - \sum_{k=1}^{n-1} \frac{\partial q}{\partial \chi_k} \chi_k \right)} \end{aligned}$$

For a production function of the form:  $Q=Q(K,L)$ ,  $\chi = \frac{K}{L}$ ,  $q(\chi) = Q(\chi,1)$ :

$$\begin{aligned} \bullet \quad \eta_K &= L^{r-1} \frac{\partial q}{\partial \chi} \\ \bullet \quad \eta_L &= L^{r-1} \left( r q - \frac{\partial q}{\partial \chi} \chi \right) \\ \bullet \quad w_K &= L^{r-1} \frac{q}{\chi} \\ \bullet \quad w_L &= L^{r-1} q \\ \bullet \quad \text{RMS}(K,L) &= \text{RMS}(K) = \frac{\frac{\partial q}{\partial \chi}}{r q - \frac{\partial q}{\partial \chi} \chi} \\ \bullet \quad \varepsilon_K &= \frac{\frac{\partial q}{\partial \chi}}{\frac{q}{\chi}} \\ \bullet \quad \varepsilon_L &= \frac{r q - \frac{\partial q}{\partial \chi} \chi}{q} \end{aligned}$$



$$\bullet \quad \sigma = \sigma_{KL} = \chi \frac{rq \frac{\partial^2 q}{\partial \chi^2} + (1-r) \left( \frac{\partial q}{\partial \chi} \right)^2}{\frac{\partial q}{\partial \chi} \left( rq - \frac{\partial q}{\partial \chi} \chi \right)}$$

A.3. Necessary and sufficient conditions for nonlinear optimization

Considering now the non-linear programming problem:

$$\begin{cases} \max(\min) f(x_1, \dots, x_n) \\ g_i(x_1, \dots, x_n) \geq 0, i = \overline{1, p} \\ x_1, \dots, x_n \geq 0 \end{cases}$$

where  $f, g_i \in C^2(D_p)$  and a solution  $\bar{x} = (\bar{x}_1, \dots, \bar{x}_n)$  the Karush-Kuhn-Tucker conditions occur:  $\exists \lambda_i \in \mathbf{R}_+, i = \overline{1, p}$  so that:

$$\begin{cases} \varepsilon \nabla f(\bar{x}_1, \dots, \bar{x}_n) + \sum_{i=1}^p \lambda_i \nabla g_i(\bar{x}_1, \dots, \bar{x}_n) = 0 \\ g_i(\bar{x}_1, \dots, \bar{x}_n) \geq 0, i = \overline{1, p} \\ \lambda_i g_i(\bar{x}_1, \dots, \bar{x}_n) = 0, i = \overline{1, p} \end{cases}$$

where  $\nabla F$  is the gradient of  $F$  defined by:  $\nabla F = \left( \frac{\partial F}{\partial x_1}, \dots, \frac{\partial F}{\partial x_n} \right)$  and  $\varepsilon = 1$  for the case of maximizing and  $\varepsilon = -1$  in the case of minimizing.

If  $f, g_i, i = \overline{1, p}$  are of class  $C^2$ , from [1] follows, for the maximizing case, the sufficiency of Karush-Kuhn-Tucker conditions takes place in the broader framework of quasi-concavity of functions  $f$  and  $g$  and, moreover, if for a solution  $\bar{x} = (\bar{x}_1, \dots, \bar{x}_n)$  one of the conditions occurs:

- $\exists k = \overline{1, n}$  such that  $\frac{\partial f}{\partial x_k}(\bar{x}) < 0$ ;
- $\exists k = \overline{1, n}$  such that  $\frac{\partial f}{\partial x_k}(\bar{x}) > 0$  and  $\bar{x}_k > 0$ ;
- $\nabla f \neq 0$ ;
- $f$  is concave.

For the problem: 
$$\begin{cases} \min f(x_1, \dots, x_n) \\ g_i(x_1, \dots, x_n) \geq 0, i = \overline{1, p} \\ x_1, \dots, x_n \geq 0 \end{cases}$$

replacing  $f$  with  $-f$  and taking into account that  $\min f(x_1, \dots, x_n) = -\max(-f(x_1, \dots, x_n))$  follows that Karush-Kuhn-Tucker conditions becomes:

$$\begin{cases} -\nabla f(\bar{x}_1, \dots, \bar{x}_n) + \sum_{i=1}^p \lambda_i \nabla g_i(\bar{x}_1, \dots, \bar{x}_n) = 0 \\ g_i(\bar{x}_1, \dots, \bar{x}_n) \geq 0, i = \overline{1, p} \\ \lambda_i g_i(\bar{x}_1, \dots, \bar{x}_n) = 0, i = \overline{1, p} \end{cases}, \lambda_i \in \mathbf{R}_+, i = \overline{1, p}$$

and sufficiency reduces to one of the cases:

- $\exists k = \overline{1, n}$  such that  $\frac{\partial f}{\partial x_k}(\bar{x}) > 0$ ;
- $\exists k = \overline{1, n}$  such that  $\frac{\partial f}{\partial x_k}(\bar{x}) < 0$  and  $\bar{x}_k > 0$ ;
- $\nabla f \neq 0$ ;
- $f$  is convex.

In the particular case of the problem of minimizing the total cost (TC) relative to a production function  $Q=Q(x_1, \dots, x_n)$  and  $p_i, i = \overline{1, n}$  - the prices of inputs:

$$\begin{cases} \min \sum_{k=1}^n p_k x_k \\ Q(x_1, \dots, x_n) - Q_0 \geq 0 \\ x_1, \dots, x_n \geq 0 \end{cases} \quad \text{the Karush-Kuhn-Tucker conditions are:}$$

$$\begin{cases} -p_k + \lambda \frac{\partial Q}{\partial x_k}(\bar{x}_1, \dots, \bar{x}_n) = 0, k = \overline{1, n} \\ Q(x_1, \dots, x_n) \geq Q_0 \\ \lambda(Q(x_1, \dots, x_n) - Q_0) = 0 \end{cases} \quad . \text{ Because } \lambda=0 \text{ implies } p_k=0 \text{ - which is absurd}$$

in economic terms, results:  $\lambda \neq 0$  then:  $\begin{cases} \lambda \frac{\partial Q}{\partial x_k}(\bar{x}_1, \dots, \bar{x}_n) = p_k, k = \overline{1, n} \\ Q(x_1, \dots, x_n) = Q_0 \end{cases}$  or, with

another expression:  $\begin{cases} \frac{\frac{\partial Q}{\partial x_1}(\bar{x}_1, \dots, \bar{x}_n)}{p_1} = \dots = \frac{\frac{\partial Q}{\partial x_n}(\bar{x}_1, \dots, \bar{x}_n)}{p_n} \\ Q(x_1, \dots, x_n) = Q_0 \end{cases}$ . Because the objective

function  $f(x_1, \dots, x_n) = \sum_{k=1}^n p_k x_k$  is affine,  $Q$  is quasi-concave and, in addition

$\frac{\partial f}{\partial x_k}(\bar{x}) = p_k > 0$  follows, from the foregoing, that these conditions are sufficient.

#### A.4. Production efficiency

Let us consider in the following two goods  $\Phi$ ,  $\Psi$  and a number of  $n$  inputs  $F_1, \dots, F_n$  available in quantities  $\bar{x}_1, \dots, \bar{x}_n$ , and the production functions of  $\Phi$  or  $\Psi$  as follows:

$$Q = Q_\Phi(x_1, \dots, x_n), \quad Q = Q_\Psi(x_1, \dots, x_n)$$

appropriate to the consumption of  $x_k$  units of factor  $F_k, k = \overline{1, n}$ . We will assume that the production functions are of class  $C^2$  inside space production  $SP$ .

We will build the Edgeworth's box consisting in a  $n$ -dimensional parallelepiped:  $[0, \bar{x}_1] \times \dots \times [0, \bar{x}_n]$  the quantities of  $\Phi$  being relative to  $O(0, \dots, 0)$  and those of  $\Psi$  relative to  $F(\bar{x}_1, \dots, \bar{x}_n)$  on the parallelepiped sides. Let consider an initial allocation of inputs for  $\Phi$  and  $\Psi$ :

$$x_\Phi = (a_1, \dots, a_n), \quad x_\Psi = (b_1, \dots, b_n)$$

where  $a_i + b_i = \bar{x}_i, i = \overline{1, n}$ . The productions appropriate to the initial allocation are:  $Q_{\Phi,0}(a_1, \dots, a_n), Q_{\Psi,0}(b_1, \dots, b_n)$  relative to  $O$  and  $F$ , respectively. Because  $b_i = \bar{x}_i - a_i, i = \overline{1, n}$  we have:  $Q_{\Psi,0}(b_1, \dots, b_n) = Q_\Psi(\bar{x}_1 - a_1, \dots, \bar{x}_n - a_n)$ . The production function of  $\Psi$  is therefore:  $\hat{Q}_\Psi = Q_\Psi(\bar{x}_1 - x_1, \dots, \bar{x}_n - x_n)$  and means the production of  $\Psi$

relative to the origin of axes. We have now:  $\frac{\partial \hat{Q}_\Psi}{\partial x_i} = -\frac{\partial Q_\Psi}{\partial x_i}, \frac{\partial^2 \hat{Q}_\Psi}{\partial x_i \partial x_j} = \frac{\partial^2 Q_\Psi}{\partial x_i \partial x_j},$

$i, j = \overline{1, n}$  therefore  $\hat{Q}_\Psi$  is also quasi-concave but with negative partial derivatives of

order 1. Considering the isoproduction hypersurfaces, it follows that (relative to O) those of  $\Phi$  is convex, while that of  $\hat{\Psi}$  is concave.

Let  $PZ_{\Phi,0} = \{(x_1, \dots, x_n) \in SP \mid Q_{\Phi}(x_1, \dots, x_n) \geq Q_{\Phi,0}\}$  - the production zone of  $\Phi$  superior to  $Q_{\Phi,0}$  and  $PZ_{\Psi,0} = \{(x_1, \dots, x_n) \in SP \mid Q_{\Psi}(x_1, \dots, x_n) \geq Q_{\Psi,0}\}$  - the production zone of  $\Psi$  superior to  $Q_{\Psi,0}$ .

Suppose now that  $\text{int}(PZ_{\Phi,0} \cap PZ_{\Psi,0}) \neq \emptyset$  (*int means the interior of the set, i.e. those points for which there is a n-dimensional cube centered in them sufficiently small side and included in the given set*).

Let now a point  $C(c_1, \dots, c_n) \in \text{int}(PZ_{\Phi,0} \cap PZ_{\Psi,0})$  and also let the straight line that passes through the origin and C. Let note  $D(d_1, \dots, d_n)$  the intersection with the isoproduction hypersurface  $Q_{\Phi}(x_1, \dots, x_n) = Q_{\Phi,0}$  and  $E(e_1, \dots, e_n)$  the intersection with the isoproduction hypersurface  $\hat{Q}_{\Psi}(x_1, \dots, x_n) = \hat{Q}_{\Psi,0}$ . We have now  $Q_{\Phi}(d_1, \dots, d_n) = Q_{\Phi,0}$  and  $\hat{Q}_{\Psi}(e_1, \dots, e_n) = \hat{Q}_{\Psi,0}$ . Because  $\Phi$  is convex we obtain that:  $Q_{\Phi}(d_1, \dots, d_n) < Q_{\Phi}(c_1, \dots, c_n)$  and  $\hat{Q}_{\Psi}$  - concave implies that  $\hat{Q}_{\Psi}(e_1, \dots, e_n) > \hat{Q}_{\Psi}(c_1, \dots, c_n)$  or  $Q_{\Psi}(\bar{x}_1 - e_1, \dots, \bar{x}_n - e_n) > Q_{\Psi}(\bar{x}_1 - c_1, \dots, \bar{x}_n - c_n)$ . After these inequalities follows that the production of each good can increase, so the initial allocation is not optimal.

We call Pareto's efficiency the situation where new production can not improve without affecting the other's production. From the foregoing, it follows that the Pareto's efficiency is obtained if the isoproduction hypersurfaces are tangent.

The condition of tangency for  $Q = Q_{\Phi}(x_1, \dots, x_n)$  and  $Q = \hat{Q}_{\Psi}(x_1, \dots, x_n) = Q_{\Psi}(\bar{x}_1 - x_1, \dots, \bar{x}_n - x_n)$  is reduced to the determination of those points  $(x_1, \dots, x_n)$

where  $\frac{\partial Q_{\Phi}}{\partial x_i} = \lambda \frac{\partial \hat{Q}_{\Psi}}{\partial x_i}$ ,  $i = \overline{1, n}$ ,  $\lambda \in \mathbf{R}$  i.e. those points where hypersurfaces intersect

and have the same tangent hyperplane (*directors parameters are proportional*).

Taking into account that  $\hat{Q}_{\Psi}(x_1, \dots, x_n) = Q_{\Psi}(\bar{x}_1 - x_1, \dots, \bar{x}_n - x_n)$  we have that:

$$\frac{\partial Q_{\Phi}}{\partial x_i} = \mu \frac{\partial Q_{\Psi}}{\partial x_i}, \quad i = \overline{1, n}, \quad \mu = -\lambda \in \mathbf{R}.$$

In marginal notation, we have:  $\eta_{\Phi,i}(x_1, \dots, x_n) = \mu \eta_{\Psi,i}(\bar{x}_1 - x_1, \dots, \bar{x}_n - x_n)$ ,  $i = \overline{1, n}$ ,  $\mu \in \mathbf{R}$ .

For two inputs (K and L) the above relations are equivalent with:  $\frac{\eta_{\Phi,K}}{\eta_{\Phi,L}} = \frac{\eta_{\Psi,K}}{\eta_{\Psi,L}}$ .

On the other hand:  $\frac{\eta_{\Phi,K}}{\eta_{\Phi,L}} = \frac{dL}{dK}|_{\Phi} = \text{RMS}_{\Phi}(K,L)$  - marginal rate of technical substitution of capital for  $\Phi$  and  $\frac{\eta_{\Psi,K}}{\eta_{\Psi,L}} = \frac{dL}{dK}|_{\Psi} = \text{RMS}_{\Psi}(K,L)$  - marginal rate of technical substitution of capital for  $\Psi$ . The upper equality becomes:  $\text{RMS}_{\Phi}(K,L) = \text{RMS}_{\Psi}(K,L)$ .

All of the points where the allocation is Pareto's efficient generates the production contract curve.

Contract curve represents all combinations of goods for which no party can maximize its production without diminishing the other's production. On the other hand, any point on the curve represents a possible allocation contracts. The problem is this: if one good will be produced in order to reach the maximum level, what will do the other?

Considering now the prices of n inputs as  $p_1, \dots, p_n$  the total cost is:  $TC = \sum_{i=1}^n p_i x_i$  and it maximize the production if it is tangent to the isoproduction hypersurface. But each good want to be produced in maximum quantity therefore:

$$\frac{\eta_{\Phi,1}(x_1, \dots, x_n)}{p_1} = \dots = \frac{\eta_{\Phi,n}(x_1, \dots, x_n)}{p_n},$$

$$\frac{\eta_{\Psi,1}(\bar{x}_1 - x_1, \dots, \bar{x}_n - x_n)}{p_1} = \dots = \frac{\eta_{\Psi,n}(\bar{x}_1 - x_1, \dots, \bar{x}_n - x_n)}{p_n}$$

or, in other words, the cost hyperplane will be tangent to both isoproduction hypersurfaces, that is it will coincide with the common tangent hyperplane.

Considering the production contract curve of the form:

$$x_1 = g_1(\lambda), \dots, x_n = g_n(\lambda), \lambda \in \mathbf{R}$$

follows:

$$\frac{\eta_{\Phi,1}(g_1(\lambda), \dots, g_n(\lambda))}{p_1} = \dots = \frac{\eta_{\Phi,n}(g_1(\lambda), \dots, g_n(\lambda))}{p_n}$$

from where:

$$p_k = \frac{\eta_{\Phi,k}(g_1(\lambda), \dots, g_n(\lambda))}{\eta_{\Phi,1}(g_1(\lambda), \dots, g_n(\lambda))} v, v > 0, k = \overline{1, n}$$

We note that prices are determined up to a multiplicative factor, which does not affect the result of the problem and can therefore consider  $v=1$ . If the initial allocation of factors of production was  $x_\Phi = (a_1, \dots, a_n)$ ,  $x_\Psi = (b_1, \dots, b_n)$  the total cost of production of  $\Phi$  is  $TC_\Phi = \sum_{k=1}^n p_k a_k$ . The new amounts of factors (which also satisfy the same total cost) involves:  $\sum_{k=1}^n p_k (a_k - x_k) = 0$ . Replacing the values of  $p_k$  into this equation:

$$\sum_{k=1}^n \frac{\eta_{\Phi,k}(g_1(\lambda), \dots, g_n(\lambda))}{\eta_{\Phi,1}(g_1(\lambda), \dots, g_n(\lambda))} (a_k - g_k(\lambda)) = 0$$

hence we will find  $\lambda \in \mathbf{R}$ . Substituting in the appropriate expressions, will result  $p_k$  and  $x_k$ ,  $k = \overline{1, n}$ .

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## A Study of Allen Production Function with Differential Geometry

Alin Cristian Ioan<sup>1</sup>

**Abstract.** In this paper we shall made an analysis of Allen production function from the differential point of view. We shall obtain some interesting results about the nature of the points of the surface and the total curvature.

**Keywords:** production functions; metric; curvature; Allen

**JEL Classification:** E23

### 1. Introduction

In the theory of production functions, all computations and phenomenons are studied for a constant level of production. In order to detect many aspects of them, a complete analysis can be made only at the entire surface.

We define on  $\mathbf{R}^2$  – the **production space** for two resources: K – capital and L - labor as  $SP = \{(K,L) \mid K,L \geq 0\}$  where  $x \in SP$ ,  $x = (K,L)$  is **the set of resources**. Because not any amount of resources are possible, we restrict the production area to a subset  $D_p \subset SP$  called **domain of production**.

It is called an **Allen production function** an application:

$$Q: D_p \rightarrow \mathbf{R}_+, (K,L) \rightarrow Q(K,L) = c\sqrt{aK^2 + 2bKL + dL^2} \in \mathbf{R}_+ \quad \forall (K,L) \in D_p, \quad a,b,c,d \in \mathbf{R}_+, c > 0$$

The production function is  $C^\infty$ -differentiable and homogenous of degree 1.

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## 2. The Differential Geometry of Allen Surface

The graph representation of a production function is a surface.

Let note in what follows:

$$(1) \quad p = \frac{\partial Q}{\partial L}, \quad q = \frac{\partial Q}{\partial K}, \quad r = \frac{\partial^2 Q}{\partial L^2}, \quad s = \frac{\partial^2 Q}{\partial L \partial K}, \quad t = \frac{\partial^2 Q}{\partial K^2}$$

We have after simple calculations:

$$(2) \quad p = \frac{c^2(bK + bL)}{Q}, \quad q = \frac{c^2(aK + bL)}{Q}, \quad r = -\frac{c^4(b^2 - ad)K^2}{Q^3}, \quad s = \frac{c^4(b^2 - ad)KL}{Q^3},$$

$$t = -\frac{c^4(b^2 - ad)L^2}{Q^3}$$

The bordered Hessian:

$$(3) \quad H_f = \begin{pmatrix} 0 & q & p \\ q & t & s \\ p & s & r \end{pmatrix} = \begin{pmatrix} 0 & \frac{c^2(aK + bL)}{Q} & \frac{c^2(bK + bL)}{Q} \\ \frac{c^2(aK + bL)}{Q} & -\frac{c^4(b^2 - ad)L^2}{Q^3} & \frac{c^4(b^2 - ad)KL}{Q^3} \\ \frac{c^2(bK + bL)}{Q} & \frac{c^4(b^2 - ad)KL}{Q^3} & -\frac{c^4(b^2 - ad)K^2}{Q^3} \end{pmatrix}$$

therefore, because:

$$(4) \quad \Delta^B_1 = \begin{vmatrix} 0 & \frac{c^2(aK + bL)}{Q} \\ \frac{c^2(aK + bL)}{Q} & -\frac{c^4(b^2 - ad)L^2}{Q^3} \end{vmatrix} = -\frac{c^4(aK + bL)^2}{Q^2} < 0,$$

$$\Delta^B_2 = \begin{vmatrix} 0 & \frac{c^2(aK + bL)}{Q} & \frac{c^2(bK + bL)}{Q} \\ \frac{c^2(aK + bL)}{Q} & -\frac{c^4(b^2 - ad)L^2}{Q^3} & \frac{c^4(b^2 - ad)KL}{Q^3} \\ \frac{c^2(bK + bL)}{Q} & \frac{c^4(b^2 - ad)KL}{Q^3} & -\frac{c^4(b^2 - ad)K^2}{Q^3} \end{vmatrix} = \frac{c^4(b^2 - ad)}{Q}$$



we obtain that in order to  $Q$  be quasiconcave (that is for any  $a \in \mathbf{R}$ ,  $Q^{-1}([a, \infty))$  is convex) we must have  $b^2 - ad > 0$ .

For a constant value of one parameter we obtain a curve on the surface, that is  $Q=Q(K, L_0)$  or  $Q=Q(K_0, L)$  are both curves on the production surface. They are obtained from the intersection of the plane  $L=L_0$  or  $K=K_0$  with the surface  $Q=Q(K, L)$ .

In the study of the surfaces, two quadratic forms are very useful.

The first fundamental quadratic form of the surface is:

$$(5) \quad g = g_{11}dL^2 + 2g_{12}dLdK + g_{22}dK^2$$

where:  $g_{11}=1+p^2$ ,  $g_{12}=pq$ ,  $g_{22}=1+q^2$ .

In our case:

$$(6) \quad g_{11} = 1 + \frac{c^2(bK + dL)^2}{aK^2 + 2bKL + dL^2}, \quad g_{12} = \frac{c^2(bK + dL)(aK + bL)}{aK^2 + 2bKL + dL^2},$$

$$g_{22} = 1 + \frac{c^2(aK + bL)^2}{aK^2 + 2bKL + dL^2}$$

The area element is:

$$(7) \quad d\sigma = \sqrt{g_{11}g_{22} - g_{12}^2} dKdL = \sqrt{\Delta} dKdL =$$

$$\sqrt{\frac{c^2(aK + bL)^2 + c^2(b^2 - ad)K^2 + (1 + c^2d)(aK^2 + 2bKL + dL^2)}{aK^2 + 2bKL + dL^2}} dKdL$$

and the surface area  $A$  when  $(K, L) \in R$  (a region in the plane  $K$ - $O$ - $L$ ) is  $A = \iint_R d\sigma dKdL$ .

The second fundamental form of the surface is:

$$(8) \quad h = h_{11}dL^2 + 2h_{12}dLdK + h_{22}dK^2$$

where:  $h_{11} = \frac{r}{\sqrt{1+p^2+q^2}}$ ,  $h_{12} = \frac{s}{\sqrt{1+p^2+q^2}}$ ,  $h_{22} = \frac{t}{\sqrt{1+p^2+q^2}}$ .

In our case:

$$(9) \quad h_{11} = -\frac{c^3(b^2 - ad)K^2}{Q^2 \sqrt{c^2(aK + bL)^2 + c^2(b^2 - ad)K^2 + (1 + c^2d)(aK^2 + 2bKL + dL^2)}},$$

$$h_{12} = \frac{c^3(b^2 - ad)KL}{Q^2 \sqrt{c^2(aK + bL)^2 + c^2(b^2 - ad)K^2 + (1 + c^2d)(aK^2 + 2bKL + dL^2)}},$$

$$h_{22} = - \frac{c^3(b^2 - ad)L^2}{Q^2 \sqrt{c^2(aK + bL)^2 + c^2(b^2 - ad)K^2 + (1 + c^2d)(aK^2 + 2bKL + dL^2)}}.$$

Considering the quantity  $\delta = h_{11}h_{22} - h_{12}^2$  we have that:

(10)  $\delta = 0$

If  $\delta > 0$  in each point of the surface, we will say that it is elliptical. Such surfaces are the hyperboloid with two sheets, the elliptical paraboloid and the ellipsoid. If  $\delta < 0$  in each point of the surface, we will say that it is hyperbolic. Such surfaces are the hyperboloid with one sheet and the hyperbolic paraboloid. If  $\delta = 0$  in each point of the surface, we will say that it is parabolic. Such surfaces are the cone surfaces and the cylinder surfaces.

From (10) we find that the production surface is parabolic.

The curvature of a curve is, from an elementary point of view, the degree of deviation of the curve relative to a straight line. Considering a surface S and an arbitrary curve through a point P of the surface who has the tangent vector v in P, let the plane  $\pi$  determined by the vector v and the normal N in P at S. The intersection of  $\pi$  with S is a curve  $C_n$  named normal section of S. Its curvature is called normal curvature.

If we have a direction  $m = \frac{dL}{dK}$  in the tangent plane of the surface in an arbitrary

point P we have that the normal curvature is given by:  $k(m) = \frac{h_{11}m^2 + 2h_{12}m + h_{22}}{g_{11}m^2 + 2g_{12}m + g_{22}}$ .

The extreme values  $k_1$  and  $k_2$  of the function  $k(m)$  are the principal curvatures of the surface in that point. The quantity  $K = k_1k_2$  is named the total curvature in the considered point. We have  $K = 0$  therefore the surface has null constant total curvature.

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**Miscellaneous****Europe 2020 Strategy's Viability under This More Volatile Economic World****Romeo-Victor Ionescu<sup>1</sup>**

**Abstract:** The global crisis accentuated the competition between the greatest world economic actors. One of them, the EU28, tried to find, as solution to support its economic growth, the Cohesion Policy. The paper is focused on the analysis of the results of the Cohesion Policy and the perspectives of the new strategy Europe 2020. In order to do these, the paper uses the latest official statistic data and divides the analysis into three periods: until 2012, during 2012-2014 and during 2015-2020. The analysis is based on three elements (a comparative trend analysis, followed by a cluster analysis and a forecast using SPSS19 software) and takes into considerations four important macroeconomic indicators. The main conclusion of the paper is that the objectives of the Europe 2020 strategy will be not achieved and the socio-economic disparities will increase across the EU in 2020. All conclusions of the analysis are supported by pertinent diagrams and statistic tables.

**Keywords:** crisis' challenges; regional clusters; economic recovery; economic disparities; economic forecast.

**JEL Classification:** R11; R12; F62; F63

**1. Introduction**

The EU28 is close to a new financial perspective which will support the new Europe 2020 Strategy focused on clever sustainable development able to decrease the socio-economic disparities between the Member States and to offer a model of successful regional integrative organization.

There are a lot of economic studies regarding the future of the EU. Some of them talk about the fragile improvement in financial market conditions (Baker S. R. et al., 2013). Other research is more pessimistic and considers that a strong rebound of the domestic demand was not to be expected (Jordà O. et al., 2012).

More specialists focused on the idea that the financial crisis-induced fall in credit supply tend to amplify investment downturns, in particular in vulnerable countries (Buca A., Vermeulen P., 2012).

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The latest approaches talk about a new global crisis connected to the weak of the international bank regulations. After Lehman Brothers' failure in 2008, the main global economic actors were defensive and destroyed the global coordination.

In order to pass this impasse, the EU leaders negotiate the union bank details and the American congressmen continue the reform of the financial sector. Maybe a very important problem is that to restore confidence in the banking sector.

Each main global economic actor tried to find national solution to face the crisis. UK, for example, applied a disastrous policy of selling the gold reserves of the country at 256-296 dollars an ounce. Meanwhile, gold prices climbed to 1205 dollars an ounce.

USA adopted a new law aimed at the financial sector (Dodd-Frank law), but EU continued to manage a permissive legislation in this sector.

On the other hand, China is closed to decrease. The total loans in China increased from 9000 billion dollars in 2008 to 23000 billion dollars in 2013. Moreover, the dependence on short-term financing on the capital markets reminiscent of the earlier collapse of U.S. bank Lehman Brothers. All these above elements talk about the risk of a new global crisis.

As an intermediate conclusion, the global crisis is far away from its end and the economic recovery process, including EU28, is very difficult. Under this less optimistic approach, the launch of the Europe 2020 Strategy tried to represent a step further on the integration road.

This paper tries to sustain the idea of greater socio-economic disparities between the Member States which contrasts with the European cohesion objective. Moreover, the analysis in this paper wants to see if the deadline of this new strategy will bring better or not socio-economic cohesion across the Member States.

## **2. Research Methodology**

According to the economics theoretical approach, a real economic analysis of a phenomenon has to be based at least three indicators. The analysis in this paper uses the following indicators: GDP growth rate, inflation rate, unemployment rate and government gross debt.

The paper uses the latest official statistic data (European Commission, 2013). The analysis follows three steps. First consists in a comparative analysis between the above four indicators during 1992-2012, in order to establish the trends.

The second step is a cluster analysis made under SPSS19 software, in order to sustain the idea that the Member States can be analyzed under three clusters. The paper uses a TwoStep cluster analysis, where the categorical variables are the above four economic indicators and the distance measure is log-likelihood. The clustering criterion is BIC (Schwarz's Bayesian Criterion). The problem is that these clusters selected for 2012 will be the same in 2020.

Finally, the analysis is focused on a forecast of the GDP growth rates till the end of 2020, in order to observe if the socio-economic disparities across the EU will or not increase. The dependent variables in this forecast are the GDP growth rates and the independent variable is time. The forecast method is ARIMA. The conclusions of the analysis are supported by pertinent statistical tables and diagrams.

### **3. The Impact of the Global Crisis on the EU28 Economy**

There were great economic disparities between the Member States during 1992-2012. This time period can be divided in two parts: the period prior to the global crisis (1992-2008) and the period 2009-2012.

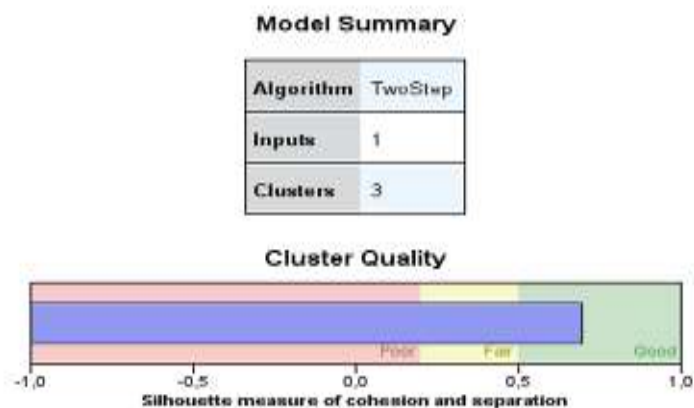
There are great disparities related to the GDP growth rate across the Member States until 2008. The highest dispersion is 1: 4.7 if we take into consideration the average growth rate during whole period and it is related to Italy and Ireland.

As a general point of view, the GDP growth rates decreased in 2009, compared to the period 1992-2008, in all Member States, which faced to negative growth rates. Poland was the exception. This negative trend was followed by a discreet recovery in 2010 and 2011.

On the other hand dispersion continued to increase from -14.8% in Lithuania, to 1.6 in Poland in 2009, from -4.9% in Greece, to 6.6% in Sweden in 2010 and from -7.1% in Greece, to 8.3% in Estonia in 2011.

2012 marked a new negative change as a result of another impact of the global crisis. As a result, the Member States (including Croatia, which adhered in 2013) could be divided into three groups: 13 states with negative GDP growth rates (Belgium, Czech Republic, Denmark, Greece, Spain, Italy, Cyprus, Hungary, Netherland, Portugal, Slovenia, Finland and Croatia), 8 states with GDP growth rates between 0% and 1% (Bulgaria, Germany, Ireland, France, Luxembourg, Austria, Romania and UK) and 7 states with GDP growth rates greater than 1% (Estonia, Latvia, Lithuania, Malta, Poland, Slovakia and Sweden). In order to test if this approach is correct, the paper uses the TwoStep cluster analysis (see Figure 1).

The result of the cluster analysis is fair-good; it means that the assumption of three clusters is correct.



**Figure 1. Cluster analysis under GDP growth rate criterion**

According to the second indicator (inflation rate), 14 Member States faced to disinflation in 2009, while the other 14 had positive inflation rates, but less than 4.0%.

The highest dispersion was between -2.7% in Ireland and 4.6% in Romania. The inflation rates increased in 2010 and 2011, but the dispersion had not the same trend.

As a result, the highest dispersion was between -2.6% in Ireland and 5.1% in Romania in 2010 and between 0.4% in Sweden and 4.8% in Romania, in 2011. Under the same economic indicator, the Member States can be divided into three groups in 2012: 3 states with inflation rates less than 1% (Ireland, Greece and Sweden), 13 states with inflation rates between 1% and 2% (Belgium, Bulgaria, Denmark, Germany, Spain, France, Latvia, Luxembourg, Netherland, Austria, Portugal, Slovenia and UK) and 12 states with inflation rates greater than 2% (Czech Republic, Estonia, Italy, Cyprus, Lithuania, Hungary, Malta, Poland, Romania, Slovakia, Finland and Croatia). On the other hand, the dispersion interval increased, from -0.1% in Sweden, to 4.7% in Hungary.

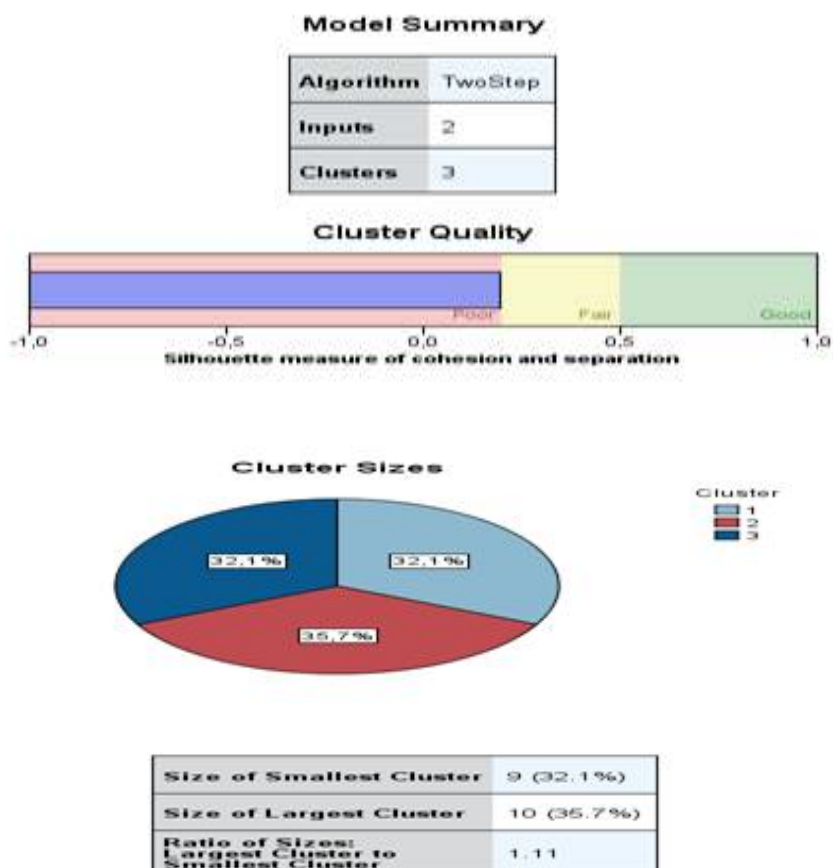
The situation will improve in 2014 from the European Commission's point of view (see Table 1) (European Commission, 2013).

**Table 1. Inflation rate forecast in the EU28 (%)**

State	2009	2010	2011	2012	2013	2014
Belgium	-1.0	1.3	2.5	1.6	0.6	0.5
Bulgaria	1.5	2.0	2.4	1.4	1.6	1.7
Czech Rep.	-0.4	0.2	1.1	2.5	1.1	0.6
Denmark	0.1	1.2	1.7	1.4	0.5	0.5
Germany	-0.8	0.2	1.5	1.1	0.8	0.7
Estonia	-0.8	1.7	4.1	3.2	2.6	2.2
Ireland	-2.7	-2.6	0.2	0.9	0.3	0.3
Greece	0.3	3.7	2.1	0.0	-1.8	-1.4
Spain	-1.2	1.0	2.1	1.4	0.7	0.0
France	-0.9	0.7	1.3	1.2	0.6	0.5
Italy	-0.2	0.6	1.9	2.3	1.0	0.7
Cyprus	-0.8	1.6	2.5	2.1	0.5	0.4
Latvia	0.3	-2.2	3.2	1.3	0.9	1.2
Lithuania	3.2	0.2	3.1	2.2	1.4	1.9
Luxembourg	-1.0	1.8	2.7	1.9	0.7	0.6
Hungary	3.0	3.7	2.9	4.7	2.6	2.3
Malta	0.8	1.0	1.5	2.2	1.2	1.2
Netherland	0.0	-0.1	1.5	1.8	1.6	0.4
Austria	-0.6	0.7	2.6	1.6	1.2	0.9
Poland	3.0	1.7	2.9	2.7	0.8	1.3
Portugal	-1.9	0.4	2.6	1.8	-0.4	0.2
Romania	4.6	5.1	4.8	2.4	3.6	2.3
Slovenia	-0.1	1.1	1.1	1.7	1.1	0.5
Slovakia	-0.1	-0.3	3.1	2.7	0.9	1.0
Finland	0.6	0.7	2.3	2.2	1.5	1.2
Sweden	0.9	0.9	0.4	-0.1	0.1	0.6
UK	1.2	2.3	3.5	1.8	1.6	1.3
Croatia	1.2	0.1	1.2	2.4	2.0	1.0

The highest inflation rate will be achieved in Hungary and Romania (2.3%), while Greece will face to disinflation (-1.4%). According to Table 1, the Member States can be divided into three groups under the inflation rate in 2014, using the same restrictions as in 2012. The cluster analysis will support this approach for 2012 and 2014 (see Figure 2).





**Figure 2. Cluster analysis under inflation rate criterion**

Almost all Member States faced to an increase of the unemployment rate during 2009-2011. In 2009, the dispersion interval was great, between 3.7% in Netherland and 18.2% in Latvia.

The economic contraction in 2010 was followed by the increase of the unemployment rates, which varied between 4.4% in Austria and 20.1% in Spain.

The highest level of the unemployment rates increased in 2011 to 21.7% in Spain, while the lowest level was achieved in Austria (4.2%). According to the unemployment rate, the disparities between the Member States allow to divide these states in 2012 into the “classic” three groups: 13 states with unemployment

rates less than 10% (Belgium, Czech Republic, Denmark, Germany, Luxembourg, Malta, Netherland, Austria, Romania, Slovenia, Finland, Sweden and UK), 13 states with unemployment rates between 10% and 20% (Bulgaria, Estonia, Ireland, France, Italy, Cyprus, Latvia, Lithuania, Hungary, Poland, Portugal, Slovakia and Croatia) and 2 states with unemployment rate greater than 20% (Greece and Spain).

The highest unemployment rate was achieved in Spain (25%) and the lowest in Austria (4.4%). The same division can be used for 2014 (see Table 2).

**Table 2. Unemployment rate forecast in the EU28 (%)**

State	2009	2010	2011	2012	2013	2014
Belgium	7.9	8.3	7.2	7.3	7.7	7.7
Bulgaria	6.8	10.3	11.3	12.2	12.2	11.9
Czech Rep.	6.7	7.3	6.7	7.0	7.6	7.3
Denmark	6.0	7.5	7.6	7.7	8.0	7.9
Germany	7.8	7.1	5.9	5.5	5.7	5.6
Estonia	13.8	16.9	12.5	10.0	9.8	9.0
Ireland	12.0	13.9	14.7	14.8	14.6	14.1
Greece	9.5	12.6	17.7	24.7	27.0	25.7
Spain	18.0	20.1	21.7	25.0	26.9	26.6
France	9.5	9.7	9.6	10.3	10.7	11.0
Italy	7.8	8.4	8.4	10.6	11.6	12.0
Cyprus	5.5	6.5	7.9	12.1	13.7	14.2
Latvia	18.2	19.8	16.2	14.9	13.7	12.2
Lithuania	13.6	18.0	15.3	13.0	11.4	9.8
Luxembourg	5.1	4.6	4.8	5.0	5.4	5.7
Hungary	10.0	11.2	10.9	10.8	11.1	11.1
Malta	6.9	6.9	6.5	6.5	6.4	6.2
Netherland	3.7	4.5	4.4	5.3	6.3	6.5
Austria	4.8	4.4	4.2	4.4	4.5	4.2
Poland	8.1	9.6	9.6	10.2	10.8	10.9
Portugal	10.6	12.0	12.9	15.7	17.3	16.8
Romania	6.9	7.3	7.4	7.0	6.9	6.8
Slovenia	5.9	7.3	8.2	9.0	9.8	10.0
Slovakia	12.1	14.5	13.6	14.0	14.0	13.6
Finland	8.2	8.4	7.8	7.7	8.0	7.9
Sweden	8.3	8.4	7.5	7.7	8.0	7.8
UK	7.6	7.8	8.0	7.9	8.0	7.8
Croatia	9.1	11.8	13.5	15.8	15.9	14.9

The problem is that the highest unemployment rate will achieve 26.6% in Spain, while the lowest unemployment rate will achieve 4.2% in Austria.

Austria and Spain had and will achieve the minimum/maximum levels of the unemployment rates.

The greatest decreases of the unemployment rates will be achieved in Latvia (from 13.7% in 2013, to 12.2% in 2014) and Lithuania (from 11.4% in 2013, to 9.8% in 2014). Great unemployment rates will be in Greece (25.7%), Portugal (16.8%), Croatia (14.9%) and Ireland (14.1%).

The same cluster analysis supports the above conclusion: the possibility of dividing Member States into three clusters with different trends (see Figure 3).



**Figure 3. Cluster analysis under unemployment rate criterion**

Last, but not the least economic indicator in this analysis is the government gross debt as % of GDP. There are some Member States which experienced high

government gross debts until 2008: Belgium (with an average debt of 109.9% of GDP), Greece (99.5%) and Italy (110.4%). In 2009, these debts increased in almost all Member States, excepting Belgium, Denmark, Netherlands, Finland and Sweden. Moreover, there are great disparities between states with low government gross debts, as Estonia (7.25) and Bulgaria (14.6% of GDP) and states with high debts, as Greece (129.7%) and Italy (116.4%).

Only three states improved their government gross debts in 2010: Estonia, Sweden and Belgium. Moreover, a peak of 148.3% of GDP was achieved by Greece in this year. In 2011, the situation became worst for Ireland, Greece, Italy and Portugal whose debts exceeded their GDP. Greece ranked the first position with a government gross debt of 170.6% of GDP.

The government gross debt values seem to put the Member States in the same three clusters in 2012 and 2014. In 2012, there were: 9 states with government gross debts less than 50% of GDP (Bulgaria, Czech Republic, Denmark, Estonia, Latvia, Lithuania, Luxembourg, Romania and Sweden), 15 states with government gross debts between 50% and 100% of GDP (Belgium, Germany, Spain, France, Cyprus, Hungary, Malta, Netherlands, Austria, Poland, Slovenia, Slovakia, Finland, UK and Croatia) and 4 states with government gross debt greater than 100% of GDP (Ireland, Greece, Italy and Portugal).

The highest government gross debt was achieved by Greece (161.6% of GDP), while the lowest one was achieved by Estonia (10.5%).

There are not major changes in the government gross debts of the above countries in 2014 (see Table 3).

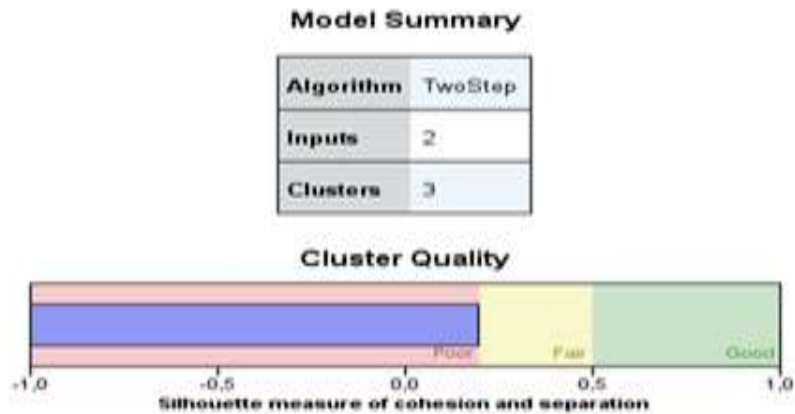
**Table 3. Government gross debt forecast in the EU28 (% of GDP)**

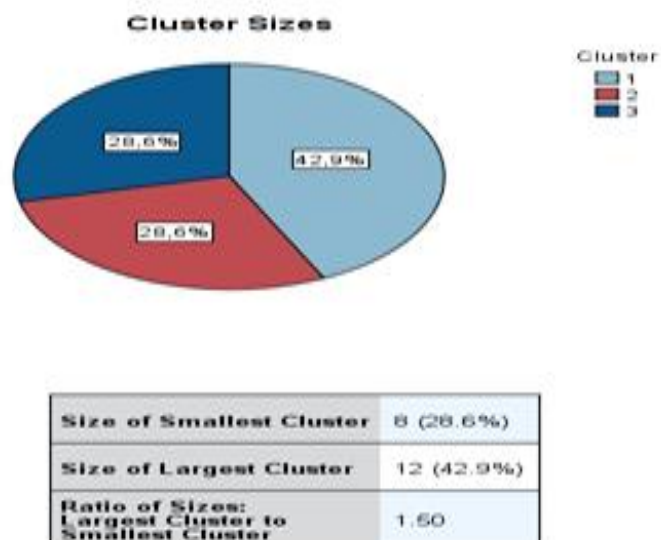
State	2009	2010	2011	2012	2013	2014
Belgium	95.7	95.5	97.8	94.8	100.8	101.1
Bulgaria	14.6	16.2	16.3	18.9	17.1	17.3
Czech Rep.	34.2	37.8	40.8	45.5	48.0	49.5
Denmark	40.7	42.7	46.4	45.6	45.9	47.3
Germany	74.5	82.5	80.5	81.6	80.7	78.3
Estonia	7.2	6.7	6.1	10.5	11.8	11.3
Ireland	64.9	92.2	106.4	117.2	122.2	120.1
Greece	129.7	148.3	170.6	161.1	175.6	175.2
Spain	53.9	61.5	69.3	88.4	95.8	101.0
France	79.2	82.3	86.0	90.3	93.4	95.0
Italy	116.4	119.2	120.7	127.1	128.1	127.1
Cyprus	58.5	61.3	71.1	86.5	93.1	97.0
Latvia	36.7	44.5	42.2	41.9	44.4	41.5
Lithuania	29.3	37.9	38.5	41.1	40.5	40.3

Luxembourg	15.3	19.2	18.3	20.5	22.2	24.1
Hungary	79.8	81.8	81.4	78.6	78.7	77.7
Malta	66.3	67.4	70.4	73.1	73.8	73.6
Netherland	60.8	63.1	65.6	70.8	73.8	75.0
Austria	69.2	72.0	72.4	74.3	75.2	74.5
Poland	50.9	54.8	56.4	55.8	57.0	57.5
Portugal	83.2	93.5	108.0	120.6	123.9	124.7
Romania	23.6	30.5	34.7	38.0	38.1	38.0
Slovenia	35.0	38.6	46.9	53.7	59.5	63.4
Slovakia	35.6	41.0	43.3	52.4	55.1	57.1
Finland	43.5	48.6	49.0	53.4	56.4	57.6
Sweden	42.6	39.5	38.4	37.7	37.3	35.5
UK	67.8	79.4	85.2	89.8	96.4	97.9
Croatia	35.7	42.2	46.7	53.6	57.4	60.1

Greece will maintain the highest debt level (175.2% of GDP) and Estonia the lowest (11.3% of GDP). The problem is that Belgium, Ireland, Spain, Italy and Portugal will face to government gross debts greater than their GDP in 2014. Moreover, France, Cyprus and UK's government gross debts will be very close to their GDP.

The cluster analysis supports the above conclusions and the Member States grouping into three distinct categories (see Figure 4).





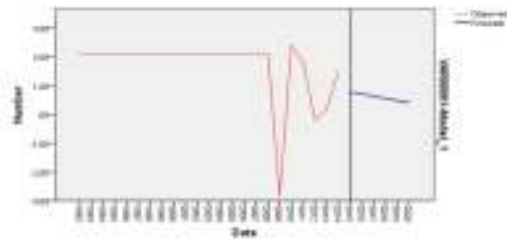
**Figure 4. Cluster analysis under government gross debt criterion**

At this moment of the analysis, we can obtain two conclusions. The first one is that the social cohesion policy didn't support the elimination of the socio-economic disparities in 2012.

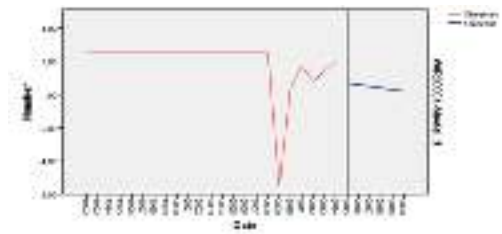
The second conclusion is based on the European Commission's forecast for 2014 and is the same as for 2012. Moreover, the Member States can be divided into "classic" three groups, which can lead to the idea of an EU with three economic speeds.

#### **4. EU28 Economy's forecast for 2020**

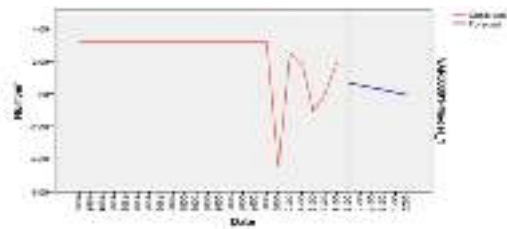
The third approach in this paper is to realize a forecast of the macroeconomic growth across the EU28 during the new financial perspective, using the SPSS19 software (see Figure 5).



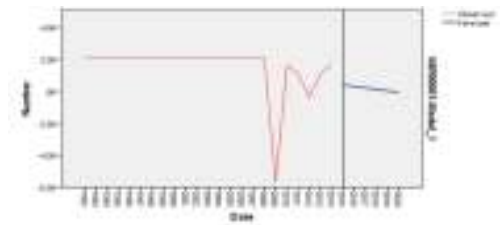
Belgium



Bulgaria



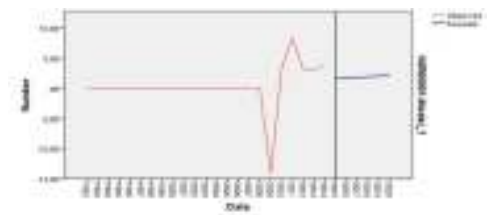
Czech Republic



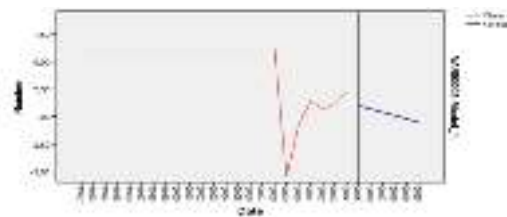
Denmark



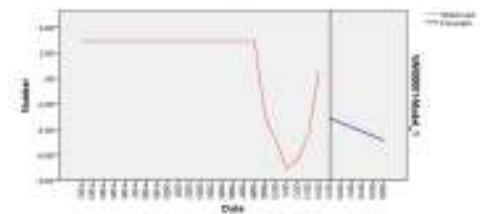
Germany



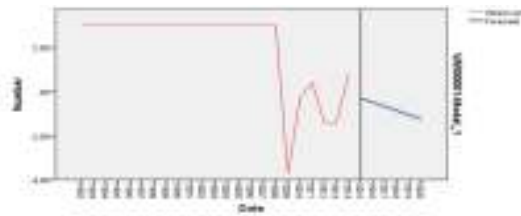
Estonia



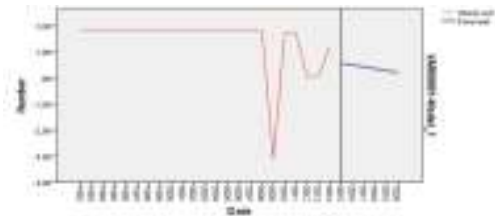
Ireland



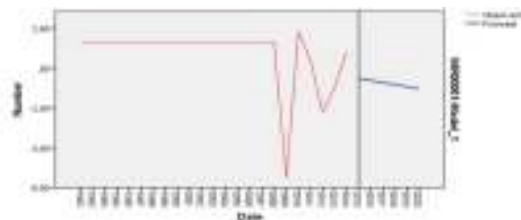
Greece



Spain



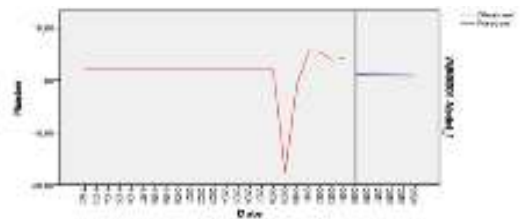
France



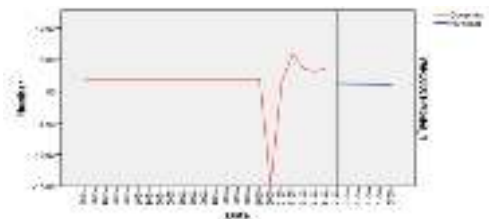
Italy



Cyprus



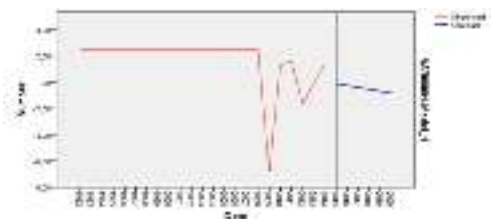
Latvia



Lithuania

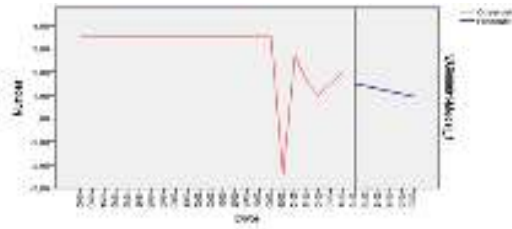


Luxembourg

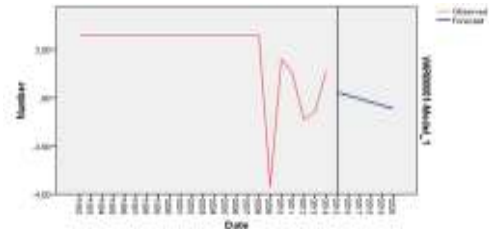


Hungary

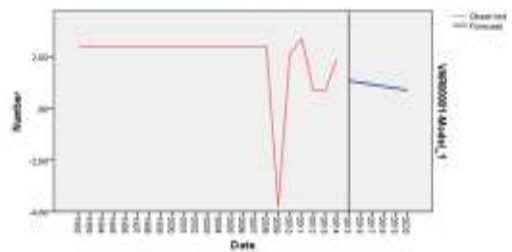




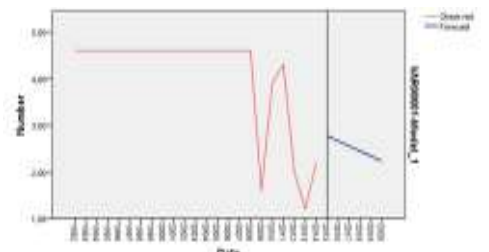
Malta



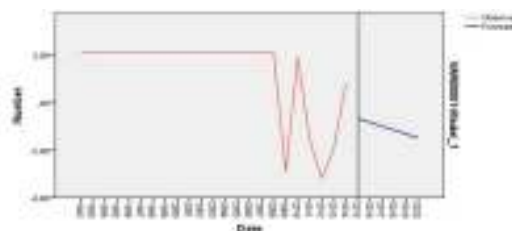
Netherland



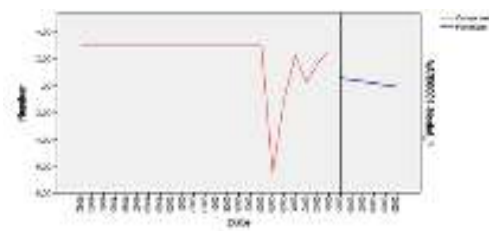
Austria



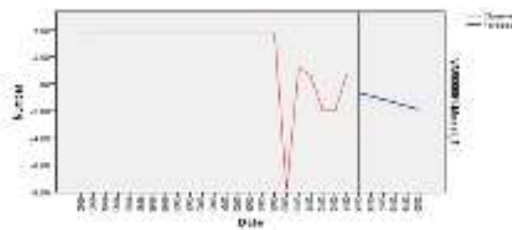
Poland



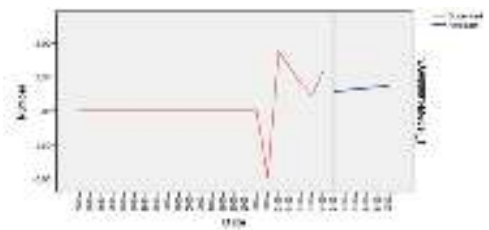
Portugal



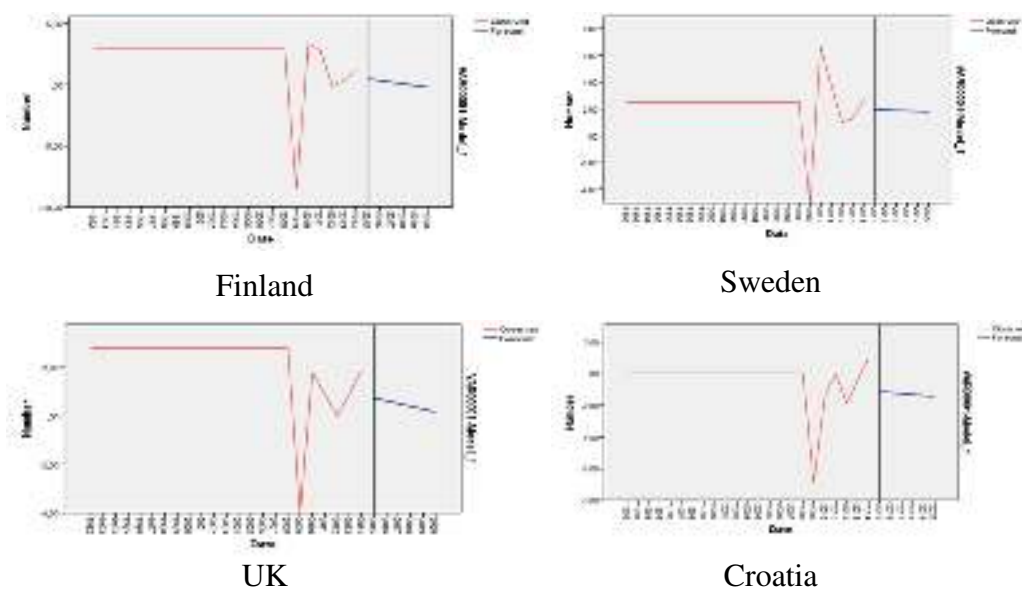
Romania



Slovenia



Slovakia



**Figure 5. GDP growth rate forecasting**

In order to improve the forecast, the analyzed time period in the paper is 20 years (1992-2012). It is completed to the official European Commission forecast for 2013-2014.

According to this forecast, the situation of the Member States in 2020, using the GDP growth rate, is presented in Table 4.

**Table 4. GDP growth rate forecast in the EU28 (%)**

State	2009	2010	2011	2012	2013	2014	2020
Belgium	-2.8	-2.4	1.8	-0.2	0.2	1.5	0.5
Bulgaria	-5.5	0.4	1.7	0.8	1.4	2.0	0.0
Czech Rep.	-4.5	2.5	1.9	-1.1	0.0	1.9	-0.5
Denmark	-5.7	1.6	1.1	-0.4	1.1	1.7	-0.3
Germany	-5.1	4.2	3.0	0.7	0.5	2.0	1.0
Estonia	-14.1	3.3	8.3	3.2	3.0	4.0	1.2
Ireland	-5.5	-0.8	1.4	0.7	1.1	2.2	-0.3
Greece	-3.1	-4.9	-7.1	-6.4	-4.4	0.6	-4.5
Spain	-3.7	-0.3	0.4	-1.4	-1.4	0.8	-1.5
France	-3.1	1.7	1.7	0.0	0.1	1.2	0.3
Italy	-5.5	1.8	0.4	-2.2	-1.0	0.8	-1.2
Cyprus	-1.9	1.3	0.5	-2.3	-3.5	-1.3	-1.9
Latvia	-17.7	-0.9	5.5	5.3	3.8	4.1	1.0
Lithuania	-14.8	1.5	5.9	3.6	3.1	3.6	0.9

Luxembourg	-4.1	2.9	1.7	0.2	0.5	1.6	0.2
Hungary	-6.8	1.3	1.6	-1.7	-0.1	1.3	1.0
Malta	-2.4	2.7	1.6	1.0	1.5	2.0	1.0
Netherland	-3.7	1.6	1.0	-0.9	-0.6	1.1	-0.5
Austria	-3.8	2.1	2.7	0.7	0.7	1.9	0.8
Poland	1.6	3.9	4.3	2.0	1.2	2.2	2.1
Portugal	-2.9	1.9	-1.6	-3.2	-1.9	0.8	-1.2
Romania	-6.6	-1.1	2.2	0.2	1.6	2.5	-0.2
Slovenia	-7.8	1.2	0.6	-2.0	-2.0	0.7	-1.8
Slovakia	-4.9	4.4	3.2	2.0	1.1	2.9	2.0
Finland	-8.5	3.3	2.8	-0.1	0.3	1.2	0.0
Sweden	-5.0	6.6	3.7	1.0	1.3	2.7	2.0
UK	-4.0	1.8	0.9	0.0	0.9	1.9	0.2
Croatia	-6.9	-1.4	0.0	-1.9	-0.4	1.0	-1.5

According to Table 4, in 2020, 13 states will achieve GDP growth rates negative (Czech Republic, Denmark, Ireland, Greece, Spain, Italy, Cyprus, Hungary, Netherland, Portugal, Romania, Slovenia and Croatia), 8 states will have GDP growth rates between 0% and 1.0% (Bulgaria, Belgium, France, Lithuania, Luxembourg, Austria, Finland and UK) and 7 states will perform GDP growth rates greater than 1.0% (Germany, Estonia, Latvia, Malta, Poland, Slovakia and Sweden).

Poland, Slovakia and Sweden will perform GDP growth rates greater or equal than 2.0% in 2014. Greece, Cyprus, Slovenia and Croatia will face to greatest negative growth rates.

An interesting observation is that the states clusters grouping realized for 2012 is under a 78.57% ratio the same in 2020, which means that the forecast made in this paper is good.

## 5. Conclusions

Economists tried to find the ideal spatial approach in order to obtain maximum socio-economic benefits. Region seems to be the best spatial concept able to support socio-economic development. The problem is that region is very complex element which can cover a little geographical area, a greater one, a country, a group of countries, a continent and even the world. As a result, the regional approaches are different and contradictory sometimes.

European Union represented a model of regional development. As a result, it tried to implement a powerful Cohesion Policy, in order to achieve prosperity across all EU28 regions and to decrease the socio-economic disparities.

The problem is that EU28 was not able to face the global crisis and to generate the same economic recovery for all its Member States. This is why the interest in regional approach becomes very important now.

In order to implement an optimal regional policy, are necessary theoretical approaches and intervention tools.

There are a lot of regional models created in order to support the efficient regional intervention. Some of them are more sophisticated than others. All these models have restrictions and limits. Moreover, the European Cohesion Policy was implemented in order to support EU as regional integrative organization as global economic actor in its competition to USA, China, Japan and the emerging economies.

This ambitious objective was not achieved. Moreover, the new strategy Europe 2020 will lead to a division of the Member States. The analysis of the latest official data supports the idea of separation into three groups of Member States. This trend is verified by cluster analysis for all four economic indicators used in the paper.

The realized forecast leads to a less aggregated organization in which the national solutions seem to be more used in order to support the economic recovery and the sustainable development on medium term.

## 6. Acknowledgements

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## Balanced Scorecard – Strategic Management Tool of Performance in Public Institutions

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**Abstract:** Balanced Scorecard (BSC) is used to achieve an operational strategic vision at all levels of the organization regarding issues related to performance, strategy, communication, resource allocation, decision-making and competitiveness. BSC was created to restrict the limits of traditional financial and management tools and ensure unity of vision and long-term action in an organization. The main advantage of the method consists in guiding managers and departments, human resources, technological and financial resources towards the strategy of the organization. Unfortunately BSC is mainly used in private companies, because high costs and lack of specialists pose a real obstacle in implementing this instrument in public institutions. Our arguments attempts to show that the Balanced Scorecard can be the most appropriate among all the management tools for the public sector.

**Key Words:** Balanced Scorecard; KPIs; strategic management; public institutions

**JEL Classification:** H83; M41

### 1. Introduction

Balanced Scorecard provides an overview of the entire business, allowing monitoring the performance of all the proposed strategic objectives, the measures taken to achieve them, the degree of staff involvement, involving all financial and non-financial aspects. Balanced Scorecard concept involves grouping key performance indicators (KPIs), financial and nonfinancial, in four perspectives. The four sizes cover financial success, market leadership, customer loyalty, capital development, business process control and, in part, the consequences for the community (*figure 1*).

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**Figure 1. Balanced Scorecard for the public sector and non-profit**

(Source: adapted from Kaplan & Norton, 1996, p. 76)

Measurement of performance, the main function of the BSC, divides the four perspectives in *result indicators* that indicate past efforts and *pilot indicators* that indicate future performance, intervening in identifying opportunities and preventing errors. Specifically, the Balanced Scorecard approach helps public managers in the implementation of the ideas contained in management plans and strategy development.

## 2. Balanced Scorecard Perspectives

In what follows, we will consider a public sector entity of the four perspectives. The goal is ultimately to provide useful information management, in decision making, both financial and nonfinancial.

**Financial Perspective**

Financial perspective shows the results of the financial terms. Unlike the private sector, public organizations do not pursue profit, but efficiency, i.e. providing services at a reduced cost (Şandor & Raboca, 2004). Any organization calculates and uses financial indicators, but this does not automatically ensure utility in making current decisions or in explaining the performance through current actions.

The main feature and, equally, the main problem of the financial indicators is that they measure the past and what is easy to measure. Although often criticized, the financial indicators should not be abandoned, but should be chosen with care and supplemented by the non-financial measures.

**Customer Perspective**

Customer perspective refers to the organization's ability to provide products or services of good quality, in a way as to please the customer, resulting in his final satisfaction. (Şandor & Raboca, 2004) Meeting consumer expectations and desires must become the fundamental objective of organization. (Kotler, 1997)

In the public sector there are a number of specific issues. Thus, measuring customer satisfaction is an indispensable element for the organization to verify that doing the right thing. A public institution position becomes somewhat difficult because customer satisfaction and citizen satisfaction, although the two concepts are indissolubly linked, they should be treated differently. Meanwhile, the challenge for the public sector is balancing the two key aspects of public services, namely the relatively low price and in the same time, high quality, including their accessibility. Also, unlike the private sector, the public sector can not afford to choose or focus on customers preferred or most profitable customers, but must consider the needs of the entire community.

**Internal Processes Perspective**

Internal perspective refers to the analysis of the internal processes of the organization, what needs to be done in order to achieve the desired results. For this we need to identify key processes and monitor them continuously so that we know where we stand (Şandor & Raboca, 2004). It is very important for entity to organize and manage its internal work from the point of view of the customer. Customer orientation should be converted into internal indicators to show how customer expectations are met. From this perspective, the organization should identify internal powers: the use of technology, excellence of products and services, easiness to modify a product or service or to create a new one are elements that allow it to maintain and penetrate the market (Albu & Albu, 2005). Good management of internal processes means continuous improvement of processes, labor productivity and ensuring flexibility, quality and providing value.

### **Learning & Growth Perspective**

Innovation and learning perspective covers the main conditions for success. Innovation and learning capacity of the organization is essential not only when it comes to maintaining the current state of facts, but also to improve it in a constantly changing environment (Şandor & Raboca, 2004). Learning and innovation are not factual elements, quantifiable; therefore they are very difficult to assess and measure, but are elements that provide identity and long-term organizational success. These indicators are included in the BSC according to their relationship with strategy, their power to influence the future performance of the organization. Organizations are not limited only to measure what is easy to measure in this BSC perspective, but will focus on important elements in terms of organizational needs, such as innovation, employee attitude evaluation, the degree of preservation and rotation of employees, experience and learning.

### **3. Implementing the Balanced Scorecard in a Public Hospital**

Worldwide Balanced Scorecard has been implemented widely in the private sector. Some authors (Wisniewski & Olafsson, 2004, pp. 602-610) believe that the Balanced Scorecard was originally conceived as a concept addressed the need for performance measurement in the private sector and not fully meet the needs of the public sector in this regard. While in private sector the managers have begun to realize the advantages of this modern tool for monitoring and improving performance, in public sector the situation is different (Vintilescu-Belciug, et alli., 2010), the phenomenon still being shy.

Even though the costs are high and the lack of specialists constitutes a real obstacle in implementing this instrument in public institutions, we will try in the following to present arguments demonstrating that *the Balanced Scorecard can be the most suitable of all the management tools for the public sector*. Application of BSC in a public hospital (Albu & Albu, 2005) may lead to results very valuable. Currently, hospitals operate in an environment characterized by increasing need for transparency, cost control, simplification and streamlining in organizing. In figure no. 2 we schematized the steps leading the implementation of the Balanced Scorecard in a public hospital.



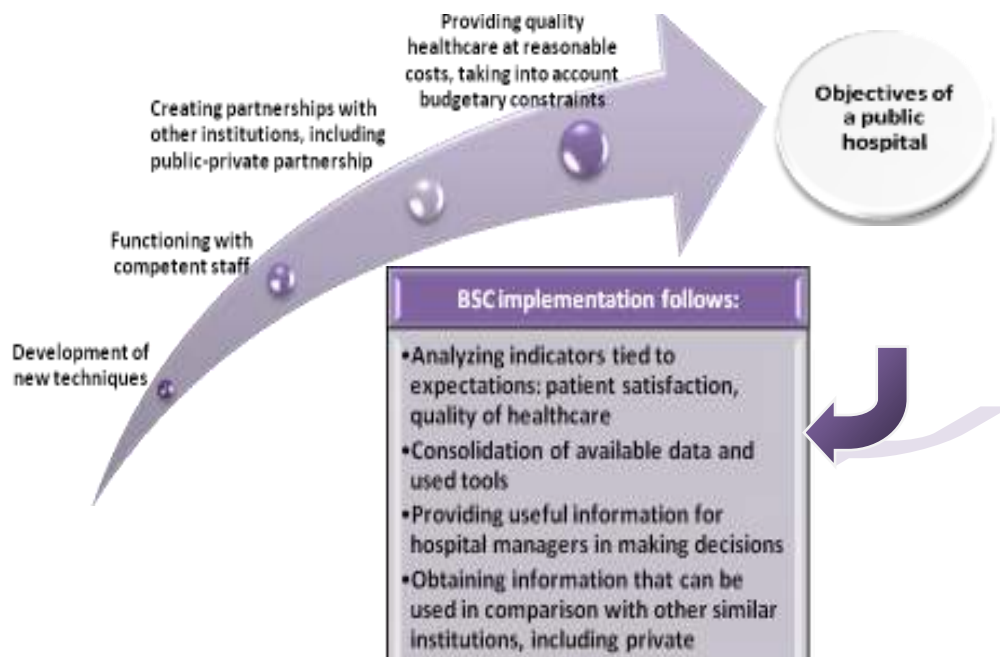


Figure 2. Steps preceding the implementation of BSC in a public hospital

(Source: own achievement)

Basically, the BSC scheme for a hospital is presented in figure no. 3.



Figure 3. Balanced Scorecard for hospital

(Source: own achievement after Albu & Albu, 2005, p. 221)

After analyzing theoretical aspects of the four perspectives, we built a model for implementation of Balanced Scorecard in a public hospital in Romania. Thus, we specifically identified the sources of data that will get financial and non-financial indicators required BSC analysis and specific objectives for each perspective individually. Finally, we determined the indicators, their measurement mode, established targets and persons in charge for objectives.

### 3.1. Financial Perspective

#### *Data Sources:*

- Strategic development plan and procurement plan;
- Revenue and expenditure budget - on the organization, on medical sections and departments;
- Budgetary execution account;
- Internal accounting system and internal stocks management system.

#### *Objectives:*

- Regular monitoring of specific economic and financial indicators:

#### Evaluation of economic and financial indicators

Target goal	Indicators	Objective <sup>1</sup>	Level	Person in charge
Optimizing the management of budget' hospital and attracting alternative sources of funding	Budget execution towards approved budget	94,97%	Tactical	CFO
	The percentage of personnel expenditures in total expenditure	59,26%		
	The percentage of medicines expenditures in total expenditure	10,03%		
	The percentage of own revenues in total revenues	12,14%		
	The average cost per day of hospitalization	Minimal		

### 3.2. Internal Processes Perspective

Hospitals are large sized organizations with a high concentration of capital and a higher inertia of internal processes. Internal processes of a hospital mainly relate:

- Strategic management of the organization;
- Operational management of information;
- HR management;

<sup>1</sup> The optimum value of the indicator or national averages according to the Public Health Ministry Order no. 1567 of 14 September 2007 approving the national averages of hospital management performance indicators published in the Official Gazette of Romania no. 683 of 8 October 2007.

- Environment care management;
- Quality services management;
- Patient rights and communication;
- Management of patient data;
- Prevention and risk management;
- Management of nosocomial infections.

*Data Sources:*

- Integrated Quality Management System (ISO standards and operating procedures);
- Internal analysis based on classification system DRG<sup>1</sup>;
- Medical statistics;
- Internal control management & risk management<sup>2</sup>.

*Objectives:*

1. Improving internal work processes;
2. Increasing the efficiency of work processes;
3. Intensifying the monitoring and control activities in each medical department to eliminate existing risks and efficient use of resources allocated by monitoring indicators for the use of services regularly:
  - Number of patients discharged;
  - Average length of stay;
  - Utilization rate of beds;
  - Complexity index of discharged cases;
  - Proportion of patients hospitalized with programming in all hospitalized patients;
  - Proportion of emergencies in all patients.
4. Development of prevention and control of resources allocated to protect against losses due to waste, abuse, errors and frauds;
5. Improving communication between hospital structures to ensure operational flow of information without distortion, so that they can be exploited effectively in the prevention and internal control;
6. Designing in each structure of performance standards for each activity, in order to use them to perform analyzes based on objective criteria for exploitation of allocated resources;
7. Establishing mutually advantageous relations with partners:

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<sup>1</sup> DRG (Diagnosis Related Groups) is a classification scheme in which similar patients are divided into diagnostic groups, homogeneous in terms of clinical disease and the resources used in treatment.

<sup>2</sup> According to the Ministry of Public Finance Order no. 946 of 4 July 2005 approving the Internal Control Code, including standards of management / internal control and public entities and for development of management control systems, published in Official Gazette of Romania no. 675 of 28 July 2005.

## Assessment of the relation with partners

Target goal	Indicators	Objective	Level	Person in charge
The level of unpaid bills to suppliers on due date	The number of unpaid bills to suppliers on due date	→ 0	Tactical	CFO
The level of uncollected sales	Uncollected bills on due date on each client	→ 0		

**3.3. Customer Perspective***Data Sources:*

- Medical statistics;
- Satisfaction questionnaires requested directly to patients and on hospital site;
- Internal studies by category of customers (patients, diagnostics and types of hospitalization).

*Objectives:*

1. Increasing specialization and diversification of medical services provided;
2. Active involvement at the community level;
3. Obtaining maximum satisfaction of patient:

## Evaluation of patient satisfaction

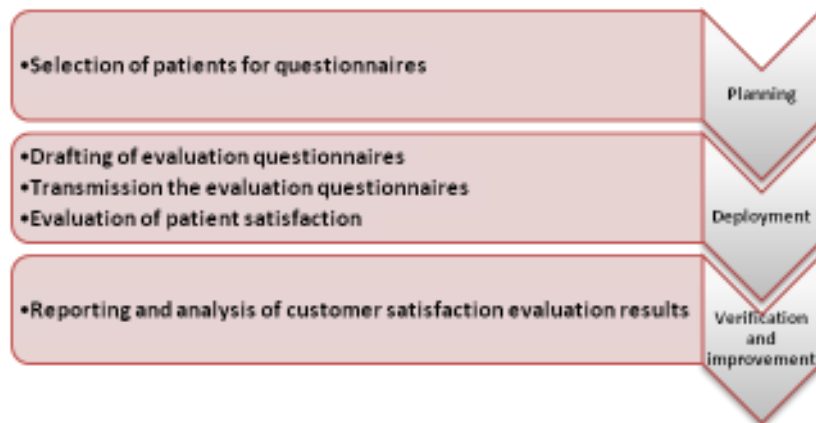
Target goal	Indicators	Objective	Level	Person in charge
Quality of healthcare services provided to patients	Number of complaints received from patients regarding the quality of services	→ 0	Strategic	Manager and
	Prejudices caused to patients in the hospital's fault or medical staff	→ 0		Medical Director
Promptitude to achieving health services	Number of cases on non-compliance the conditions set with patients	→ 0	Strategic	Chief of quality management
	Number of situations in which exceeded the delivery deadlines stipulated	→ 0		

#### 4. Getting patient feedback

Evaluation of patient feedback

Target goal	Indicators	Objective	Level	Person in charge
Patient satisfaction	Patient satisfaction index by: <ul style="list-style-type: none"> <li>- Number of suggestions received from patients;</li> <li>- Percentage of complaints considered objective by the medical board;</li> <li>- Number of complaints resolved in favor of patients.</li> </ul>	Minim 95%	Strategic	Manager and Medical Board
	Analysis of patient satisfaction questionnaires and taking appropriate measures through: <ul style="list-style-type: none"> <li>- Number of completed questionnaires;</li> <li>- Number of improvement measures taken.</li> </ul>	→ 100%		

The patient satisfaction evaluation process in the hospital is shown in figure no. 4



**Figure 4. Description of evaluation process of patient satisfaction in hospital**

(Source: own achievement)

Increasing the quality of services provided by continuous monitoring of quality indicators:

- In-hospital mortality rate;
- Nosocomial infections rate;
- Rate of patients readmitted within 30 days of discharge;
- Index of concordance between the diagnosis at admission and discharge diagnosis;

- Percentage of patients transferred to other hospitals of all patients hospitalized;
- Number of patient complaints recorded.

## 5. Learning & Growth Perspective

### *Data Sources:*

- Training statistics;
- Continuing medical education (CME points): training courses, participation in medical conferences, published articles, etc.;
- Annual evaluation of individual professional competence and performance:

#### ***For executive positions:***

- knowledge and experience;
- promptness and efficiency in achieving the tasks set out in the job description;
- quality of work and activities;
- assuming responsibilities by responsiveness, availability to extra effort, perseverance, objectivity, discipline;
- intensity of involving in use of equipment and materials without exceeding the norms of consumption;
- adaptation to the complexity of work, initiative and creativity.

#### ***For management positions (in addition to the executive positions):***

- knowledge of the purpose, objectives and role of the compartment throughout organization;
- ability to make optimal decisions and taking responsibility in connection therewith;
- ability to organize and coordinate the activity of compartment;
- ability to create a stimulating environment, without conflicts and good collaboration with other departments.

### *Objectives:*

1. Providing employment positions with qualified personnel with specialized training necessary to perform the duties set out in the job description;
2. Providing continued training of staff;
3. Increasing the degree of dissemination of successful practices;
4. Developing communication skills and competences;
5. Regular evaluation of employees in regarding the fulfillment of duties and responsibilities:

Regular evaluation of staff

Target goal	Indicators	Objective	Level	Person in charge
Employees evaluation results	Share of employees who have received the qualification « <i>Very Good</i> » or « <i>Good</i> » in the annual evaluation of employees in total evaluated employees	→ 100%	Tactical	HR Chief

Monitoring the performance of employees in each department and medical sectors using quantitative and qualitative indicators, specific and relevant, including with regard to economy, efficiency and effectiveness:

Evaluation of HR management

Target goal	Indicators	Objective	Level	Person in charge
Human resource management performance	Average number of patients discharged on a doctor	291	Strategic	Manager and Medical Director
	Average number of patients discharged to a nurse	76		
	Average number of consultations performed in ambulatory on a doctor	4.571		
	Average number of consultations on a doctor in emergency care unit	167		
Adequacy of establishment plan hospital	Proportion of doctors in all staff	10,49%	Tactical	HR Chief
	Proportion of medical staff in all hospital staff employed	61,33%		
	Proportion of medical staff with higher education of all medical personnel	22,77%		

Training, awareness and competence of staff covers the following aspects:

*At hiring:*

- planning the necessary staff (staff establishment, staff normative);
- Evaluation, employment and training at hiring;
- Verification and improvement by upgrading staffing needs.

*For employed staff:*

- Planning by establishing training needs, planning and preparing training courses;
- Development of training courses, evaluation of the trainees, evidence of training courses and staff awareness;
- Verification and improvement by assessing the effectiveness of training and competence of staff, updating training needs.

## 6. Conclusions

We appreciate the fact that BSC was created and it is used to restrict the limits of and traditional financial and management tools and to ensure unity of vision and long-term action in the organization. The main advantage of the method consists in guiding managers, departments, human resources, technological and financial toward the strategy organization. In Romania, where organizational climate is quite poor, there is every chance that this system should be regarded only as an additional form of control of staff, so be sabotaged by employees. On the other hand, the interest of public institutions managers did not go too often toward performance, at least in terms of BSC experience. A few public institutions are concerned about customer satisfaction, accountability to the citizen and efficiency. Using the Balanced Scorecard in the public sector is shown to be beneficial since improves and enhances aspects of performance, strategy, alignment, communication, resource allocation, decision-making and competitiveness. Balanced Scorecard is a tool built to harmonize actions and strategic plans into a consistent control system. Since the BSC philosophy is to learn from own actions, teamwork and follow up strategy, this tool puts a heavy emphasis on clear communication of objectives and priorities. In short, being flexible and dynamic, BSC shows what must be done.

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## About Andrica's Conjecture

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**Abstract:** The paper establishes an equivalence of the Andrica's conjecture in the direction of an increase of the difference of square root of primes by a power of a ratio of two consecutive primes.

**Keywords:** Andrica's conjecture; prime

**JEL Classification:** E23

### 1. Introduction

Even prime number theory dates back to ancient times (see the Rhind papyrus or Euclid's Elements) it retains its topicality and fascination to any mathematician due to numerous issues remain unresolved to this day.

A number  $p \in \mathbb{N}$ ,  $p \geq 2$  is called prime if its only positive divisors are 1 and  $p$ . The remarkable property of primes is that any nonzero natural number other than 1 can be written as a unique product (up to a permutation of factors) of prime numbers to various powers.

If there is not a formula, for the moment, generating prime numbers, there exist a lot of attempts (many successful, in fact) to determine some of their properties.

Unfortunately, many results are at the stage of conjectures (theorems that seem to be valid, but remained unproven yet).

A famous conjecture relative to prime numbers is that of Dorin Andrica. Denoting by  $p_n$  - the  $n$ -th prime number ( $p_1=2$ ,  $p_2=3$ ,  $p_3=5$  etc.), Andrica's conjecture ([1]) states that:

$$\sqrt{p_{n+1}} - \sqrt{p_n} < 1 \quad \forall n \geq 1$$

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Noting  $g_n = p_{n+1} - p_n$  – the prime gap that is the difference between two successive prime numbers, the conjecture can be written in the form:

$$g_n < 2\sqrt{p_n} + 1 \quad \forall n \geq 1$$

The equivalence is immediate because

$$g_n = p_{n+1} - p_n < 2\sqrt{p_n} + 1 \Leftrightarrow p_{n+1} < (\sqrt{p_n} + 1)^2 \text{ which is obviously equivalent to: } \\ \sqrt{p_{n+1}} - \sqrt{p_n} < 1.$$

Even if Andrica's conjecture is weaker than that resulting from Oppermann's conjecture which states that  $g_n < \sqrt{p_n}$  the attempts to prove have not been successful to this moment.

In the following, we shall prove a theorem of equivalence of Andrica's conjecture with another conjecture of increasing the difference of square root of consecutive primes with a power of their ratio.

## 2. Main Theorem

### Theorem

Let  $p_n$  the  $n$ -th prime number. The following statements are equivalent for  $n \geq 5$ :

1.  $\sqrt{p_{n+1}} - \sqrt{p_n} < 1$ ;
2.  $\exists \alpha \geq 0$  such that:  $\sqrt{p_{n+1}} - \sqrt{p_n} < \left(\frac{p_n}{p_{n+1}}\right)^\alpha$ .

### Proof

$$\underline{2 \Rightarrow 1} \text{ Because } \frac{p_n}{p_{n+1}} < 1 \text{ follows that: } \left(\frac{p_n}{p_{n+1}}\right)^\alpha < \left(\frac{p_n}{p_{n+1}}\right)^0 = 1.$$

1  $\Rightarrow$  2 If we take the logarithm in the relationship, it becomes:

$$\ln(\sqrt{p_{n+1}} - \sqrt{p_n}) < \alpha \ln\left(\frac{p_n}{p_{n+1}}\right) \Leftrightarrow \ln(\sqrt{p_{n+1}} - \sqrt{p_n}) < \alpha(\ln p_n - \ln p_{n+1}) \Leftrightarrow \\ \frac{\ln(\sqrt{p_{n+1}} - \sqrt{p_n})}{\ln p_{n+1} - \ln p_n} < -\alpha \Leftrightarrow$$

$$\frac{\ln(\sqrt{p_{n+1}} - \sqrt{p_n})}{\ln \sqrt{p_{n+1}} - \ln \sqrt{p_n}} < -2\alpha.$$

Let now the function:

$$f: (a, \infty) \rightarrow \mathbf{R}, f(x) = \frac{\ln(x-a)}{\ln x - \ln a} \text{ with } a > 2$$

We have now:  $f'(x) = \frac{x(\ln x - \ln a) - (x-a)\ln(x-a)}{x(x-a)(\ln x - \ln a)^2}$ .

Noting  $g(x) = x(\ln x - \ln a) - (x-a)\ln(x-a)$  we get:  $g'(x) = \ln x - \ln a - \ln(x-a) = h(x)$ .

Because  $h'(x) = -\frac{a}{x(x-a)} < 0$  we have that  $h$  is strictly decreasing. How

$h(a+1) = \ln(a+1) - \ln a > 0$ ,  $h(a+2) = \ln \frac{a+2}{2a} < 0$ , follows that  $\exists \xi \in (a+1, a+2)$  such that:

$h(\xi) = 0$  that is:

$$(1) \ln \xi - \ln a = \ln(\xi - a)$$

and after the monotony:  $h(x) > 0 \forall x < \xi$  and  $h(x) < 0 \forall x > \xi$ .

Because  $g' = h$  we obtain that  $g$  is strictly increasing on  $(a, \xi)$  and strictly decreasing on  $(\xi, \infty)$ .

But  $\lim_{x \rightarrow a} g(x) = 0$ ,  $\lim_{x \rightarrow \infty} g(x) = -\infty$ ,  $g(\xi) = \xi(\ln \xi - \ln a) - (\xi - a)\ln(\xi - a)$  and from (1):

$$g(\xi) = a \ln(\xi - a) > 0$$

It results that  $\exists \eta > \xi > a+1$  such that:  $g(\eta) = 0$  that is:

$$(2) \eta(\ln \eta - \ln a) = (\eta - a)\ln(\eta - a)$$

From monotonicity, we obtain that:  $g(x) > 0 \forall x \in (a, \eta)$  and  $g(x) < 0 \forall x \in (\eta, \infty)$ .

Therefore,  $f$  is strictly increasing on  $(a, \eta)$  and strictly decreasing on  $(\eta, \infty)$ .

As  $f(a+1) = 0$ ,  $f(\eta) = \frac{\ln(\eta - a)}{\ln \eta - \ln a}$ , and from (2):  $f(\eta) = \frac{\eta}{\eta - a} > 0$ ,  $\lim_{x \rightarrow \infty} f(x) = 1$  it follows that:  $f(x) < 0 \forall x \in (a, a+1)$ .

From hypothesis 1 (Andrica's conjecture), we have:  $\sqrt{p_{n+1}} - \sqrt{p_n} < 1$  and noting:  
 $x = \sqrt{p_{n+1}}$ ,  $a = \sqrt{p_n}$  we have:  $x \in (a, a+1)$ .

Be so:  $\alpha = -\frac{1}{2} \sup_n \frac{\ln(\sqrt{p_{n+1}} - \sqrt{p_n})}{\ln \sqrt{p_{n+1}} - \ln \sqrt{p_n}} \geq 0$ . The statement 2 is now obvious. Q.E.D.

### 3. Determination of the Constant $\alpha$

Using the Wolfram Mathematica software, in order to determine the constant  $\alpha$  (for the first 100000 prime numbers):

```
alfa:=1000.
```

```
For[n=5,n<100000,n++,alfa1=Log[Prime[n]/Prime[n+1],Sqrt[Prime[n+1]]]-
```

```
Sqrt[Prime[n]]];alfa=Min[alfa1,alfa];
```

```
Print[N[alfa,1000]]
```

we found that the first 1000 decimals are:

```

alpha=2.281103221027218229423232822443286599017584100009984588220198770
7555016956079556125523902095828355521972189294678153434257651743655
1726362923880678160746758935639657414508211919857643587356785905060
6949765734239636479131860224037385179731071495942268916184068058269
6177512720818817490245596293254286947503092584044428951753449292269
7389861883902366389124093450412998636957454073445598539595694269058
9958313834222509660041292032936969131875482708936809950924268932852
6997409051212237021151286038659114545358509297427153361178689719192
7351545951228711831887776623070206318211691345478696428460823105785
4097720248263859745150386851610652371395957541613534650131084885714
3384952309151433452360237879606095184289487480462134479859039214689
8208626417098165357501169756266653666858624450374358872935039206880
8501947089500551068277858220736778549760090107511452195156335525727
1942429070573882246361510791556298086569047772964502980196653160300
34642716170057957313704524610621876879383627332120315454057552219

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## **The History of the Environmental Conditions Research in the Brăila Plain**

**Anca Gabriela Turtureanu <sup>1</sup>**

**Abstract:** The Romanian Plain has generally represented the study subject of research in the field of Geography and especially of the environment geography and resources. Due to the structure and the geological movements of the climate factors, the hydrography in the area of the Braila Plain has represents a subject of many geographers' interest. The divagation and the leaving of the water course phenomena of some rivers, the genesis and the classification of lake basins, underground waters, the links that are established between these hydrographical units, the water balance, the water chemism, the existence of the oil and gas resources made the researchers study them as a whole.

**Keywords:** research, environmental conditions

**JEL Classification:** Q300; Q390

### **1. Introduction**

The Romanian Plain and especially its North-Eastern part of the relief unit in Romania where the most frequent areas with important salt accumulations are met, easily soluble in the lakes, underground waters or soil. The process is favored by several main factors: the dry climate with long drought periods, the semiendoreic area, the poorly fragmented and poorly drained relief, the existence in the Subcarpathians of saliferous rocks formations, from which salt could be transported through surface or underground waters.

The presence of numerous salted lakes in the Eastern part of the Romanian Plain, in the northern part of Ialomita, especially in the Braila Plain, has sparked the interest of many researchers, geologists, geographers, chemists or doctors even since the last century.

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## 2. Research History

The number of the scientific papers has significantly grown since the end of the XIX-th century.

Amongst the first contributions, we can emphasize the ones of Carol Davilla (1873), I. Romniceanu (1887), Gr. Stefanescu (1888), J. C. Apostoleanu (1884), G. M. Murgoci (1957) and others.

In 1884, J.C. Apostoleanu wrote about the Salt Lake in Braila, emphasizing the fact that ever since the last century, the issue of the therapeutic waters in this area has sparked the researchers' interest, who always tried to understand the phenomenon, and also to find solutions for their use.

In 1907, the "*Raport asupra lucrărilor facute de secția agrogeologică în 1906-1907*" volume claims that "Mr. Em. I. Protopopescu-Pache has researched the Ialomita Plain, Baragan, south from the Bucuresti-Fetesti railway and the Mostistea Valley"... "Mr. Murgoci, the head of the section"... "studied all by himself the Rusetu Crown Domain and the region of the salt lakes in Tatratru, the area around Braila and the Salt Lake"... (Murgoci, 1957).

G. M. Murgoci claims within this volume that "Calmatui Valley is nothing else than an old water course of Buzau, which divagated towards East, probably on the Iencei Valley and then towards north-east, across some parallel valleys, such as Batrana (with Jirlau), Infundu (with Amara), Boldu (with Balta Alba). (Murgoci, 1957).

Further, he argues that "the valleys and the plain have always been slimed, thus forming large cones of dejection where waters coming from the hills have spread. The waters that originated from the Buzau, Ramnicului Valley, etc. were spreading in more strong water courses, integrating more areas between them. One of the courses – Buzau – was Calamtui, another might have been Buzoielul with Ianca and another one the actual Buzau" (Murgoci, 1957).

"Another consequence of the sinking of the plain and of the sliming of the small rivers meadows was the filling of the tributary valleys of Ialomita, Calmatui, Buzau, etc., creating alongside the old tributary valley a series of river estuaries and even lakes, which nowadays became salty, such as: Strachina, Amara, Fundata, Saratuica, Jilava, Cotorca alongside Ialomita, Costeiu, Caineni, Jirlau, Balta Alba, Amara alongside Buzau" (Murgoci, 1957).

G. M. Murgoci further claimed that "Salt lakes are associated to two groups of depressions: the ones on the old, crossed and obstructed from a larger draining course; others are in depression in the middle of fields, at the meeting points of the prehistoric dunes in the loess area. They might also correspond to some river beds

of some water courses, which were subsequently obstructed, covered and transformed through the advance of the dunes or through the loess overlying. Their water though is an underground water, which emerges in deeper depression compared to the superior level of waters.

The underground water, fueled by atmospheric precipitations and poorly from the rivers that come from the hills, is full of soluble salts, due to the reactions that happen in the loose soil and underground, rich in many alkaline substances; depressions, in front of the atmosphere focuses more and creates the salt lakes.

Together with D. R. Ruscescu, G.M. Murgoci developed a map of phreatic waters, which will be referred to for much time from now on.

These statements of G.M. Murgoci and the one regarding the fact that he supposes a flow of Danube more western from the actual one through Baragan on the Jigalia Valley have also been approved by G. Valsan and N. Popp, but fought by v. Mihailescu.

Pana A. (1911), in his volume, „*Cursul inferior al Călmățuiului*”, develops the hypotheses launched by Gheorghe Murgoci regarding the flow of the Buzau Valley in the plain on the actual valley of Calmatui and Râmnicu Sărat. Vâlsan G., (1915) in his PhD thesis, named „*Câmpia Română-Contributiuni de geografie fizică*”, proposes an synthetically apparently general approach of the elements that are specific to the plain area, which in reality represent an analysis model of the relief in the plain area, emphasizing the paleogeographical evolution.

G. Valsan, in the “*Câmpia Romana. Contributiuni de geografie fizica*” volume, issued in 1915, continues the ideas displayed by G. M. Murgoci, but brings new geomorphological arguments and new statements regarding the fact that “Calmatuiul, which has a lower volume compared to Sarata, seems to be a poor steppe stream, lost in the middle of this sad meadow like in a coat that is too large for its poor being. The waters that start running it near Buzau, would have been long time absorbed by the humidity voracious land or by the sun beams of the summer, if it wouldn't have received underground supplies from Buzau. Phreatic waters also fuel the several lakes especially under the right shores of the meadow in the old bays carved in deserted meandres. The only activity of Calmatui is the desertion of the several slimed ravines, that nowadays seem to be grinds which are higher than the rest of the plain, and in the nearby of the Danube where they form a very complicated small meadow where it deepens with approximately 2 meters. This small meadow is the result of the effort of Calmatui to create an own equilibrium profile in the cone of dejection of the meadow where it wanders, spreading it into the land of the Balta towards the nearby of the Danube flow.” (Valsan, 1971).



When talking about underground waters, G. Valsan uses the phreatic waters map developed by G.M. Murgoci.

The complex problems of Baragan from the geological perspective are underlined by E. Liteanu (1956) who made the first ranking of the underground waters between Arges and Siret, based on the significant extent of the psamo-psephitic, which represent the most important hydrogeological reservoir and establishes a series of hydrogeological and hydrogeochemical links between the waters of the salt lakes and phreatic waters.

A wide and complex study with significant value regarding this region, which emphasized the morphohydrographical climate conditions, the draining issues, in the case of the issues regarding the humidity excess has been carried out by P. Gastescu, I. Zavoianu, O. Bogdan, A. Breier, B. Driga.

Alongside this study, a reference map was developed - "*Sectorul de Nord-Est al Campiei Romane - excesul de umiditate*".

Research upon the hydrologic regime of the Danube was carried out by dr. Petre Gastescu in the paper called "Cateva aspecte ale bilantului hidrologic al lacurilor din Lunca Dunarii", S. Hancu, C. Bondar, M. Podani, I. Zavoianu

Regarding the issue of the lakes in Braila Plain, a wide research field was opened both regarding the origin of the lake basin and their chemism.

N. Florea, in the study called "*Geochimia si valorificarea apelor din Campia Romana de N-E*" (1976) proposes the description of the natural waters in the northern part of the Baragan regarding the salt accumulation, the examination of the issue regarding the balneare, industrial and agricultural value of the waters.

Data about the lakes in this area is found in the synthesis work on the lakes in Romania, which emphasizes the fact that the presence of the lakes in the Romanian Plain, contrary to the unfavorable climate conditions is especially related to the hydrographic network (Gastescu, 1971).

The origin and the genesis of the salts in the phreatic waters and the lakes in the Braila Plain have been explained under different approaches by Gr. Stefanescu (1888), L. Mrazec (1901), G. M. Murgoci (1907). The hypotheses they brought are explained, supported or fought with persuasive arguments by P. Petrescu (1940), E. Liteanu si C. Ghenea (1962), P. Marosi (1967), N. Florea (1970) si P. Gastescu (1971).

P. Gastescu (1971) gives arguments supporting the fact that the saltiness in this area depends on the underground and phreatic waters (study on the Caineni lake), but also on the climate conditions and the role of the lake basins as slow salt concentration and preservation areas.

I. Ujvari (1972) takes again the issues regarding the hydrographic features (the flow, solid flows, hydrochemical particularities, etc.) and also the deserted flows of Buzau, the underground draining and the economic value of the water resources in the area.

Regarding the study of the underground waters in this area, significant contributions were brought by E. Liteanu, P. Marosi, A. Pricajan and A. Brandabur.

Amongst the first ones that begin the analysis of the environment factors we can also enumerate St. Hepites (1882), who made studies on the climate in the Romanian Plain.

E. Oteteleseanu (1914), C. D. Elefteriu (1921), O. Neacsu (1964), I. Gugiuman (1964) also write about the climate in the North-Eastern Romanian Plain.

A benchmark in the field remains the work called "Potentialul climatic al Baraganului" (O. Bogdan, 1980) which emphasizes the climatic potential of Baragan, based on significant field research and interpretations of the links between the climate, hydrographic, local and regional factors.

Vegetation and fauna were studied by I. Serbanescu (1965), R. Calinescu (1969), D. Ivan (1979) and N. Costache (1996) elaborates a study regarding the biogeographical regionalization of Romania.

Soils, with all their particular features specific to the area of the Braila Plain were analyzed by Cernescu (1961), C. Chirita (1967), C. Paunescu (1967), D. Teaci (1967), N. Florea (1976).

Wide research, especially on the soils and the phreatic waters has been carried at SCCASS Braila by a group of agricultural and hydrotechnic engineers, amongst which we can name I. Visinescu, M. Zamfir, etc. together with I. Cojocar, they published in 2001 the volume „Insula Mare a Brailei. Reabilitare hidroameliorativa”, which widely displays all the activities as well as the analyses and the conclusions they have reached. (I. Andronache, 2008).

After 1950, research institutes have developed (IGAR, INMH, IPACH, ISPIF, ICITID etc.), wherein the most important studies regarding the complex arrangement of the Danube Meadow and monographic studies were carried out (*Zona de varsare a Dunarii - monografie hidrologica, Dunarea intre Bazias si Ceatal Izmail, Geografia Vaii Dunarii Romanesti* etc.).

### 3. Conclusions

Nowadays, the Romanian Plain is scientifically well known under all aspects (geophysical and geological, geomorphological and geographical ones, climate ones, hydrographic and hydrogeological, pedological and agro-ameliorative, geotechnical and geobotanical ones), through different individual or collective research, carried out by different specialty institutes or departments.

Besides the surface research, hundreds and even thousands of hydrogeological and geotechnical drillings were made, which impose their synthesizing and interpretation on lithological and stratigraphic complexes.

By valuing the tradition of the Romanian naturalist school, which set the basis of the scientific knowledge of the Romanian Plain at the end of the XIX-th century, the XX-th century and the first part of our century, research has continued within the Doctoral Schools of the Romanian Academy or within the universities that took again and developed the research according to the actual requirements.

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