**Miscellaneous**

# On Certain Conditions for Generating Production Functions - I

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**Abstract**: The article is the first in a series that will treat underlying conditions to generate a production function. The importance of production functions is fundamental to analyze and forecast the various indicators that highlights different aspects of the production process. How often forgets that these functions start from some premises, the article comes just meeting these challenges, analyzing different initial conditions. On the other hand, where possible, we have shown the concrete way of determining the parameters of the function.

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## 1 Introduction

Theory of production functions is vitally important in microeconomic analysis.

The need of economic phenomena mathematization, not only from a desire to give legitimacy to scientific economic theory but rather, to draw conclusions and prediction of enterprise activity required a careful analysis of them.

Well-thought literature profile, but especially practical applications encountered in all kinds of handouts, printed or online, we drew a number of issues that sometimes are neglected (probably considered insignificant) or omitted with true intent.

The first issue found by us is that of verification of sufficient conditions (not always necessary, but depending on the actual nature of the problem) as a function to be truly of production.

Another aspect which seems essential is the practical applicability. One question that could be asked of any student from any part of the Earth, is: "Departing from a

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series of discrete data, how you will generate the output and, especially, what kind of production function will choose?"

By own researches, I realized that maybe over 90% of production functions presented in teaching applications are of Cobb-Douglas type (requiring, however, the constancy of elasticity), the remainder being more or less created artificial (often they even unverified existing conditions).

It might object here that the learning exercises aims to increase math skills with these functions. The problem is not this, but what follow...

I rarely saw concrete applications, showing clearly how to practically apply these functions. Without this approach, the theory remains dry, with beautiful graphics (as an aside, all graphs of production looks pretty much the same, what will result in the following) and without practical application.

Following these minimum considerations, we will try in the following pages to generate major production functions based on practical conditions (the approach being not new, meeting in original papers), but systematized and then explaining in each case how can apply them practically.

## 2 General Notions

In what follows, we assume that resources are infinitely divisible, which implies the use of specific tools of mathematical analysis to analyze specific phenomena.

We thus define on $\mathbf{R}^n$ the space of production for n fixed resources as:

$$SP=\{(x_1,...,x_n) \mid x_i \geq 0,\ i=\overline{1,n}\}$$

where $x \in SP$, $x=(x_1,...,x_n)$ is an ordered set of resources (inputs).

Because within a production process, depending on the nature of applied technology, but also its specificity, not any amount of resources possible, we will restrict the production area to a subset $D_p \subset SP$ called production domain.

It is now called production function (output) an application:

$$Q: D_p \rightarrow \mathbf{R}_+,\ (x_1,...,x_n) \rightarrow Q(x_1,...,x_n) \in \mathbf{R}_+\ \forall (x_1,...,x_n) \in D_p$$

For an effective and complex mathematical analysis of a production function we will require a number of axioms (not all essential) both its scope and its definition.

A1. The production domain $D_p$ is convex i.e. $\forall x=(x_1,...,x_n)$, $y=(y_1,...,y_n) \in D_p$ $\forall \lambda \in [0,1]$ follows $(1-\lambda)x+\lambda y=((1-\lambda)x_1+\lambda y_1,...,(1-\lambda)x_n+\lambda y_n) \in D_p$.

Axiom A1 only mean that in the process of changing of the inputs from a level x to y, the linear shift is achieved through a series of successive steps which keeps them in the field of production, so by default the possibility of using the production function chosen. The condition could relax here, requiring domain to be, for example, connected by arches, that to be a continuous path between any two n-uple inputs.

A2. Q(0,0,...,0)=0

The axiom reflects a common sense assumption namely that in the absence of any input can not get any output.

A3. The production function is continuous.

Continuity, in purely mathematical sense, represents that for any fixed point $\left(\overline{x}_1,...,\overline{x}_n\right)$ of the production domain $D_p$ and any string of inputs $(y_k)_{k\geq 1}$, $y_k=\left(y_1^k,...,y_n^k\right)$ which converges to $\left(\overline{x}_1,...,\overline{x}_n\right)$ (or otherwise $y_i^k \to \overline{x}_i \quad \forall i=\overline{1,n}$ ) the production $Q\left(y_1^k,...,y_n^k\right)$ converges to $Q\left(\overline{x}_1,...,\overline{x}_n\right)$.

More simply, the continuity of the production function means that for two sets of resources $(x_1,...,x_n)$ and $(y_1,...,y_n)\in D_p$ close enough, result outputs $Q(x_1,...,x_n)$ and $Q(y_1,...,y_n)$ close enough. In other words, a very small change of inputs lead to a reasonable production obtained.

An axiom, not necessarily required, but particularly useful for obtaining significant results (using differential calculus) is:

A4. The production function is of class $C^2(D_p)$ i.e. admits 2nd order continous partial derivatives.

The condition of belonging to the class $C^2$ may seem, at first glance, restrictive, but is not really. All basic functions (constant, power, exponential, logarithmic, trigonometric functions as those obtained from them by arithmetic operations of addition, subtraction, multiplication, division, power lifting, composing or reversal) are of $C^\infty$ class (implicitly of class $C^2$) on the definition domain i.e. have their partial derivatives of any order and these are continues. As a function of class $C^k$, $k\geq 0$ is continuous implies that axiom A3, given that accept A4, is a simple consequence of the latter, so it can be removed.

What is actually at least $C^1$ class differentiability? If for a continuous function means, at an immediately approach (without much mathematical rigor) that its graph is not „broken" on the definition domain, the derivativability of class $C^1$ means that it does not have „corners" or „folds", the graph being smooth. In addition, for example in a corner point (for functions of one variable – different left and right derivatives) we can not make predictions, the behavior at left/right not anticipates the behavior at right/left.

A5. The production function is monotonically increasing in each variable.

A5 axiom states that in "ceteris paribus" hypotesis, $\forall i=\overline{1,n}$ if $x_i \geq y_i$ then> $Q(\bar{x}_1,...\bar{x}_{i-1},x_i,\bar{x}_{i+1},\bar{x}_n) \geq Q(\bar{x}_1,...\bar{x}_{i-1},y_i,\bar{x}_{i+1},\bar{x}_n)$ $\forall \bar{x}_k \geq 0$, $k=\overline{1,n}$, $k \neq i$ such that $(\bar{x}_1,...\bar{x}_{i-1},x_i,\bar{x}_{i+1},\bar{x}_n)$, $(\bar{x}_1,...\bar{x}_{i-1},y_i,\bar{x}_{i+1},\bar{x}_n) \in D_p$. If the function Q is at least $C^1(D_p)$ the character of monotonically increasing becomes $\frac{\partial Q}{\partial x_i} \geq 0$, $i=\overline{1,n}$. In terms of a "classic" production function with two variables: K – capital and L - labor, we have: $\frac{\partial Q}{\partial K} \geq 0$, $\frac{\partial Q}{\partial L} \geq 0$.

Also from the axiom A5 result, as an immediate consequence, that if $x_1 \geq y_1,...,x_n \geq y_n$ then: $Q(x_1,x_2,...,x_n) \geq Q(y_1,x_2,...,x_n) \geq Q(y_1,y_2,...,x_n) \geq ... \geq Q(y_1,y_2,...,y_n)$. It is obvious that the relationship occurs only if the nature of the inequalities between components is the same for all of them.

A condition often referred to in the definition of the production function is:â

A6. The production function is quasi-concave.

The quasi-concavity of a function means:

$$Q(\lambda x+(1-\lambda)y) \geq \min(Q(x),Q(y)) \ \forall \lambda \in [0,1] \ \forall x,y \in R_p$$

Geometrically speaking, a quasi-concave function has property to be above the lowest values recorded at the end of a certain segment. The property is equivalent to the convexity of the set $Q^{-1}[a,\infty)$ $\forall a \in \mathbf{R}$, where $Q^{-1}[a,\infty)=\{x \in R_p \mid Q(x) \geq a\}$.

What does the quasi-concavity so? Convexity of the set $Q^{-1}[a,\infty)$ lies in that if $Q(x) \geq a$, $Q(y) \geq a$ then $Q((1-\lambda)x+\lambda y) \geq a$. This specifies, in conjunction with the axiom A1, that the transition from one set of inputs x to y is at a production level equal to or greater than a specified lower limit. Neither this condition would not necessarily be required, existing situations (for example, the transition to a market economy of the former communist states) the refurbishment (thus changing the structure of inputs) was made with temporary dip in the level of production. But as economic analysis, most often refers (unfortunately) to the processes that are somewhat stabilized, we will retain this condition.

Considering so a production function $Q:D_p \to \mathbf{R}_+$, $(x_1,...,x_n) \to Q(x_1,...,x_n) \in \mathbf{R}_+$ $\forall (x_1,...,x_n) \in D_p$ let the bordered Hessian matrix:

$$H^B(f)=\begin{pmatrix} 0 & \frac{\partial Q}{\partial x_1} & \frac{\partial Q}{\partial x_2} & \cdots & \frac{\partial Q}{\partial x_n} \\ \frac{\partial Q}{\partial x_1} & \frac{\partial^2 Q}{\partial x_1^2} & \frac{\partial^2 Q}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 Q}{\partial x_1 \partial x_n} \\ \frac{\partial Q}{\partial x_2} & \frac{\partial^2 Q}{\partial x_2 \partial x_1} & \frac{\partial^2 Q}{\partial x_2^2} & \cdots & \frac{\partial^2 Q}{\partial x_2 \partial x_n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{\partial Q}{\partial x_n} & \frac{\partial^2 Q}{\partial x_n \partial x_1} & \frac{\partial^2 Q}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 Q}{\partial x_n^2} \end{pmatrix}$$

and $\Delta_k^B$ - the boarded principal diagonal determinants formed with the first (k+1) rows and columns of the matrix $H^B(f)$. We have the following theorem:

**Theorem** If Q is a quasi-concave function then $(-1)^k \Delta_k^B \geq 0$, k=$\overline{1,n}$ . If $(-1)^k \Delta_k^B > 0$ then Q is quasi-concave function.

Notes from the theorem that if at least one determinant is null we have not ensured the existence of quasi-concavity.

For classical production functions Q=Q(K,L) the sufficient condition for quasi-concavity becomes: $\begin{vmatrix} 0 & \frac{\partial Q}{\partial K} & \frac{\partial Q}{\partial L} \\ \frac{\partial Q}{\partial K} & \frac{\partial^2 Q}{\partial K^2} & \frac{\partial^2 Q}{\partial K \partial L} \\ \frac{\partial Q}{\partial L} & \frac{\partial^2 Q}{\partial K \partial L} & \frac{\partial^2 Q}{\partial L^2} \end{vmatrix} > 0$ therefore:

$$2\frac{\partial Q}{\partial K}\frac{\partial Q}{\partial L}\frac{\partial^2 Q}{\partial K \partial L} - \left(\frac{\partial Q}{\partial K}\right)^2 \frac{\partial^2 Q}{\partial L^2} - \left(\frac{\partial Q}{\partial L}\right)^2 \frac{\partial^2 Q}{\partial K^2} > 0.$$

Recall, near the end of this introduction, that a function is called homogeneous if $\exists r \in \mathbf{R}$ such that: $Q(\lambda x_1,...,\lambda x_n)=\lambda^r Q(x_1,...,x_n)$ $\forall \lambda \in \mathbf{R}^*$. r is called the degree of homogeneity of the function.

We say that a production function $Q:D_p \to \mathbf{R}_+$ is with constant return to scale if $Q(\lambda x_1,...,\lambda x_n)=\lambda Q(x_1,...,x_n)$ (so homogeneous of first degree), with increasing return to scale if $Q(\lambda x_1,...,\lambda x_n)>\lambda Q(x_1,...,x_n)$ and with decreasing return to scale if $Q(\lambda x_1,...,\lambda x_n)<\lambda Q(x_1,...,x_n)$ $\forall \lambda \in (1,\infty)$ $\forall (x_1,...,x_n) \in D_p$. The fact that a return to production is at constant scale means that the production has the same multiplication factor with those of the two factors. Similarly, the return of increasing (decreasing) scale production is multiplied by a factor higher (lower) than that of inputs.

We will note below for functions Q=Q(K,L): $\chi=\frac{K}{L}$.

In what follows we will analyze production functions of the form: Q=Q(K,L)

## 3 Main Indicators of a Production Function

Let a production function:

$$Q:D_p\to\mathbf{R}_+,\ (x_1,...,x_n)\to Q(x_1,...,x_n)\in\mathbf{R}_+\ \forall(x_1,...,x_n)\in D_p$$

We will call the marginal productivity relative to a production factor $x_i$: $\eta_{x_i}=\frac{\partial Q}{\partial x_i}$ and represents the trend of variation of production at the variation of the factor $x_i$. In particular, for a production function of the form: Q=Q(K,L) we have $\eta_K=\frac{\partial Q}{\partial K}$ - called the marginal productivity of capital and $\eta_L=\frac{\partial Q}{\partial L}$ - called the marginal productivity of labor.

If the output is given by discrete values, we define: $\eta_{x_i}=\frac{\Delta Q}{\Delta x_i}$ meaning the mean variation of the production on the interval of length $\Delta x_i$.

We call also the average productivity relative to a production factor $x_i$: $w_{x_i}=\frac{Q}{x_i}$ and represents the value of production at the consumption of a unit of factor $x_i$.

In particular, for a production function of the form: Q=Q(K,L) we have: $w_K=\frac{Q}{K}$ - called the productivity of capital, and $w_L=\frac{Q}{L}$ - the productivity of labor.

From [4], we have that in the general case of the variation of all inputs, for $k_1$ units of input 1,...,$k_n$ units of input n, and Q(0,...,0)=0:

$$Q(k_1,...,k_n)=k_1\int_0^1\eta_{x_1}(k_1t,...,k_nt)dt+...+k_n\int_0^1\eta_{x_n}(k_1t,...,k_nt)dt$$

In particular, for Q=Q(K,L) we have: $Q(K,L)=K\int_0^1\eta_K(Kt,Lt)dt+L\int_0^1\eta_L(Kt,Lt)dt$.

Again, from [7], considering the factors i and j with i≠j, we define the restriction of production area: $P_{ij}=\{(x_1,...,x_n) \mid x_k=a_k=\text{const}, k=\overline{1,n}, k\neq i,j, x_i,x_j \in D_p\}$ relative to the two factors when the others have fixed values and $D_{ij}=\{(x_i,x_j) \mid (x_1,...,x_n)\in P_{ij}\}$ - the domain of production relative to factors i and j.

Defining $Q_{ij}:D_{ij}\to\mathbf{R}_+$ - the restriction of the production function to the factors i and j, i.e.: $Q_{ij}(x_i,x_j)=Q(a_1,...,a_{i-1},x_i,a_{i+1},...,a_{j-1},x_j,a_{j+1},...,a_n)$ we obtain that $Q_{ij}$ define a surface in $\mathbf{R}^3$ for every pair of factors (i,j).

We call partial marginal rate of technical substitution of the factors i and j, relative to $D_{ij}$ (caeteris paribus), the opposite change in the amount of factor j to substitute a variation of the quantity of factor i in the situation of conservation production level and note: $RMS(i,j,\overline{x})=-\frac{dx_j}{dx_i}=\frac{\eta_{x_i}\big|_{D_{ij}}}{\eta_{x_j}\big|_{D_{ij}}}$ in an arbitrary point $\overline{x}=\left(\overline{x}_1,...,\overline{x}_n\right)$. We define also ([7]) the global marginal rate of substitution between the i-th factor and the others as: $RMS(i,\overline{x})=\frac{\eta_{x_i}(\overline{x})}{\sqrt{\sum_{\substack{j=1\\ j\neq i}}^{n}\eta_{x_j}^2(\overline{x})}}$. The global marginal rate of technical substitution is the minimum (in the meaning of norm) of changes in consumption of factors so that the total production remain unchanged.

In particular, for a production function of the form: Q=Q(K,L) we have:

$$RMS(K,L)=\frac{\eta_K}{\eta_L},\ RMS(L,K)=\frac{\eta_L}{\eta_K}$$

It is called elasticity of production in relation to a production factor $x_i$: $\varepsilon_{x_i}=\frac{\frac{\partial Q}{\partial x_i}}{\frac{Q}{x_i}}=\frac{\eta_{x_i}}{w_{x_i}}$ - the relative variation of production at the relative variation of factor $x_i$. In particular, for a production function of the form: Q=Q(K,L) we have $\varepsilon_K=\frac{\frac{\partial Q}{\partial K}}{\frac{Q}{K}}=$

$\frac{\eta_K}{w_K}$ - called the elasticity of production in relation to the capital and $\varepsilon_L = \frac{\frac{\partial Q}{\partial L}}{\frac{Q}{L}} =$

$\frac{\eta_L}{w_L}$ - the elasticity factor of production in relation to the labor.

If the production function is homogenous of degree r, after Euler's relation: $\sum_{i=1}^{n} x_i \frac{\partial Q}{\partial x_i} = rQ$ we obtain that $\sum_{i=1}^{n} \varepsilon_{x_i} = r$.

## 4 Conditions of Marginal Productivity

### 4.1. $\eta_K$ =constant=$\alpha$, $\eta_L$ ≠constant

In this case, we have: $Q(K,L) = K\int_0^1 \eta_K(Kt, Lt)dt + L\int_0^1 \eta_L(Kt, Lt)dt = K\int_0^1 \alpha dt + L\int_0^1 \eta_L(Kt, Lt)dt = \alpha K + Lg(K,L)$. Because $\frac{\partial Q}{\partial K} = \alpha$ we have that $\frac{\partial g}{\partial K} = 0$ that is $g=g(L)$. Therefore: $Q(K,L)=\alpha K+f(L)$. Now $\frac{\partial Q}{\partial L} = f'(L) \neq 0 \Rightarrow f \neq$ constant.

The conditions from the axioms become:

- $Q(0,0)=0 \Rightarrow f(0)=0$
- f – continuous
- $f \in C^2(D_p)$
- $\frac{\partial Q}{\partial K} = \alpha > 0$
- $\frac{\partial Q}{\partial L} = f'(L) > 0$
- $2\frac{\partial Q}{\partial K}\frac{\partial Q}{\partial L}\frac{\partial^2 Q}{\partial K \partial L} - \left(\frac{\partial Q}{\partial K}\right)^2 \frac{\partial^2 Q}{\partial L^2} - \left(\frac{\partial Q}{\partial L}\right)^2 \frac{\partial^2 Q}{\partial K^2} = -\alpha^2 f''(L) > 0 \Rightarrow f''(L) < 0$

After these considerations we obtain that $\alpha>0$ and f is a monotonically increasing, strictly concave differentiable function of class at least two and vanishing in 0.

If now Q is homogenous, we have: $\exists r\in\mathbf{R}$: $Q(\lambda K,\lambda L)=\lambda^r Q(K,L)$ that is: $\alpha\lambda K+f(\lambda L)=\lambda^r(\alpha K+f(L))$.

If $r\neq 1\Rightarrow \alpha\lambda K+f(\lambda L)=\alpha\lambda^r K+\lambda^r f(L)\Rightarrow K=\dfrac{\lambda^r f(L)-f(\lambda L)}{\alpha\lambda\left(1-\lambda^{r-1}\right)}$ .Because K and L are independent variables follows K=constant therefore contradiction. We have r=1 that is: $f(\lambda L)=\lambda f(L)$, f being linear: $f(L)=\beta L$. We obtained: $Q=\alpha K+\beta L$ – the linear production function. Let note in this case that Q is quasi-concave even though $f''(L)=0$ for $f(L)=\beta L$.

For the linear production function, the determination of the parameters is very simple (using Least Square Method).

Let $(K_i,L_i,Q_i)_{i=1,\ldots,n}$ values of the capital, labor and production at the moments 1 to n. The minimum condition of the expression: $E=\sum_{i=1}^{n}\left(\alpha K_i+\beta L_i-Q_i\right)^2$ (relative to $\alpha$ and $\beta$) becomes:

$$\begin{cases}\dfrac{1}{2}\dfrac{\partial E}{\partial\alpha}=\alpha\sum_{i=1}^{n}K_i^2+\beta\sum_{i=1}^{n}K_iL_i-\sum_{i=1}^{n}K_iQ_i=0\\[2mm] \dfrac{1}{2}\dfrac{\partial E}{\partial\beta}=\alpha\sum_{i=1}^{n}K_iL_i+\beta\sum_{i=1}^{n}L_i^2-\sum_{i=1}^{n}L_iQ_i=0\end{cases}$$

therefore:

$$\begin{cases}\alpha=\dfrac{\sum_{i=1}^{n}K_iQ_i\sum_{i=1}^{n}L_i^2-\sum_{i=1}^{n}L_iQ_i\sum_{i=1}^{n}K_iL_i}{\sum_{i=1}^{n}K_i^2\sum_{i=1}^{n}L_i^2-\left(\sum_{i=1}^{n}K_iL_i\right)^2}\\[4mm] \beta=\dfrac{\sum_{i=1}^{n}L_iQ_i\sum_{i=1}^{n}K_i^2-\sum_{i=1}^{n}K_iQ_i\sum_{i=1}^{n}K_iL_i}{\sum_{i=1}^{n}K_i^2\sum_{i=1}^{n}L_i^2-\left(\sum_{i=1}^{n}K_iL_i\right)^2}\end{cases}$$

### 4.2. $\eta_L$ =constant=$\alpha$, $\eta_K$ ≠constant

Like previous, we obtain (permuting K with L): $Q(K,L)=\alpha L+f(K)$ with f satisfying the same conditions like above. The determination of the parameters is as above.

**4.3.** $\eta_K$ **=constant=α,** $\eta_L$ **=constant**=β

Q(K,L)= $K\int_0^1 \eta_K(Kt,Lt)dt + L\int_0^1 \eta_L(Kt,Lt)dt = K\int_0^1 \alpha dt + L\int_0^1 \beta dt$ =αK+βL – the linear production function. The determination of the parameters is as above.

**4.4.** $\eta_K = \alpha\chi^{\beta} = \alpha\frac{K^{\beta}}{L^{\beta}}$

Q(K,L)= $K\int_0^1 \eta_K(Kt,Lt)dt + L\int_0^1 \eta_L(Kt,Lt)dt = K\int_0^1 \alpha\frac{K^{\beta}}{L^{\beta}}dt + L\int_0^1 \eta_L(Kt,Lt)dt =$

$\frac{\alpha K^{\beta+1}}{L^{\beta}} + g(K,L)$.

But $\frac{\partial Q}{\partial K} = \alpha\frac{K^{\beta}}{L^{\beta}} \Rightarrow \frac{\alpha(\beta+1)K^{\beta}}{L^{\beta}} + \frac{\partial g}{\partial K} = \alpha\frac{K^{\beta}}{L^{\beta}}$ from where: $\frac{\partial g}{\partial K} = -\alpha\beta\frac{K^{\beta}}{L^{\beta}} \Rightarrow$ $g = -\frac{\alpha\beta}{L^{\beta}}\int K^{\beta}dK = -\frac{\alpha\beta}{L^{\beta}}\left(\frac{K^{\beta+1}}{\beta+1} + f(L)\right)$ therefore: Q(K,L)= $\frac{\alpha K^{\beta+1}}{(\beta+1)L^{\beta}} - \frac{\alpha\beta}{L^{\beta}}f(L) =$ $\frac{\alpha}{\beta+1}K^{\beta+1}L^{-\beta} + h(L)$ where h(L)= $-\frac{\alpha\beta}{L^{\beta}}f(L)$.

The conditions from the axioms become:

- h – continuous
- $h \in C^2(D_p)$
- $\frac{\partial Q}{\partial K} = \alpha K^{\beta}L^{-\beta} > 0 \Leftrightarrow \alpha > 0$
- $\frac{\partial Q}{\partial L} = -\frac{\alpha\beta}{\beta+1}K^{\beta+1}L^{-\beta-1} + h'(L) > 0$
- $2\frac{\partial Q}{\partial K}\frac{\partial Q}{\partial L}\frac{\partial^2 Q}{\partial K\partial L} - \left(\frac{\partial Q}{\partial K}\right)^2\frac{\partial^2 Q}{\partial L^2} - \left(\frac{\partial Q}{\partial L}\right)^2\frac{\partial^2 Q}{\partial K^2} > 0 \Leftrightarrow$

$\beta\left(\frac{\alpha K^{\beta+1}L^{-\beta-1}}{\beta+1} + h'(L)\right)^2 + \alpha K^{\beta+1}L^{-\beta}h''(L) < 0$

After these considerations we obtain that α>0 and h has the properties:

- $h'(L) > \frac{\alpha\beta}{\beta+1} K^{\beta+1} L^{-\beta-1}$
- $\beta\left(\frac{\alpha K^{\beta+1} L^{-\beta-1}}{\beta+1} + h'(L)\right)^2 + \alpha K^{\beta+1} L^{-\beta} h''(L) < 0$

If now, we want that the function be homogenous, we have:

$Q(\lambda K,\lambda L) = \lambda \frac{\alpha}{\beta+1} K^{\beta+1} L^{-\beta} + h(\lambda L) = \lambda^r Q(K,L) = \lambda^r \frac{\alpha}{\beta+1} K^{\beta+1} L^{-\beta} + \lambda^r h(L)$ that is:

$$\left(\lambda^r - \lambda\right)\frac{\alpha}{\beta+1} K^{\beta+1} L^{-\beta} = h(\lambda L) - \lambda^r h(L)$$

If r≠1 we find that: $K^{\beta+1} = \frac{(\beta+1)}{\alpha\left(\lambda^r - \lambda\right)L^{\beta}}\left(h(\lambda L) - \lambda^r h(L)\right)$ that is K depends from L – contradiction.

We have therefore: r=1, that is: $h(\lambda L) = \lambda h(L)$, h being linear: $h(L)=\rho L$.

The production function becomes (after obvious notations):

$Q(K,L) = \frac{\alpha}{\beta+1} K^{\beta+1} L^{-\beta} + \rho L$ - Bruno production function with $\beta \in (-1,0)$ (after the above conditions), $\rho>0$, $\alpha>0$.

Let now $(K_i,L_i,Q_i)_{i=1,\dots,n}$ values of the capital, labor and production at the moments 1 to n. The minimum conditions of the expression: $E=\sum_{i=1}^{n}\left(\frac{\alpha}{\beta+1} K_i^{\beta+1} L_i^{-\beta} + \rho L_i - Q_i\right)^2$ (relative to α, β and ρ) are very difficult to be solve (and is not relevant because the existence of this function requires the particular form of $\eta_K$), therefore we shall determine first, the discrete values of $\eta_K = \frac{\Delta Q}{\Delta K}$ that is: $\eta_{K,p} = \frac{Q_{p+1} - Q_p}{K_{p+1} - K_p}$, p= $\overline{1,n-1}$ and after, from the initial condition, that $\eta_K = \alpha\frac{K^{\beta}}{L^{\beta}}$ we have that: $\ln\eta_K = \ln\alpha + \beta\ln\frac{K}{L}$. Let now $E_1=\sum_{p=1}^{n-1}\left(\Phi + \beta\ln\frac{K_p}{L_p} - \ln\eta_{K,p}\right)^2$ where $\Phi = \ln\alpha$.

The Least Square Method gives us:

$$\begin{cases} \frac{1}{2}\frac{\partial E_1}{\partial \Phi} = (n-1)\Phi + \beta\sum_{p=1}^{n-1}\ln\frac{K_p}{L_p} - \sum_{p=1}^{n-1}\ln\eta_{K,p} = 0 \\ \frac{1}{2}\frac{\partial E_1}{\partial \beta} = \Phi\sum_{p=1}^{n-1}\ln\frac{K_p}{L_p} + \beta\sum_{p=1}^{n-1}\left(\ln\frac{K_p}{L_p}\right)^2 - \sum_{p=1}^{n-1}\ln\frac{K_p}{L_p}\ln\eta_{K,p} = 0 \end{cases}$$

therefore:

$$\begin{cases} \Phi^* = \dfrac{\sum_{p=1}^{n-1}\ln\eta_{K,p}\sum_{p=1}^{n-1}\left(\ln\frac{K_p}{L_p}\right)^2 - \sum_{p=1}^{n-1}\ln\frac{K_p}{L_p}\sum_{p=1}^{n-1}\ln\frac{K_p}{L_p}\ln\eta_{K,p}}{(n-1)\sum_{p=1}^{n-1}\left(\ln\frac{K_p}{L_p}\right)^2 - \left(\sum_{p=1}^{n-1}\ln\frac{K_p}{L_p}\right)^2} \\ \beta^* = \dfrac{(n-1)\sum_{p=1}^{n-1}\ln\frac{K_p}{L_p}\ln\eta_{K,p} - \sum_{p=1}^{n-1}\ln\eta_{K,p}\sum_{p=1}^{n-1}\ln\frac{K_p}{L_p}}{(n-1)\sum_{p=1}^{n-1}\left(\ln\frac{K_p}{L_p}\right)^2 - \left(\sum_{p=1}^{n-1}\ln\frac{K_p}{L_p}\right)^2} \end{cases}$$

and $\alpha^* = e^{\Phi^*}$.

After these: Q(K,L)= $\frac{\alpha^*}{\beta^*+1}K^{\beta^*+1}L^{-\beta^*} + \rho L$. The determination of $\rho$ can be determined in the following way. Let note: $\Omega_i = Q_i - \frac{\alpha^*}{\beta^*+1}K_i^{\beta^*+1}L_i^{-\beta^*}$, i=$\overline{1,n}$ and the condition that the expression: $E_2 = \sum_{i=1}^{n}(\rho L_i - \Omega_i)^2$ be minimum. We have therefore $\frac{1}{2}\frac{dE_2}{d\rho} = \sum_{i=1}^{n}L_i(\rho L_i - \Omega_i) = 0$ therefore: $\rho^* = \frac{\sum_{i=1}^{n}L_i\Omega_i}{\sum_{i=1}^{n}L_i^2}$ where at least one $L_i \neq 0$. Finally: Q(K,L)= $\frac{\alpha^*}{\beta^*+1}K^{\beta^*+1}L^{-\beta^*} + \rho^* L$. Let note here that because $\rho^*$=constant we must have that the values: $\frac{\Omega_i}{L_i}$, i=$\overline{1,n}$ must be approximately constant. To inquire this we can use the 3σ-rule that is in the interval (M-3σ,M+3σ)= $M\left(1-3\frac{\sigma}{M},1+3\frac{\sigma}{M}\right)$ lies over 89% values, where M is the average of these. Therefore, we shall compute for the values $\frac{\Omega_i}{L_i}$, i= $\overline{1,n}$ the average: M= $\frac{\sum_{i=1}^{n}\frac{\Omega_i}{L_i}}{n}$ and the standard deviation σ= $\frac{\sqrt{n\sum_{i=1}^{n}\left(\frac{\Omega_i}{L_i}\right)^2-\left(\sum_{i=1}^{n}\frac{\Omega_i}{L_i}\right)^2}}{n}$. If the value $\frac{\sigma}{M}$ is sufficiently small we can assume that $\rho^*$ is almost a constant and the determination is as in the upper.

**4.5.** $\eta_L=\alpha\chi^\beta=\alpha\frac{K^\beta}{L^\beta}$

Because the relation can pe written as: $\eta_L=\alpha\frac{L^{-\beta}}{K^{-\beta}}$ we shall proceed as in 4.4. and we shall obtain (permuting K with L and replacing β with -β): Q(K,L)= $\frac{\alpha}{1-\beta}K^\beta L^{1-\beta}+h(K)$.

The conditions from the axioms become:

- h – continuous
- $h\in C^2(D_p)$
- $\frac{\partial Q}{\partial L}=\alpha K^\beta L^{-\beta}>0 \Leftrightarrow \alpha>0$
- $\frac{\partial Q}{\partial K}=\frac{\alpha\beta}{1-\beta}K^{\beta-1}L^{1-\beta}+h'(K)>0$
- $2\frac{\partial Q}{\partial K}\frac{\partial Q}{\partial L}\frac{\partial^2 Q}{\partial K\partial L}-\left(\frac{\partial Q}{\partial K}\right)^2\frac{\partial^2 Q}{\partial L^2}-\left(\frac{\partial Q}{\partial L}\right)^2\frac{\partial^2 Q}{\partial K^2}>0\Leftrightarrow$

$$-\beta\left(\frac{\alpha L^{1-\beta}K^{\beta-1}}{1-\beta}+h'(K)\right)^2+\alpha L^{1-\beta}K^\beta h''(K)<0$$

After these considerations we obtain that α>0 and h has the properties:

- $h'(K) > -\frac{\alpha\beta}{1-\beta} L^{1-\beta} K^{\beta-1}$
- $-\beta\left(\frac{\alpha L^{1-\beta} K^{\beta-1}}{1-\beta} + h'(K)\right)^2 + \alpha L^{1-\beta} K^{\beta} h''(K) < 0$

If now, we want that the function be homogenous, we have, as previous: r=1, that is: $h(\lambda L) = \lambda h(L)$, h being linear: h(L)=ρL, the production function becoming (after obvious notations):

Q(K,L)= $\frac{\alpha}{1-\beta} L^{1-\beta} K^{\beta} + \rho K$ - Bruno production type function with $\beta \in (0,1)$ (after the above conditions), ρ>0, α>0.

Let now $(K_i, L_i, Q_i)_{i=1,...,n}$ values of the capital, labor and production at the moments 1 to n. We shall determine first, the discrete values of $\eta_L = \frac{\Delta Q}{\Delta L}$ that is: $\eta_{L,p} = \frac{Q_{p+1} - Q_p}{L_{p+1} - L_p}$, p=$\overline{1, n-1}$ and after, from the initial condition, that $\eta_L = \alpha \frac{L^{-\beta}}{K^{-\beta}}$ we have that: $\ln \eta_L = \ln \alpha - \beta \ln \frac{L}{K}$. Let now E1=$\sum_{p=1}^{n-1}\left(\Phi - \beta \ln \frac{L_p}{K_p} - \ln \eta_{L,p}\right)^2$ where Φ =ln α. The Least Square Method gives us (as upper):

$$\begin{cases} \Phi^* = \dfrac{\sum_{p=1}^{n-1} \ln \eta_{L,p} \sum_{p=1}^{n-1}\left(\ln \frac{L_p}{K_p}\right)^2 - \sum_{p=1}^{n-1} \ln \frac{L_p}{K_p} \sum_{p=1}^{n-1} \ln \frac{L_p}{K_p} \ln \eta_{L,p}}{(n-1)\sum_{p=1}^{n-1}\left(\ln \frac{L_p}{K_p}\right)^2 - \left(\sum_{p=1}^{n-1} \ln \frac{L_p}{K_p}\right)^2} \\ \beta^* = -\dfrac{(n-1)\sum_{p=1}^{n-1} \ln \frac{L_p}{K_p} \ln \eta_{L,p} - \sum_{p=1}^{n-1} \ln \eta_{L,p} \sum_{p=1}^{n-1} \ln \frac{L_p}{K_p}}{(n-1)\sum_{p=1}^{n-1}\left(\ln \frac{L_p}{K_p}\right)^2 - \left(\sum_{p=1}^{n-1} \ln \frac{L_p}{K_p}\right)^2} \end{cases}$$

and $\alpha^* = e^{\Phi^*}$.

After these: Q(K,L)= $\frac{\alpha^*}{1-\beta^*}L^{1-\beta^*}K^{\beta^*}+\rho K$. The determination of $\rho$ can be determined in the following way. Let note: $\Omega_i = Q_i - \frac{\alpha^*}{1-\beta^*}L_i^{1-\beta^*}K_i^{\beta^*}$, i=$\overline{1,n}$ and the condition that the expression: E2= $\sum_{i=1}^{n}(\rho K_i - \Omega_i)^2$ be minimum. We have therefore $\frac{1}{2}\frac{dE_2}{d\rho} = \sum_{i=1}^{n} K_i(\rho K_i - \Omega_i) = 0$ therefore: $\rho^* = \frac{\sum_{i=1}^{n} K_i\Omega_i}{\sum_{i=1}^{n} K_i^2}$ where at least one $K_i \neq 0$. Finally: Q(K,L)= $\frac{\alpha^*}{1-\beta^*}L^{1-\beta^*}K^{\beta^*}+\rho^* K$. The demarche relative to the constancy of $\rho^*$ is similarly to 4.4.

**4.6.** $\eta_K = \frac{aK^\alpha + bL^\beta}{K^\gamma}$, $\eta_L = \frac{cK^\alpha + dL^\beta}{L^\gamma}$

Q(K,L)= $K\int_0^1 \eta_K(Kt, Lt)dt + L\int_0^1 \eta_L(Kt, Lt)dt =$

$$K\int_0^1 \frac{aK^\alpha t^\alpha + bL^\beta t^\beta}{K^\gamma t^\gamma}dt + L\int_0^1 \frac{cK^\alpha t^\alpha + dL^\beta t^\beta}{L^\gamma t^\gamma}dt =$$

$$aK^{\alpha-\gamma+1}\int_0^1 t^{\alpha-\gamma}dt + bK^{1-\gamma}L^\beta\int_0^1 t^{\beta-\gamma}dt + cK^\alpha L^{1-\gamma}\int_0^1 t^{\alpha-\gamma}dt + dL^{1+\beta-\gamma}\int_0^1 t^{\beta-\gamma}dt =$$

$$K^\alpha\left(aK^{1-\gamma} + cL^{1-\gamma}\right)\int_0^1 t^{\alpha-\gamma}dt + L^\beta\left(bK^{1-\gamma} + dL^{1-\gamma}\right)\int_0^1 t^{\beta-\gamma}dt$$

4.6.1. If $\alpha$-$\gamma \neq$-1, $\beta$-$\gamma \neq$-1 then: Q(K,L)= $\frac{1}{\alpha-\gamma+1}K^\alpha\left(aK^{1-\gamma}+cL^{1-\gamma}\right) + \frac{1}{\beta-\gamma+1}L^\beta\left(bK^{1-\gamma}+dL^{1-\gamma}\right)$.

The conditions from the axioms become:

- $\frac{\partial Q}{\partial K} = aK^{\alpha-\gamma} + \frac{c\alpha}{\alpha-\gamma+1}K^{\alpha-1}L^{1-\gamma} + \frac{b(1-\gamma)}{\beta-\gamma+1}K^{-\gamma}L^\beta > 0$

- $\frac{\partial Q}{\partial L}=\frac{c(1-\gamma)}{\alpha-\gamma+1}K^{\alpha}L^{-\gamma}+\frac{b\beta}{\beta-\gamma+1}K^{1-\gamma}L^{\beta-1}+dL^{\beta-\gamma}>0$
- $2\frac{\partial Q}{\partial K}\frac{\partial Q}{\partial L}\frac{\partial^2 Q}{\partial K\partial L}-\left(\frac{\partial Q}{\partial K}\right)^2\frac{\partial^2 Q}{\partial L^2}-\left(\frac{\partial Q}{\partial L}\right)^2\frac{\partial^2 Q}{\partial K^2}>0$

If now, we want that the function be homogenous (of degree r), we have:

$$Q(\lambda K,\lambda L)=\lambda^{\alpha-\gamma+1}\frac{1}{\alpha-\gamma+1}K^{\alpha}\left(aK^{1-\gamma}+cL^{1-\gamma}\right)+\lambda^{\beta-\gamma+1}\frac{1}{\beta-\gamma+1}L^{\beta}\left(bK^{1-\gamma}+dL^{1-\gamma}\right)=$$

$$\lambda^{r}Q(K,L)=\lambda^{r}\frac{1}{\alpha-\gamma+1}K^{\alpha}\left(aK^{1-\gamma}+cL^{1-\gamma}\right)+\lambda^{r}\frac{1}{\beta-\gamma+1}L^{\beta}\left(bK^{1-\gamma}+dL^{1-\gamma}\right)$$ that is:

$$\left(\lambda^{\alpha-\gamma+1}-\lambda^{r}\right)\frac{1}{\alpha-\gamma+1}K^{\alpha}\left(aK^{1-\gamma}+cL^{1-\gamma}\right)+\left(\lambda^{\beta-\gamma+1}-\lambda^{r}\right)\frac{1}{\beta-\gamma+1}L^{\beta}\left(bK^{1-\gamma}+dL^{1-\gamma}\right)=0$$

4.6.1.a. If now $r\neq\beta-\gamma+1$ we have: $\frac{\lambda^{\alpha-\gamma+1}-\lambda^{r}}{\lambda^{\beta-\gamma+1}-\lambda^{r}}=-\frac{\alpha-\gamma+1}{\beta-\gamma+1}\frac{L^{\beta}\left(bK^{1-\gamma}+dL^{1-\gamma}\right)}{K^{\alpha}\left(aK^{1-\gamma}+cL^{1-\gamma}\right)}$.

4.6.1.a.i. If $r=\alpha-\gamma+1$ we obtain: $L^{\beta}\left(bK^{1-\gamma}+dL^{1-\gamma}\right)=0$ that is L= constant – contradiction or $K^{1-\gamma}=-\frac{d}{b}L^{1-\gamma}$ - contradiction with the independence of K and L (if $b\neq 0$) or $dL^{1-\gamma}=0$ (b=0) which is true only of d=0. But in this case, we have that: $\eta_K=aK^{\alpha-\gamma}$, $\eta_L=cK^{\alpha}L^{-\gamma}$ and $Q(K,L)=\frac{aK^{\alpha-\gamma+1}+cK^{\alpha}L^{1-\gamma}}{\alpha-\gamma+1}=$ $pK^{\alpha-\gamma+1}+qL^{\alpha-\gamma+1}\left(\frac{K}{L}\right)^{\alpha}$ with obvious notations.

Let now $(K_i,L_i,Q_i)_{i=1,\dots,n}$ values of the capital, labor and production at the moments 1 to n. We shall determine first, the discrete values of $\eta_K=\frac{\Delta Q}{\Delta K}$, $\eta_L=\frac{\Delta Q}{\Delta L}$ that is: $\eta_{K,p}=\frac{Q_{p+1}-Q_p}{K_{p+1}-K_p}$, $\eta_{L,p}=\frac{Q_{p+1}-Q_p}{L_{p+1}-L_p}$, $p=\overline{1,n-1}$ and after, from the initial condition, that $\eta_K=aK^{\alpha-\gamma}$, $\eta_L=cK^{\alpha}L^{-\gamma}$ we have that: $\ln\eta_K=\ln a+(\alpha-\gamma)\ln K$ and $\ln\eta_L=\ln c+\alpha\ln K-\gamma\ln L$. Let now first $E_1=$

$\sum_{p=1}^{n-1}\left(\Phi+\alpha\ln K_p-\gamma\ln L_p-\ln\eta_{L,p}\right)^2$ where $\Phi$ =ln c and The Least Square Method gives us (as upper):

$$\begin{cases}(n-1)\Phi+\alpha\sum_{p=1}^{n-1}\ln K_p-\gamma\sum_{p=1}^{n-1}\ln L_p=\sum_{p=1}^{n-1}\ln\eta_{L,p}\\ \Phi\sum_{p=1}^{n-1}\ln K_p+\alpha\sum_{p=1}^{n-1}\left(\ln K_p\right)^2-\gamma\sum_{p=1}^{n-1}\ln K_p\ln L_p=\sum_{p=1}^{n-1}\ln\eta_{L,p}\ln K_p\\ \Phi\sum_{p=1}^{n-1}\ln L_p+\alpha\sum_{p=1}^{n-1}\ln K_p\ln L_p-\gamma\sum_{p=1}^{n-1}\left(\ln L_p\right)^2=\sum_{p=1}^{n-1}\ln\eta_{L,p}\ln L_p\end{cases}$$

from where we shall find: $c^*=e^{\Phi^*}$, $\alpha^*$, $\gamma^*$.

After these: $\ln\eta_K=\ln a+\left(\alpha^*-\gamma^*\right)\ln K$. For the determination of "a", let note here that because ln a is constant we must have that the values: $\ln\eta_{K,i}-\left(\alpha^*-\gamma^*\right)\ln K_i$, i=$\overline{1,n}$ must be approximately constant. To inquire this we can use the 3σ-rule that is in the interval (M-3σ,M+3σ)= $M\left(1-3\frac{\sigma}{M},1+3\frac{\sigma}{M}\right)$ lies over 89% values, where M is the average of these. Therefore, we shall compute for the values $\ln\eta_{K,i}-\left(\alpha^*-\gamma^*\right)\ln K_i$, i=$\overline{1,n}$ the average: M=$\frac{\ln\eta_{K,i}-\left(\alpha^*-\gamma^*\right)\ln K_i}{n}$ and the standard deviation σ= $\frac{\sqrt{n\sum_{i=1}^{n}\left(\ln\eta_{K,i}-\left(\alpha^*-\gamma^*\right)\ln K_i\right)^2-\left(\sum_{i=1}^{n}\ln\eta_{K,i}-\left(\alpha^*-\gamma^*\right)\ln K_i\right)^2}}{n}$. If the value $\frac{\sigma}{M}$ is sufficiently small we can assume that ln a is almost a constant and the determination is as in the upper. Let note also $a^*$ this value.

Now we have Q(K,L)=$\frac{a^*K^{\alpha^*-\gamma^*+1}+c^*K^{\alpha^*}L^{1-\gamma^*}}{\alpha^*-\gamma^*+1}$ with obvious notations.

4.6.1.a.ii. If r≠α-γ+1 we have that $0\neq f(\lambda)=\frac{\lambda^{\alpha-\gamma+1}-\lambda^r}{\lambda^{\beta-\gamma+1}-\lambda^r}=-\frac{\alpha-\gamma+1}{\beta-\gamma+1}\frac{L^\beta\left(bK^{1-\gamma}+dL^{1-\gamma}\right)}{K^\alpha\left(aK^{1-\gamma}+cL^{1-\gamma}\right)}$ - contradiction with the fact that for constant K and L we shall have f=constant which is impossible.

4.6.1.b. Returning, now for r=β-γ+1 we have: $\left(\lambda^{\alpha-\gamma+1}-\lambda^{r}\right)\frac{1}{\alpha-\gamma+1}K^{\alpha}\left(aK^{1-\gamma}+cL^{1-\gamma}\right)=0$.

4.6.1.b.i. For α-γ+1≠r we shall obtain that the equality becomes true only if a=c=0 (as upper) that is: $\eta_K=bK^{-\gamma}L^{\beta}$, $\eta_L=dL^{\beta-\gamma}$, Q(K,L)= $\frac{bK^{1-\gamma}L^{\beta}+dL^{\beta-\gamma+1}}{\beta-\gamma+1}$ = $pL^{\beta-\gamma+1}+qK^{\beta-\gamma+1}\left(\frac{K}{L}\right)^{-\beta}$. The deterination of the parameters can be done like in 4.6.1.a.i replacing α with β, K with L, a with d and c with b.

4.6.1.b.ii. If α-γ+1=r we have an identity. In this case: α=β=r+γ-1 and: $\eta_K=aK^{\alpha-\gamma}+bK^{-\gamma}L^{\alpha}$, $\eta_L=cK^{\alpha}L^{-\gamma}+dL^{\alpha-\gamma}$, Q(K,L)= $\frac{aK^{\alpha-\gamma+1}+dL^{\alpha-\gamma+1}+cK^{\alpha}L^{1-\gamma}+bL^{\alpha}K^{1-\gamma}}{\alpha-\gamma+1}$. The determination of the parameters in this case is a little bit difficult because α and γ lies also at power of K and L and at the denominator of Q.

If, in particular, α=γ= $\frac{1}{2}$ we shall have: Q(K,L)= $aK+b\sqrt{K}\sqrt{L}+dL$ - Diewert production function (homogenous of degree 1).

4.6.2. If α-γ=-1 or β-γ=-1 then the integral becomes -∞ which is a contradiction which the nature of production.

**4.7.** $\eta_K=aK^{\alpha}L^{\beta}$, $\eta_L=bK^{\gamma}L^{\delta}$

In this case Q(K,L)= $K\int_0^1\eta_K(Kt,Lt)dt+L\int_0^1\eta_L(Kt,Lt)dt=$ $K\int_0^1 at^{\alpha+\beta}K^{\alpha}L^{\beta}dt+L\int_0^1 bt^{\gamma+\delta}K^{\gamma}L^{\delta}dt$

4.7.1. If α+β≠-1 and γ+δ≠-1 we have: Q(K,L)= $\frac{aK^{\alpha+1}L^{\beta}}{\alpha+\beta+1}+\frac{bK^{\gamma}L^{\delta+1}}{\gamma+\delta+1}$.

The conditions from the axioms become:

- $\frac{\partial Q}{\partial K}=aK^{\alpha}L^{\beta}$ >0 that is a>0

- $\frac{\partial Q}{\partial L} = bK^{\gamma}L^{\delta}$ >0 that is b>0
- $2\frac{\partial Q}{\partial K}\frac{\partial Q}{\partial L}\frac{\partial^2 Q}{\partial K\partial L} - \left(\frac{\partial Q}{\partial K}\right)^2\frac{\partial^2 Q}{\partial L^2} - \left(\frac{\partial Q}{\partial L}\right)^2\frac{\partial^2 Q}{\partial K^2} =$

$a^2b(2\beta-\delta)K^{2\alpha+\gamma}L^{2\beta+\delta-1} - ab^2\alpha K^{\alpha+2\gamma-1}L^{\beta+2\delta}$ >0 that is: $a(2\beta-\delta)K^{\alpha}L^{\beta-1} - b\alpha K^{\gamma-1}L^{\delta}$ >0

If now, we want that the function be homogenous (of degree r), we have:

Q($\lambda$K,$\lambda$L)= $\lambda^{\alpha+\beta+1}\frac{aK^{\alpha+1}L^{\beta}}{\alpha+\beta+1} + \lambda^{\gamma+\delta+1}\frac{bK^{\gamma}L^{\delta+1}}{\gamma+\delta+1} = \lambda^r\frac{aK^{\alpha+1}L^{\beta}}{\alpha+\beta+1} + \lambda^r\frac{bK^{\gamma}L^{\delta+1}}{\gamma+\delta+1}$ that is:

$\left(\lambda^{\alpha+\beta+1} - \lambda^r\right)\frac{aK^{\alpha+1}L^{\beta}}{\alpha+\beta+1} + \left(\lambda^{\gamma+\delta+1} - \lambda^r\right)\frac{bK^{\gamma}L^{\delta+1}}{\gamma+\delta+1} = 0$ .

If $r \neq \gamma+\delta+1$ we obtain: $\frac{\lambda^{\alpha+\beta+1} - \lambda^r}{\lambda^{\gamma+\delta+1} - \lambda^r} = -\frac{b(\alpha+\beta+1)K^{\gamma-\alpha-1}L^{\delta-\beta+1}}{a(\gamma+\delta+1)}$ .

If $\alpha+\beta+1 \neq r$ and $\alpha+\beta \neq \gamma+\delta$ the expression from left depends from $\lambda$ which is a contradiction with the right.

If $\alpha+\beta+1=r$ we shall find that: $b(\alpha+\beta+1)K^{\gamma-\alpha-1}L^{\delta-\beta+1}$ =0 that is $\alpha+\beta+1=r=0$ which is a contradiction with the hypothesis.

If $\alpha+\beta=\gamma+\delta\neq-1$ we have: $bK^{\gamma-\alpha-1}L^{\delta-\beta+1} = -a$ that is a contradiction with the variability of K and L.

We have therefore: $r=\gamma+\delta+1$ and with the same arguments $r=\alpha+\beta+1$. In this case the production function is homogenous and has the expression: Q(K,L)= $\frac{aK^{\alpha+1}L^{\beta} + bK^{\alpha+\beta-\delta}L^{\delta+1}}{\alpha+\beta+1}$. With new notations: Q(K,L)= $AK^{\mu}L^{\beta} + BK^{\mu+\beta-\varepsilon}L^{\varepsilon}$. For A=0 or B=0 we obtain the classical Cobb-Douglas production function.

The determination of the parameters follows obviously (like upper) from the conditions: $\eta_K = aK^{\alpha}L^{\beta}$, $\eta_L = bK^{\gamma}L^{\delta}$ .

4.7.2. If $\alpha+\beta=-1$ or $\gamma+\delta=-1$ then the integral becomes $-\infty$ which is a contradiction which the nature of production.

## 5 Conditions of Marginal Rate of Substitution

**5.1. RMS(K,L)=** $\frac{a\chi+b}{c\chi+d}$ **where** $\chi=\frac{K}{L}$, Q being homogenous of degree 1.

Because Q(K,L)= $Q(\chi L, L)=LQ(\chi,1)$ we will note $q(\chi)=Q(\chi,1)$ and we have: Q(K,L)=L $q(\chi)$.

Now: $\eta_K=\frac{\partial Q}{\partial K}=Lq'(\chi)\frac{\partial\chi}{\partial K}=q'(\chi)$ and $\eta_L=\frac{\partial Q}{\partial L}=q(\chi)+Lq'(\chi)\frac{\partial\chi}{\partial L}=q(\chi)-\chi q'(\chi)$ .

We have RMS(K,L)= $\frac{\eta_K}{\eta_L}=\frac{q'(\chi)}{q(\chi)-\chi q'(\chi)}=\frac{a\chi+b}{c\chi+d}$.

In this case: $\left[a\chi^2+(b+c)\chi+d\right]q'(\chi)=(a\chi+b)q(\chi)$ which it can be written as:

$\frac{q'(\chi)}{q(\chi)}=\frac{a\chi+b}{a\chi^2+(b+c)\chi+d}$ if $a\chi^2+(b+c)\chi+d\neq 0$.

Integrating: $\ln q(\chi)=\int\frac{a\chi+b}{a\chi^2+(b+c)\chi+d}d\chi$

<u>5.1.1.</u> If $(b+c)^2-4ad<0$ we have that:

$$\ln q(\chi)=\frac{1}{2}\ln\left|a\chi^2+(b+c)\chi+d\right|+\frac{b-c}{\sqrt{4ad-(b+c)^2}}\operatorname{arctg}\frac{2a\chi+b+c}{\sqrt{4ad-(b+c)^2}}+C,\ C\in\mathbf{R}$$

therefore: $q(\chi)=C\sqrt{\left|a\chi^2+(b+c)\chi+d\right|}e^{\frac{b-c}{\sqrt{4ad-(b+c)^2}}\operatorname{arctg}\frac{2a\chi+b+c}{\sqrt{4ad-(b+c)^2}}}$, $C\in\mathbf{R}_+^*$.

Expressing in function of K and L, we find that:

Q(K,L)= $C\sqrt{\left|aK^2+(b+c)KL+dL^2\right|}e^{\frac{b-c}{\sqrt{4ad-(b+c)^2}}\operatorname{arctg}\frac{2aK+(b+c)L}{L\sqrt{4ad-(b+c)^2}}}$, $C\in\mathbf{R}_+^*$.

In particular, for b=c we have: Q(K,L)= $C\sqrt{\left|aK^2+2bKL+dL^2\right|}$ , $C\in\mathbf{R}_+^*$ with $b^2$-ad<0 – the Allen production function.

5.1.2. If $(b+c)^2-4ad=0$ we have that: $\ln q(\chi)=\ln\left|\chi+\frac{b+c}{2a}\right|-\frac{b-c}{2a\chi+b+c}+C$ therefore:

$$q(\chi)=C\left|\chi+\frac{b+c}{2a}\right|e^{-\frac{b-c}{2a\chi+b+c}},\ C\in\mathbf{R}_+^*.$$

Finally: Q(K,L)= $q(\chi)=C\left|K+\frac{b+c}{2a}L\right|e^{-\frac{(b-c)L}{2aK+(b+c)L}}$

5.1.3. $(b+c)^2-4ad>0$ we have that:

$$\ln q(\chi)=\frac{1}{2}\ln\left|a\chi^2+(b+c)\chi+d\right|+\frac{b-c}{2a}\int\frac{1}{(\chi-\chi_1)(\chi-\chi_2)}d\chi=$$

$\frac{1}{2}\ln\left|a\chi^2+(b+c)\chi+d\right|+\frac{b-c}{2a}\frac{1}{\chi_1-\chi_2}\ln\left|\frac{\chi-\chi_1}{\chi-\chi_2}\right|$ where $\chi_1$ and $\chi_2$ are the real roots of $a\chi^2+(b+c)\chi+d=0$.

We have now: $q(\chi)=C\sqrt{\left|a\chi^2+(b+c)\chi+d\right|}\left|\frac{\chi-\chi_1}{\chi-\chi_2}\right|^{\frac{b-c}{2a(\chi_1-\chi_2)}}$ , $C\in\mathbf{R}_+^*$. and finally:

Q(K,L)= $C\sqrt{\left|aK^2+(b+c)KL+dL^2\right|}\left|\frac{K-\chi_1L}{K-\chi_2L}\right|^{\frac{b-c}{2a(\chi_1-\chi_2)}}$ , $C\in\mathbf{R}_+^*$.

In particular, for b=c we have: Q(K,L)= $C\sqrt{\left|aK^2+2bKL+dL^2\right|}$ , $C\in\mathbf{R}_+^*$ with $b^2-ad>0$ – the Allen production function.

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# The Economic Trojan Horse is Actually a German Horse

**Romeo-Victor Ionescu**[1]

**Abstract:** The paper is focused on the immigrants' impact on the EU's economy in the context of the latest immigrant crisis generated by Germany and France. The analysis in the paper covers not only the economic negative effects, but the social effects as well. The scientific approach is based on the latest official data. A distinct part of the paper deals with forecasting procedures able to point out the powerful negative impact of the immigrants on the labor market and public finances on short and medium terms. The main conclusion of the paper is that Germany is not able to manage this immigrant crisis and it will try to solve the problem putting pressure on other Member States or translating the crisis management to the global organism, as Davos Conference, for example.

**Keywords:** migrant distribution keys; relocation scheme; risk of poverty or social exclusion; unemployment rate; labor market.

**JEL Classification:** E24; F22; F66; I32; J61

## 1. General Approach

The immigrant crisis becomes the greatest challenge in the EU's history. The dimension of this migration is impossible to quantify. Moreover, the phenomenon is far away of stopping.

Germany's initial availability to receive Syrian migrants represented the beginning of an exodus with unbelievable economic, social, political and military implications.

Moreover, it was absolutely obvious to anyone that Germany assumed EU's leadership and forced other Member States to apply its migrant policy.

The situation is so bad that Germany threatened other Member States to cut the financial assistance from the European Funds. It was an unprecedented action in the EU's history.

The final result was a European document voted by the Home Affairs ministers,

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which covered the migrants' distribution and redistribution across the Member States (European Commission, September 2015).

The worst estimations talk about one million migrants in the first year, but more specialists are more pessimistic. The basic idea is that present migrants are not only war's victims and do not come only from Syria and options connected to those Member States where they want to arrive: only the most developed countries. This is why Germany, France and Northern Member States supported the migrants' distribution process.

According to this process, four distribution keys were used in order to quantify the capacity of the Member States to absorb refugees and to integrate them then. These keys are quantified according to: the size of the population (40%), total GDP (40%), the number of asylum applications and resettled refugees per 1 million inhabitants over 2010-2014 (10%) and the unemployment rate (10%).

## 2. Literature - Critical Overview

There are on many scientific papers focused on the present migration trends. One of them describes the population growth and the less-skilled migrant workers as the main effects of the immigrants' flows (Card, 2007).

Other specialists focused on the historical overview of the immigration in Europe. This approach is followed by an analysis of the migrants' advantages and disadvantages on the European labor market (Dustmann & Frattini, 2011).

The immigration as an economic phenomenon is the theme of another research. This approach is followed by an analysis of the immigrants' effects on labor markets and public finances of host Member States, especially from Northern Europe (Kerr & Kerr, 2011).

An interesting research focuses on long-term immigration characteristics in Europe. The paper covers interesting aspects as the following: access to citizenship, asylum seeking, border enforcement, amnesties and policies to attract talent (Rica, Glitz & Ortega, 2013).

## 3. Immigrants' Relocation Schemes vs Immigrants' History in Europe

According to the above four distribution keys the Members States' implication on immigrants receiving is presented in Table 1.

**Table 1. European relocation scheme (key value-%)**

| Member State | Key | Member State | Key |
|---|---|---|---|
| Austria | 2.62 | Belgium | 2.91 |
| Bulgaria | 1.25 | Croatia | 1.73 |
| Cyprus | 0.39 | Czech Republic | 2.98 |
| Estonia | 1.76 | Finland | 1.72 |
| France | 14.17 | Germany | 18.42 |
| Greece | 1.90 | Hungary | 1.79 |
| Italy | 11.84 | Latvia | 1.21 |
| Lithuania | 1.16 | Luxembourg | 0.85 |
| Malta | 0.69 | Netherlands | 4.35 |
| Poland | 5.64 | Portugal | 3.89 |
| Romania | 3.75 | Slovakia | 1.78 |
| Slovenia | 1.15 | Spain | 9.10 |
| Sweden | 2.92 | | |

Looking to Table 1, some remarks have to be done. Denmark, Ireland and UK are not object of the relocation scheme because they didn't take part in the adoption by the Council of this scheme. All these three countries are developed economies.

The use of the size of the population as main component of the relocation key can lead to strange situations. Romania, for example has to receive more immigrants than Sweden, even that the economic development in Sweden is higher than in Romania. According to the latest official statistical data, the size of the population was 19,511,000 persons in Romania (United Nations, 2015) and 9,838,480 persons in Sweden (Statistics Sweden, 2015). On the other hand, the GDP per capita was 21426 USD (IMF, 2015) in Romania and 47319 USD in Sweden (IMFa, 2015), as well (see Figure 1).

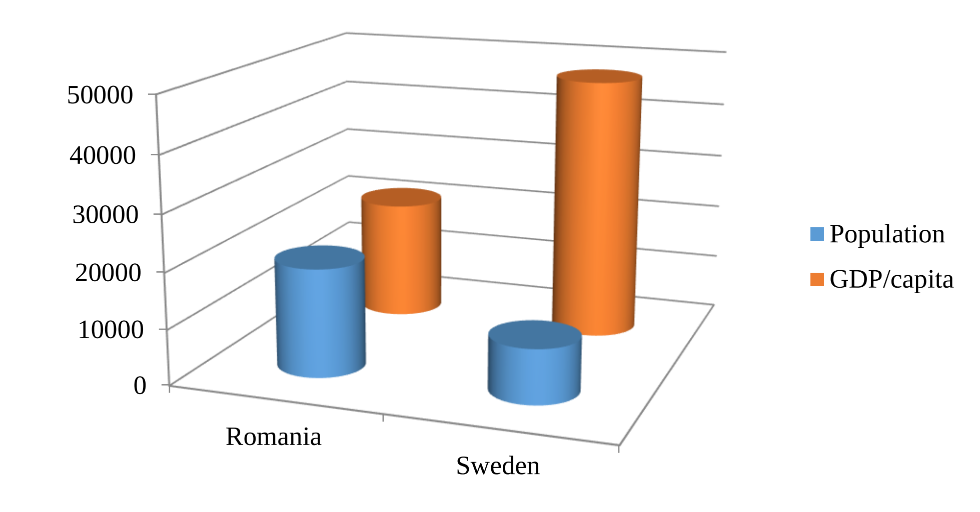



**Figure 1. Selected data for Romania and Sweden**

*Source: Personal contribution*

Germany, France and Italy have to receive the greatest number of immigrants. Those who support the immigrant receiving in these Member States talk about that their tradition in having immigrants, but we are not sure that these traditions are good enough to cover the immigrants' integration in the European economy and society.

According to the latest official statistical data, Germany and France have the greatest Muslim population across the EU28. The greatest part of them is immigrants. The main question is if these two countries succeeded in integrating those immigrants into the European society's standards and on the European labor market, as well?

The answer to this question is NO!!! For the example, 40.1% of the non- EU - born population in the EU28 was at risk of poverty or social exclusion in 2014. This is why Eurostat implemented a new statistical indicator: AROPE (risk of poverty or social exclusion). This indicator had a negative trend at least from 2005 for whole EU inhabitants (European Commission, 2016).

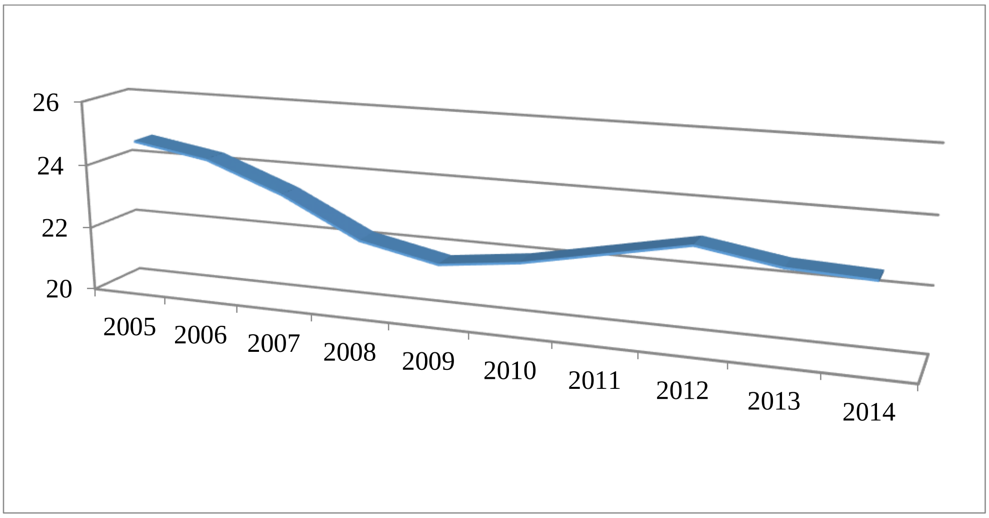



**Figure 2. Evolution of people AROPE by broad group of country of citizenship, EU-28 (%)**

*Source: Personal contribution using Eurostat data*

The situation is worst for the young people at risk of poverty and social exclusion who achieved 43.8% of young people aged 16-29 in the EU for foreign-born in 2013.

One of the elements which supported this situation is the income distribution. The average income for EU nationals was higher (16716 Euros) than for foreign citizens (14580 Euros) in 2014 (see Table 2).

**Table 2. Median income by groups of country of citizenship (Euro)**

| | Nationals (20-64) | Foreign citizens (20-64) | EU citizens (20-64) | Non-EU citizens (20-64) |
|---|---|---|---|---|
| EU28 | 16716 | 14580 | 17938 | 12633 |
| Belgium | 24364 | 15797 | 21286 | 11640 |
| Bulgaria | 3648 | 4090 | | |
| Czech Rep. | 8151 | 8195 | 7904 | 8504 |
| Denmark | 29931 | 22317 | 24716 | 19983 |
| Germany | 21041 | 17565 | 20957 | 15850 |
| Estonia | 8619 | 6098 | 6185 | 6098 |
| Ireland | 21345 | 18521 | 19106 | 14167 |

| | | | | |
|---|---|---|---|---|
| Greece | 8167 | 4848 | 6240 | 4456 |
| Spain | 14451 | 8396 | 9938 | 7466 |
| France | 22088 | 15191 | 21214 | 13648 |
| Croatia | 5566 | 5489 | | 4703 |
| Italy | 17151 | 11539 | 12294 | 11471 |
| Cyprus | 15991 | 11584 | 12062 | 10753 |
| Latvia | 5846 | 5075 | | 5050 |
| Lithuania | 5426 | 4536 | | 5206 |
| Luxembourg | 40293 | 29157 | 30222 | 23518 |
| Hungary | 4688 | 3860 | 4206 | |
| Malta | 13727 | 13373 | 14462 | 12183 |
| Netherlands | 22168 | 18401 | 22015 | 14999 |
| Austria | 25966 | 16925 | 20715 | 16079 |
| Poland | 5511 | 6504 | | 4679 |
| Portugal | 8613 | 6519 | 8480 | 6427 |
| Romania | 2325 | | | |
| Slovenia | 12382 | 7624 | 10018 | 7540 |
| Slovakia | 7335 | 7426 | 5976 | |
| Finland | 25662 | 19062 | 23343 | 15817 |
| Sweden | 29334 | 19459 | 24315 | 17250 |
| UK | 22979 | 20038 | 20110 | 20002 |

*Source: Personal contribution using Eurostat data*

According to Table 2, the greatest gaps between average income of nationals and foreign citizens are in Luxembourg, Sweden, Austria, Belgium, Denmark and France. There are no data for Romania, while the foreign citizens' average income is higher than nationals' income in Czech Republic, Poland and Slovakia.

On the other hand, 31.5% of the foreign citizens aged 20-64 faced to risk of poverty in 2014 (see Table 3).

**Table 3. Risk of poverty rate (%, 2014)**

| | Nationals (20-64) | Foreign citizens (20-64) | EU citizens (20-64) | Non-EU citizens (20-64) |
|---|---|---|---|---|
| EU28 | 15.8 | 31.5 | 23.8 | 37.6 |
| Belgium | 10.5 | 38.9 | 25.0 | 58.7 |
| Bulgaria | 18.3 | 6.0 | | 7.2 |
| Czech Rep. | 8.8 | 11.2 | 15.5 | 7.1 |
| Denmark | 13.2 | 28.2 | 20.1 | 32.7 |
| Germany | 17.1 | 23.2 | 18.5 | 29.2 |
| Estonia | 17.4 | 29.9 | 22.2 | 30.0 |
| Ireland | 14.2 | 17.7 | 11.9 | 42.1 |
| Greece | 21.0 | 47.0 | 25.0 | 51.0 |
| Spain | 20.0 | 47.6 | 36.8 | 53.5 |
| France | 11.5 | 35.5 | 22.6 | 42.8 |
| Croatia | 17.5 | 25.2 | | 30.9 |
| Italy | 17.7 | 35.4 | 33.5 | 36.3 |
| Cyprus | 10.2 | 28.2 | 23.0 | 36.8 |
| Latvia | 17.3 | 22.8 | | 23.0 |
| Lithuania | 17.4 | 28.9 | | 29.6 |
| Luxembourg | 9.5 | 22.2 | 19.4 | 36.7 |
| Hungary | 14.1 | 7.0 | 7.6 | |
| Malta | 12.7 | 20.5 | 16.6 | 26.9 |
| Netherlands | 12.0 | 19.5 | 12.4 | 26.8 |
| Austria | 9.4 | 33.1 | 29.5 | 35.8 |
| Poland | 16.3 | 6.2 | | 7.7 |
| Portugal | 18.6 | 30.0 | 24.4 | 31.2 |
| Romania | 23.1 | | | |
| Slovenia | 12.4 | 42.9 | 33.8 | 44.5 |
| Slovakia | 12.0 | 11.0 | 18.0 | |
| Finland | 11.9 | 28.3 | 17.7 | 36.2 |
| Sweden | 12.3 | 38.7 | 31.6 | 46.2 |
| UK | 14.6 | 19.4 | 18.0 | 21.4 |

*Source: Personal contribution using Eurostat data*

Across the EU28, the average risk of poverty rate for foreign citizens was 31.5% in 2014. Some Member States faced to higher poverty rates for foreign citizens: Spain (47.6%), Greece (47.0%), Slovenia (42.9%), Belgium (38.9%) and Sweden (38.7%). The lowest poverty rates were in Bulgaria (6.0%), Poland (6.2%) and Hungary (7.0%). Romania has no data connected to this indicator, even that the Romanians' rate of poverty was the greatest one across the EU28 (23.1%) in the same year.

On the other hand, the housing and living conditions of migrants are not good enough. Migrants live in households with very low work intensity (Eurostat, 2016).

According to the above analysis, the first intermediary conclusion is that EU was not able to succeed in integration immigrants even before the new wave from 2015.

## 4. Immigrants' Impact on the European Economy

Interesting scientific forecasts related to EU Muslim population's trend lead to a strange conclusion. According to US-based Pew Forum on Religion & Public Life, the Muslim population in the EU28 will increase substantially until 2030 (Simon, 2011). These forecasts were realized under the presumption that that the present demographical tendency will continue (see Table 4).

**Table 4. EU Muslim population up to 2030 (million persons)**

| | Muslim population 2010 | % total population | Muslim population 2030 | % total population |
|---|---|---|---|---|
| Austria | 0.475 | 5.7 | 0.799 | 9.3 |
| Belgium | 0.638 | 6.0 | 1.149 | 10.2 |
| Bulgaria | 1.002 | 13.4 | 1.016 | 15.7 |
| Croatia | 0.056 | 1.3 | 0.054 | 1.3 |
| Czech Rep. | 0.004 | - | 0.004 | - |
| Denmark | 0.226 | 4.1 | 0.317 | 5.6 |
| Estonia | 0.002 | 0.1 | 0.002 | 0.1 |
| Finland | 0.042 | 0.8 | 0.105 | 1.9 |
| France | 4.704 | 7.5 | 6.860 | 10.3 |
| Germany | 4.119 | 5.0 | 5.545 | 7.1 |
| Greece | 0.527 | 4.7 | 0.772 | 6.9 |
| Hungary | 0.025 | 0.3 | 0.024 | 0.3 |
| Ireland | 0.043 | 0.9 | 0.125 | 2.2 |
| Italy | 1.583 | 2.6 | 3.199 | 5.4 |
| Latvia | 0.002 | 0.1 | 0.002 | 0.1 |
| Lithuania | 0.003 | 0.1 | 0.002 | 0.1 |

| | | | | |
|---|---|---|---|---|
| Luxembourg | 0.011 | 2.3 | 0.014 | 2.3 |
| Malta | 0.001 | 0.3 | 0.001 | 0.3 |
| Netherlands | 0.914 | 5.5 | 1.365 | 7.8 |
| Poland | 0.020 | 0.1 | 0.019 | 0.1 |
| Portugal | 0.065 | 0.6 | 0.065 | 0.6 |
| Romania | 0.073 | 0.3 | 0.073 | 0.4 |
| Slovakia | 0.004 | 0.1 | 0.004 | 0.1 |
| Slovenia | 0.049 | 2.4 | 0.049 | 2.4 |
| Spain | 1.021 | 2.3 | 1.859 | 3.7 |
| Sweden | 0.451 | 4.9 | 0.993 | 9.9 |
| UK | 2.869 | 4.6 | 5.567 | 8.2 |

*Source: Personal contribution using Eurostat data*

According to data from Table 4, the Muslim population will have minor impact on labor market in Czech Republic, Estonia, Latvia, Lithuania, Poland and Slovakia in 2030. The Muslim population will stay constant as percentage of total population in Croatia, Hungary, Luxembourg, Malta, Portugal and Slovenia during 2010-2030. The other Member States will face to an increase of the Muslim population as part of the total population.

Nowadays, Germany and France have the largest Muslim population in the EU28. About 3.5 million Muslims live in Germany, but only 20% of them have German citizenship (Euro-Islam.info, 2016). As a result, the first intermediate conclusion of this chapter is that Muslim population will increase powerfully in the EU. And this forecast was realized under normal demographic conditions.

Nowadays, the German and French immigrant policy leads to supplementary high flows. These new immigrants support unemployment rate's increasing in the receiving Member States. In Germany, for example, the Muslim population unemployment rate is twice as high compared to non-Germans and it achieved 30% in some lands (European Commission, 2016). Moreover, the Muslim population will achieve 20 million in Germany within the next five years. In 2015, 1.5 million asylum seekers entered in Germany and their number will increase in 2016. At least ¾ of them have no qualifications (Eurostat, 2016). The president of the Bavarian Association of Municipalities considered that the Muslim population in Germany represents "a demographic shift of epic proportions, one that will change the face of Germany forever" (Soeren, 2015).

The second intermediate conclusion of this chapter is that the demographic structure of the German population will be change dramatically by the Muslim immigrants in the next five years. The economic impact of the Muslim immigrants is absolutely great. On 22.01.2016, the Vice-Chancellor of Germany Sigmar Gabriel declared that "80% of the refugees do not have any qualifications. An increasing proportion of them are illiterate." This is the real challenge for the EU labor market. According to the latest official data, the EU unemployment achieved 23.2 million persons in 2015, which corresponded to a rate of 9.5% (European Commission a, 2015). Under the previous assumption that the immigrants will achieve 3 million persons during 2015-2016, the EU28 will face to an unemployment rate of 10.6% at the end of 2016. The real problem is that the domestic employment increase will put under pressure the same unemployment rate in the Member States and the negative effect will be higher in 2016 (see Table 5).

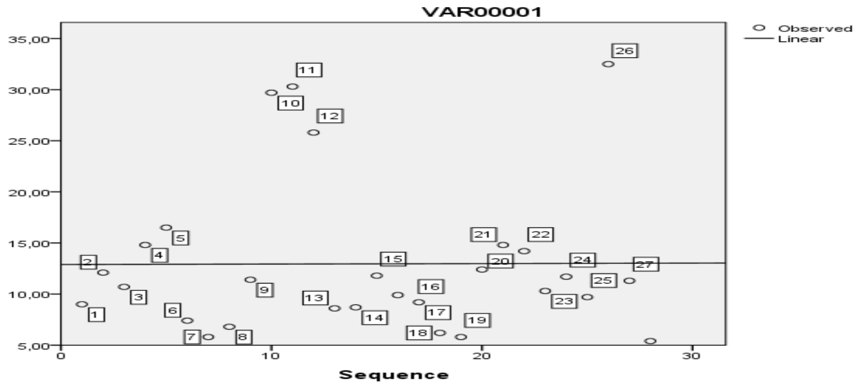
**Table 5. EU labor market dynamics (%)**

| | Employment growth rate 2015 | Unemployment rate 2015 | Employment growth rate 2016 | Unemployment rate 2016* |
|---|---|---|---|---|
| Austria | 0.7 | 6.1 | 0.8 | 9.0 |
| Belgium | 0.6 | 8.6 | 0.7 | 12.1 |
| Bulgaria | 0.3 | 10.1 | 0.3 | 10.7 |
| Cyprus | 0.2 | 15.6 | 1.2 | 14.8 |
| Croatia | 0.6 | 16.2 | 0.7 | 16.5 |
| Czech Rep. | 1.3 | 5.2 | 0.2 | 7.4 |
| Denmark | 0.9 | 6.1 | 1.0 | 5.8 |
| Estonia | 1.1 | 6.5 | -0.6 | 6.8 |
| Finland | -0.4 | 9.6 | 0.3 | 11.4 |
| France | 0.3 | 10.4 | 0.5 | 29.7 |
| Germany | 0.5 | 4.7 | 0.6 | 30.3 |
| Greece | 0.4 | 25.7 | -0.6 | 25.8 |
| Hungary | 1.8 | 7.1 | 1.1 | 8.6 |
| Ireland | 2.0 | 9.5 | 1.5 | 8.7 |
| Italy | 1.0 | 12.2 | 1.0 | 11.8 |
| Latvia | 0.2 | 10.1 | 0.4 | 9.9 |
| Lithuania | 1.5 | 9.4 | 0.2 | 9.2 |
| Luxembourg | 2.6 | 5.9 | 2.5 | 6.2 |
| Malta | 2.4 | 5.8 | 2.0 | 5.8 |
| Netherlands | 1.2 | 6.9 | 1.1 | 12.4 |
| Poland | 1.0 | 7.6 | 0.6 | 14.8 |
| Portugal | 1.1 | 12.6 | 0.8 | 14.2 |
| Romania | 0.3 | 6.7 | 0.4 | 10.3 |
| Slovakia | 1.8 | 11.6 | 1.2 | 11.7 |
| Slovenia | 0.6 | 9.4 | 0.5 | 9.7 |
| Spain | 2.8 | 22.3 | 2.5 | 32.5 |
| Sweden | 1.3 | 7.7 | 1.6 | 11.3 |
| UK | 1.7 | 5.4 | 1.0 | 5.4 |

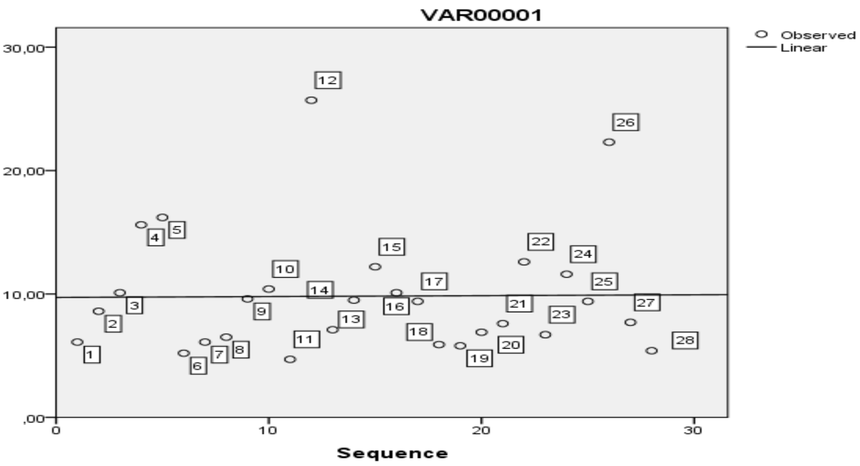
*under the assumption of the relocation immigrants' schemes. Denmark, Ireland and UK take not part of this process. Greece and Italy are transit countries.

According to Table 5, 20 Member States will face to higher unemployment rates in 2016 compared to 2015 as a result of the immigrant process. For some Member States, including both which supported this process, the unemployment rates will grow powerfully.

This process will support the disparities increasing across the EU28 (see Figure 3).




2016




2015

**Figure 3. Unemployment's disparities in 2015 and 2016**

*Source: Personal contribution using Eurostat data*

Figure 3 supports the idea of increasing disparities between the Member States as a result of immigrants' inputs. Moreover, Germany and France can face to unexpected negative effects on their labor markets.

On the other hand, the German government has to allocate 20 billion Euros for immigrants in 2016. The financial efforts focused on immigrants' support may be unrealistic for many Member States.

## 5. Conclusions

Germany adopted wrong strategy connected to immigrants' flows. It wanted to cover the lack of labor supply on German labor market and to become an important actor in the conflict regions. This approach was not good and the present negative results are far away of finishing.

Germany and France operated as leaders of the EU28 and imposed refugees' quotes to the other Member States, even that they didn't want this.

Nowadays, EU28 faces to a new important challenge – refugees' crisis – and has not viable solution for it. This crisis came over the Greek crisis and the whole EU structural crisis, as well.

The whished advantages for the German economy from the refugees' crisis change into dangerous challenge not only for Germany. EU28 is closed to enter into dangerous economic, social, political and military crisis.

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# The Entrepreneurial Intention of University Students: The Case of a University of Technology in South Africa

**Chux Gervase Iwu[1], Ikechukwu O Ezeuduji[2], Chuks Eresia-Eke[3], Robertson Tengeh[4]**

**Abstract:** This quantitative study was executed from a realist's ontological perspective and its epistemological leaning is towards that of an empiricist. The study essentially sought to determine the existence or otherwise of entrepreneurial intentions among the students. Ample emphasis needs to be placed on entrepreneurship education and practical entrepreneurship schemes (such as mentorship programmes) if developing countries are to realise the goal of having a productive and virile youth population, which would represent a significant shift from today's yawning youth unemployment position. The study collected data in a cross-sectional manner from a random sample of 150 students drawn from a leading South African University of Technology. In analyzing the data, there was recourse to the use of descriptive as well as inferential statistics. Interestingly, results show no statistically significant relationships between students' entrepreneurial intention and selected socio-demographic variables such as age, gender, culture, etc. While we acknowledge that the results of this study emerged from a sample of 150 students of a particular university and therefore betray the concept of generalization, we are equally confident that the findings have significant implications for developing economies around the world including South Africa.

**Keywords**: entrepreneurship; entrepreneurship intention; entrepreneurship education; self-employment; sub-Saharan Africa

**JEL Classification**: L260; O550; R110

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## 1 Introduction

Engaging in entrepreneurial activity is considered one of the ways of becoming self-employed. Becoming self-employed is seen as a means of sustaining oneself and consequently providing an income for others through employment of those with the capacity to add value to the business. The foregoing is partly what entrepreneurship is about. As reported by Global Entrepreneurship Monitor (GEM, 2014), most sub-Saharan African countries (especially those characterized as factor-driven economies) have recorded relatively impressive total early-stage entrepreneurial activity (TEA) in recent times. Unfortunately, South Africa is excluded from this list, largely because the extent of entrepreneurial engagement in South Africa is considered to be very low (see Table 1). The reasons for this are related to the issues of insufficient capital, poor business management ability and a general lack of infrastructural support (Gwija et al., 2014). With reference to the issue of poor business management acumen, very much like other competencies, it has been argued that education is necessary. However if the intervention of education, in this case specific to entrepreneurship is to belch reasonable results, then a pre-knowledge of the state/nature of entrepreneurial intentions of students will be critical. Perhaps in very few countries is this need more amplified than it is in South Africa where as has been revealed by the Global Entrepreneurship Monitor (GEM) reports, entrepreneurial activity is quite low across the entire spectrum (see Table 1)

**Table 1. Phases of entrepreneurial activity in the GEM economies in 2014; by geographic region (% of population Aged 18-64)**

| Region and economies | | Nascent entrepreneurship rate | New business ownership rate | Early-stage entrepreneurial activity (TEA) | Established business ownership rate | Discontinuation of businesses (% of TEA) |
|---|---|---|---|---|---|---|
| Africa | Angola | 9.5 | 12.4 | 21.5 | 6.5 | 15.1 |
| | Botswana | 23.1 | 11.1 | 32.8 | 5.0 | 15.1 |
| | Burkina Faso | 12.7 | 9.7 | 21.7 | 17.7 | 10.8 |
| | Cameroon | 26.4 | 13.7 | 37.4 | 11.5 | 17.7 |
| | **South Africa** | **3.9** | **3.2** | **7.0** | **2.7** | **3.9** |
| | Uganda | 8.9 | 28.1 | 35.5 | 35.9 | 21.2 |
| | Average (unweighted) | 14.1 | 13.0 | 26.0 | 13.2 | 14.0 |

*Source: Global Entrepreneurship Monitor Report. 2014. p. 34 (Adapted)*

To be sure, entrepreneurship education is not the only factor that spurs entrepreneurship intention. There are others. These include cultural factors (European Commission, 2012); a range of person-related infrastructural and socioeconomic issues (Matlay, 2005) as well as an individual's overall personality (Ciavarella et al., 2004). Cultural characteristics and how they impact the place of work and entrepreneurial engagement have for long been the focus of several studies.[1] The common denominator in these studies is that the culture of one's locale has a significant influence on one's worldview. Essentially, certain communities are known to explore opportunities and are more driven to achieve goals than others. This seems to be the premise upon which it has been argued that an individual's cultural context can influence his intention to become entrepreneurial (Liñán & Chen, 2009). Flowing from this is the likelihood of a 'more inspired individual' taking an interest in an entrepreneurial activity that may be considered as a challenging venture by another individual. In fact, Shinnar et al (2012) note that it is possible for an individual to perceive institutional, political, economic and or personal obstacles as insurmountable and opt for stable employment rather than pursue a career in entrepreneurship.

While entrepreneurship intention studies have gained traction in the last few years, there is no empirical evidence of its status among University of Technology (UoT) students especially in South Africa. The significance of this kind of study in a UoT can be derived from the purpose of UoTs in South Africa. UoTs are among others expected 'to be the place where practice-based learning takes place with the aim of producing job creators and addressing society's problems' (Gibbon, 2008; Scott, 2005; Asmal, 2002)[2]. It is important to note that UoTs (previously Technikons) are relatively a new phenomenon in South Africa's public university system. They offer practice based learning in the fields of technology as well as vocationally-driven diplomas and degrees in engineering, and business. Thus, UoTs have a fundamental role to play in alleviating pressures of poverty, inequality and unemployment. Research by Pihie and Akmaliah (2009) posited that 'there is a need for universities to enhance their teaching strategies in order to improve entrepreneurial self-efficacy and desire among students to opt for entrepreneurship as a career choice'. The purpose of this study is therefore to investigate factors related to the entrepreneurial intention of students of a University of Technology in South Africa. To this end, the study poses the following research questions:

[1] See (Hofstede, 2001; McClelland, 1961; Weber, 1930).

[2] This description paraphrases the different aims of universities of technology in South Africa. Sources: http://www.che.ac.za/sites/default/files/publications/d000101_UofT_Scott_22Sept2005.pdf; http://www.cepd.org.za/files/pictures/Vol16%20No1.pdf.
http://www.dhet.gov.za/Reports%20Doc%20Library/New%20Institutional%20landscape%20for%20Higher%20Education%20in%20South%20Africa.pdf.

1. Do relationships exist between the socio-demographic characteristics of students and their entrepreneurial intention?
2. What 'meanings' do students attach to entrepreneurship and its development?
3. What do students perceive as the prime motivators and inhibitors of entrepreneurial intention?
4. Is there a relationship between entrepreneurship education and students' decision to become entrepreneurs?
5. Is it likely that the students' perceived entrepreneurial inclinations and 'enablers' have relationships to their entrepreneurial intentions?

## 2. Literature Review

The need for the continued emergence of entrepreneurs in developing economies cannot be over-emphasized. Young people often described as the future of a society, present a veritable pool of individuals that may invariably become entrepreneurs. This may be the reason why studies have investigated the concept of entrepreneurial intentions (Drennan, Kennedy & Renfrow, 2005) among students (Wu & Wu, 2008) in universities across the world. The trend is also noticeable in Africa.

According to Eresia-Eke and Gunda (2015), the current complexion of the global socio-economic landscape suggests that national economic success particularly in Africa tends to be dictated by the extent of entrepreneurial activity. Indeed, economies need to be entrepreneurial (Amos & Alex, 2014) and this is only possible through the emergence of individual entrepreneurs (Gurbuz & Aykol, 2008). This to a large extent underlines the value of studies on entrepreneurial intentions.

Fayolle and Liñán (2014) opine that entrepreneurial intention has become 'a consolidated area of research within the field of entrepreneurship'. Due to this, a number of studies have been conducted on entrepreneurial intentions in both developed and developing economies (Amos & Alex, 2014). Expectedly, with these studies, new knowledge emerges but more questions arise that need to be addressed (Fayolle & Liñán, 2014).

Entrepreneurial intention is defined by Bird (1988) as a state of mind that directs an individual's attention and action towards self-employment as compared to pursuing employment prospects in an existing organisation. Essentially, the notion of entrepreneurial intention is therefore related to the desire to own a business or become self-employed (Thompson, 2009). This desire for business start-up or self-employment may be associated with issues unlimited to individual and societal factors among others. Regardless of what the underlying reasons may be, Krueger and Brazeal (1994) contend that entrepreneurship-oriented intentions can be considered as useful precursors of entrepreneurial action. This is the premise upon which models of planned behaviour become instructive as they cement the founding rationale for any study of entrepreneurial intentions. Indeed, Eresia-Eke and Gunda (2015) argue that intentions precede and can predict behaviour. So the knowledge of the entrepreneurial intentions (EI) of students should contribute to the determination of the extent to which they are likely to opt for entrepreneurship as a career option. Among other models, Ajzen's (1991) model of planned behaviour is quite predominant (Fayolle & Liñán, 2014). Generally, Ajzen's model and Shapero-Krueger's entrepreneurial event model (Krueger, Reilly & Carsrud, 2000) have been useful for the EI discourse.

Ajzen's (1991) model proposes that there is some interplay between subjective norms, perceived behavioural control and attitude towards a behaviour that are associated with the development of EI which in turn then informs the entrepreneurship behaviour of the individual. While subjective norms describe societal expectations of individual conformance to 'acceptable' standards, perceived behavioural control is concerned with the individual's perception of the level of control that an individual can exercise over resources required to become self-employed. Attitude towards a behaviour points to the extent to which the individual views a particular behaviour as favourable or otherwise.

Shapero-Krueger's entrepreneurial event model suggests that EI is dependent on five constructs namely those of specific desirables, perceived self-efficacy, perceived desirability, propensity to act and perceived feasibility. Even though the constructs from the two models may be somewhat different, they are quite compatible and overlapping (Piperopoulos & Dimov, 2015). Both models seem to suggest that intentions are formed on the basis of certain individual or societal factors. This is the line towed by this study as it seeks to examine the existence of relationships (or lack of it) between selected individuals and societal independent variables and the dependent variable, EI among students in a South African university.

The approach of examining the relationship between factors associated with the individual and how they relate to EI has been applied in previous studies with each researcher opting to focus on certain variables that were deemed useful for that particular study. According to Lee and Wong (2004), the intention to display 'certain behaviour is shaped and affected by a plethora of factors such as needs, values, wants, habits and beliefs'. This position is supported by Ajzen (1991) as well as Liñán and Chen (2006) who relate intention to cognitive variables and situational factors respectively.

Given the peculiar idiosyncrasies of countries and societies, and the fact that intentions seem to be shaped by a number of different factors, it would seem inappropriate to imagine that factors that are found to be significantly related to EI in one environment would necessarily demonstrate the same association in another environment. It is therefore not surprising that though many studies have focused on EI, the production of mixed results has been the trend. In other words, there is no clear congruence in the results generated by EI studies. This underlines the need and value of EI studies, that are specific to particular populations in particular countries.

## 3. Research Method and Design

The research questions required individual and quantified responses from students; therefore questionnaire survey was an ideal means of getting such information (Veal, 2011). A respondent-completed structured questionnaire[1] method was employed to obtain information from 150 students at a South African university of technology. Simple random sampling of students was done in their recreational spaces. Respondents were students within business and non-business programmes. Of the 150 administered questionnaires, 115 completed questionnaires were suitable for analysis.

The questionnaire was designed to investigate response heterogeneities amongst students. Items were introduced to investigate entrepreneurship ‘meanings’, motivations, barriers, intents, ‘influencers’, inclinations, ‘enablers’ and students’ profile. Questions were set as categorical and ranked variables, based on the type of question being asked, with ranked variables set mostly on a 5-point Likert scale. The scale ranged from 1 (strongly agree/very positively/very likely/very highly) to 5 (strongly disagree/very negatively/very unlikely/very lowly). The students’ profiles were sorted into categorical variables.

IBM’s SPSS version 23 software (IBM Corporation, 2013) was used for statistical analysis. The first stage of data analysis employed descriptive statistics to derive percentage frequencies of responses. Pearson Chi-square tests, Spearman’s correlation tests, and Mann-Whitney tests were later used at the second stage of analysis, to reveal relationships between variables to answer research questions. Pearson Chi-square test was used to check for relationships between categorical variables; Spearman’s correlation test was employed to check for relationships between ranked variables; and Mann-Whitney test was used to explore relationships between categorical and ranked variables (Veal, 2011). All statistical tests were done at a 95% confidence interval.

[1] Adapted from Bateman and Crant (1993) Proactive Personality Scale.

### 3.1. Results and Discussion

#### *3.1.1 Respondents' Profile and Entrepreneurial Intention*

Table 2 below depicts participants' profile as well as their responses to items in the questionnaire which addressed entrepreneurial intention. The reporting of participants' profile is in line with the first research question which was: Do relationships exist between the socio-demographic characteristics of students and their entrepreneurial intention?

**Table 2. Profile of the respondents (n=115)**

| Variable | Category | Frequency (%) |
|---|---|---|
| Age group | < 21<br>21-24<br>> 24 | 25.7<br>63.7<br>10.6 |
| Gender | Male<br>Female | 38.1<br>61.9 |
| Cultural group | Black South African<br>Coloured South African<br>Indian South African<br>White South African<br>Black Immigrant<br>Other Immigrant | 32.1<br>30.3<br>2.8<br>20.2<br>10.0<br>4.6 |
| Level of study | National Diploma<br>Bachelor<br>Masters | 80.6<br>15.0<br>4.4 |
| Study discipline | Entrepreneurship & Business<br>Non-Business | 38.3<br><br>61.7 |
| **Taken any entrepreneurship-specific course?** | **Yes**<br>**No** | **50.4**<br>**49.6** |
| **Expected year of graduation** | **2014 - 2015**<br>**2016 - 2017**<br>**2018 - 2019** | **61.4**<br>**37.7**<br>**0.9** |
| **Society's level of entrepreneurship encouragement** | **High or very high**<br>**Neither high nor low**<br>**Low or very low** | **47.4**<br>**38.5**<br>**14.1** |
| **Society's entrepreneurial failure level of tolerance** | **Low or very low**<br>**Neither high nor low**<br>**High or very high** | **26.3**<br>**51.0**<br>**22.7** |
| **Entrepreneurial intention during study** | **Yes**<br>**No**<br>**I do not know** | **62.7**<br>**24.5**<br>**12.8** |

| | | |
|---|---|---|
| **Entrepreneurial intention just after graduation** | **Yes**<br>**No**<br>**I do not know** | **53.1**<br>**28.1**<br>**18.8** |
| Entrepreneurial intention long after graduation | Yes<br>No<br>I do not know | 55.4<br>32.6<br>12.0 |
| Respondents who started a new venture during their studies | Yes<br>No | 39.8<br>60.2 |
| Work experience | Yes<br>No | 73.6<br>26.4 |
| Family member who owns a business | Yes<br>No | 67.9<br>32.1 |

Two questions were asked in the questionnaire to determine a clear entrepreneurial intention: a. 'do you wish to eventually start your own business or to become self-employed? ('during your studies, just after graduation, or a long time after graduation'); and b. 'while studying, have you started a new venture, an organisation, or a business?'. Students' responses to these questions are also shown in Table 2. Results reveal no statistically significant relationships between students' entrepreneurial intention and socio-demographic variables. This research outcome supports the finding of Mohd et al (2015) and Mat et al (2015) that no significant relationship exists between students' family business background and gender on the one hand and EI on the other hand. However this is not in line with the findings of Zhang et al (2014) who stated that males have higher EI than females.

Respondents also indicated their choice of business size or sector for career advancement. About 57% of them prefer a large company, about 35% of them prefer a small or medium-sized company, while about 12% of the respondents prefer to work in the public sector, and about 5% prefer to advance their careers in a non-profit sector. Black South Africans and other Black students prefer large companies (Chi-square, p=0.009), while Coloured and White South Africans prefer small or medium-sized company for career advancement (Chi-square, p=0.014).

Results also indicate that respondents of age group 21 - 24 have more work experience (Chi-square, p = 0.018) than other age groups. Those expecting to graduate in 2016-2017 also indicated that they had more work experience (Chi-square, p = 0.001) than those who intend to graduate in other years. Those expecting to graduate in 2014 – 2015 and 2016 – 2017 were more positive that work experience influenced their intention to embark upon an entrepreneurial career (Chi-square, p=0.000) than those who intend to graduate in other years. Coloured and White South African students are more inclined to start a new venture during their studies, than other cultural groups (Chi-square, p=0.05). Black

South Africans and other black students admitted to having work experience (Chi-square, p=0.008) than other cultural groups. White South African students within the sample indicated that they had family members who own a business than other cultural groups (Chi-square, p=0.004). Respondents were also asked if they were likely to get financial help from family members if they need help. About 27% of the respondents said this is 'very likely', about 37% of them stated it is 'likely', about 17% of them were neutral, stating 'neither likely nor unlikely', about 9% of them stated this is 'unlikely', and 11% of them declared this 'very unlikely'.

### 3.1.2. *Meanings Attached to Entrepreneurship and Factors Influencing its Development*

In order to answer research question 2: what 'meanings' do students attach to entrepreneurship and its development?, Table 3 below indicates the students' percentage of agreement to different questions posed by this study to ascertain 'meanings' that they have of entrepreneurship.

**Table 3. Percentage of respondents' association of entrepreneurship 'meanings' (n=115)**

| Statements of entrepreneurship 'meanings' | Percentage of agreement |
|---|---|
| Creating an own business | 74.1 |
| Launching and developing a project or an activity | 34.8 |
| Organising and managing own business | 48.2 |
| Taking risks | 50.0 |
| Creating a non-profit association or a co-operative | 14.3 |
| Increasing capital and wealth | 36.0 |
| Developing a new product or service | 43.8 |
| An entrepreneur is a man or woman of action for whom knowledge is a secondary concern | 24.1 |
| An entrepreneur is an inventor | 50.0 |
| Money is the only thing that an entrepreneur needs | 7.1 |

From the foregoing, most students view entrepreneurship as creating an own business, and half the student sample view entrepreneurs as inventors and as a risk-laden process. Essentially, the notion of entrepreneurial intention is related to the desire to own a business or become self-employed (Thompson, 2009).

Table 4 below shows the students' percentage of agreement to different questions posed by this study to ascertain their perceptions of factors influencing the development of entrepreneurship in the world economy.

**Table 4. Factors influencing the development of entrepreneurship in the world economy (n=115)**

| **Statements** | **Percentage of agreement** |
|---|---|
| The characteristics of people (potential entrepreneurs) | 48.2 |
| The political situation (political system, ideologies, etc.) | 39.3 |
| The economic conditions (level of inflation, tax system, state of economy, etc.) | 48.2 |
| The educational system (the availability of appropriate courses, recognition of creativity, etc.) | 50.0 |
| A positive climate for innovation in businesses and institutions, easy access to resources, motivational systems, etc. | 37.5 |
| A system of support (mentoring, advice, personalised support, sponsorship, etc.) | 42.9 |

Among other factors, students perceive the educational system, the characteristics of people and economic conditions as the most important factors influencing the development of entrepreneurship in the world economy. Research results from Hattab (2014), Solesvik et al (2014), and Zhang et al (2014) suggest that entrepreneurship education is very important for EI. Eresia-Eke and Gunda (2015) posit that the current complexion of the global socio-economic landscape suggests that national economic success particularly in Africa tends to be dictated by the extent of entrepreneurial activity. Economies need to be entrepreneurial (Amos & Alex, 2014) and this is only possible through the emergence of individual entrepreneurs (Gurbuz & Aykol, 2008).

In this research, respondents of age group less than 21, agreed more to the statement 'a system of support (mentoring, advice, personalised support, sponsorship, etc.' as being an important factor influencing the development of entrepreneurship in the world economy (Chi-square, p=0.035). Those that expect to graduate in 2014 - 2015 agreed more than others to the statement 'a positive climate for innovation in businesses and institutions, easy access to resources, motivational systems, etc.' as being an important factor influencing the development of entrepreneurship in the world economy (Chi-square, p = 0.013). White South African students agreed more to 'the educational system (the availability of appropriate courses, recognition of creativity, etc.)' as being an important factor influencing the development of entrepreneurship in the world economy, than other cultural groups (Chi-square, p=0.033). Coloured and White

South African students agreed more to the statement 'a positive climate for innovation in businesses and institutions, easy access to resources, motivational systems, etc.' as an important factor influencing the development of entrepreneurship in the world economy, than other cultural groups (Chi-square, p=0.028).

### *3.1.3. Motivations and Barriers to Becoming An Entrepreneur*

With regard to question 3, we found that about 52% of the participants perceive motivations for entrepreneurship to be 'personal fulfillment', about 35% view it as a way to 'become own boss', 34% see it as an avenue 'to make money', and about 29% consider it as a way of 'taking up a challenge'. Coloured, White South Africans and other Black students were more inclined to consider entrepreneurship as an avenue for 'personal fulfillment' than other cultural groups' respondents (Chi-square test, p=0.026).

Regarding the main barriers to becoming an entrepreneur, about 61% of students surveyed view 'a lack of financial resources' as a major barrier, while about 41% perceive 'a lack of support and assistance', as major barriers. Other barriers indicated by the subjects include 'unfavourable economic conditions' (about 39%); 'a lack of profitable opportunities' (about 24%), and about 13% perceive 'complex procedures for creating and managing a business' as main barriers. Black immigrants agreed more to 'a lack of support and assistance' to be a main barrier to becoming an entrepreneur, than other cultural groups' respondents (Chi-square test, p=0.000).

According to Lee and Wong (2004), the intention to display certain behaviour (EI for example) is shaped and affected by a plethora of factors such as needs, values, wants, habits and beliefs. This is supported by Ajzen (1991) who relates intention to cognitive variables. Liñán and Chen (2006) are also of the view that intentions are dictated by situational factors.

### *3.1.4. Academic Influencers and Entrepreneurship*

To be able to answer research question 4: Is there a relationship between entrepreneurship education and students' decision to become entrepreneurs?, we asked the students to indicate their disciplines and also to rate the degree of influence their academic activities exerted. We found that about 38% of the respondents were from Entrepreneurship and Business Management, compared to about 62% of respondents in non-business management related courses. Table 5 depicts respondents' ratings in percentage of entrepreneurial influencers. Interestingly, results reveal no statistically significant relationships between business students' and non-business students' entrepreneurial intentions. In a broad sense, students who take entrepreneurship-specific courses (whether business or non-business students) agreed more to wishing to eventually start their own businesses just after graduation, than a long time after graduation (Chi-square, p=0.000). This result points out that entrepreneurship education has a high significant stimulation towards students' decision to become entrepreneurs.

**Table 5. Respondents' ratings in percentage of entrepreneurial influencers (n=115)**

| Influencers | Very positive | Positive | Neither positive nor negative | Negative | Very Negative |
|---|---|---|---|---|---|
| Lectures | 32.2 | 52.5 | 13.6 | 0.0 | 1.7 |
| Views of a Professor | 25.0 | 42.9 | 25.0 | 7.1 | 0.0 |
| Team exercise | 8.9 | 41.1 | 37.5 | 8.9 | 3.6 |
| Business simulation or case studies | 20.7 | 48.3 | 25.8 | 5.2 | 0.0 |
| Views of classmates | 15.8 | 29.8 | 42.1 | 12.3 | 0.0 |
| Guest speakers | 19.3 | 36.8 | 29.9 | 10.5 | 3.5 |
| Independent or individual exercises | 29.8 | 43.9 | 24.5 | 1.8 | 0.0 |
| Work experience | 32.7 | 37.6 | 26.7 | 2.0 | 1.0 |

Lectures, views of an academic, business simulation or case studies, guest speakers, individual exercises and work experience, all contribute highly to influence students to become entrepreneurs. These results also point out that entrepreneurship education has a high significant stimulation towards students' decision to become entrepreneurs.

Students who declared that they wish to eventually start their own business 'just after graduation' also agreed that 'guest speakers' at the university are quite a positive influence to their entrepreneurial intentions (Mann-Whitney, p=0.042).

### *3.1.5. Entrepreneurial Inclinations and Enablers*

In order to answer research question 5: is it likely that the students' perceived entrepreneurial inclinations and 'enablers' have relationships to their entrepreneurial intentions? Respondents were asked to rate their level of agreement to entrepreneurial inclination statements (Table 6) and entrepreneurial thinking and enablers' statements (Table 7). Essentially, the notion of entrepreneurial intention is related to the desire to own a business or become self-employed (Thompson, 2009). This desire for business start-up or self-employment may be associated with issues unlimited to individual and societal factors among others. Regardless of what the underlying reasons are, Krueger and Brazeal (1994) contend that entrepreneurship-oriented intentions can be considered as useful precursors of entrepreneurial action.

**Table 6. Respondents' ratings in percentage of entrepreneurial inclination statements (n=115)**

| Statements | Strongly agree | Agree | Unsure | Disagree | Strongly disagree |
|---|---|---|---|---|---|
| I am constantly on the lookout for new ways to improve my life | 59.6 | 38.5 | 1.9 | 0.0 | 0.0 |
| I feel driven to make a difference in my community, and maybe in the world | 45.9 | 35.8 | 17.4 | 0.9 | 0.0 |
| I tend to let others take the initiative to start new projects | 16.7 | 34.3 | 24.0 | 21.3 | 3.7 |
| Wherever I have been, I have been a powerful force for constructive change | 18.1 | 46.7 | 30.4 | 3.8 | 1.0 |
| I enjoy facing and overcoming obstacles to my ideas | 38.0 | 49.1 | 11.0 | 1.9 | 0.0 |
| Nothing is more exciting than seeing my ideas turn into reality | 70.6 | 26.6 | 2.8 | 0.0 | 0.0 |
| If I see something that I do not like, I fix it | 29.0 | 48.6 | 17.7 | 4.7 | 0.0 |
| No matter what the odds, if I believe in something, I will make it happen | 37.1 | 38.1 | 22.9 | 1.9 | 0.0 |
| I love being a champion for ideas even against others' opposition | 34.6 | 44.9 | 18.6 | 1.9 | 0.0 |
| I excel against others' opposition | 20.4 | 43.5 | 30.6 | 4.6 | 0.9 |
| I am always looking for better ways to do things | 39.6 | 47.2 | 9.4 | 3.8 | 0.0 |
| If I believe in an idea, no obstacle will prevent me from making it happen | 32.7 | 49.0 | 17.3 | 1.0 | 0.0 |
| I love to challenge the status quo | 37.4 | 38.3 | 23.4 | 0.9 | 0.0 |
| When I have a problem, I tackle it head-on | 22.6 | 53.8 | 19.8 | 2.8 | 1.0 |
| I am great at turning problems into opportunities | 20.4 | 48.1 | 30.6 | 0.9 | 0.0 |

| | | | | | |
|---|---|---|---|---|---|
| I can spot a good opportunity long before others can | 18.7 | 42.1 | 30.8 | 8.4 | 0.0 |
| If I see someone in trouble, I help out in any way I can | 38.5 | 52.3 | 9.2 | 0.0 | 0.0 |

Results generally show that respondents strongly agreed or agreed to entrepreneurial inclinations' statements. However, turning these inclinations into reality (starting a new business) is a challenge (as 'a lack of financial resources', 'a lack of support and assistance', and 'unfavourable economic conditions', show themselves in this study to be major barriers). Results show a positive correlation between the statement 'home country encourages entrepreneurship' on the one hand, and

- 'wherever I have been, I have been a powerful force for constructive change' (Correlation, p=0.013);
- 'I am great in turning problems into opportunities' (Correlation, p=0.012), and
- 'if I see someone in trouble, I help in any way I can' (Correlation, p=0.026), on the other hand.

There are also positive correlations between the question 'to what extent has your work experience influenced your intention to embark upon an entrepreneurial career?' on the one hand, and

- 'I feel driven to make a difference in my community and maybe the world' (Correlation, p=0.043);
- 'wherever I have been, I have been a powerful force for constructive change' (Correlation, p=0.014);
- 'I enjoy facing and overcoming obstacles to my ideas' (Correlation, p=0.002);
- 'I excel against others' opposition' (Correlation, p=0.022);
- 'if I believe in an idea, no obstacle will prevent me from making it happen' (Correlation, p=0.006), and
- 'I love to challenge the status quo' (Correlation, p=0.001) on the other hand.

The results of this study show statistically significant relationships between some of the students' perceived entrepreneurial inclinations and 'enablers' on the one hand, and clear entrepreneurial intentions on the other. Respondents who agreed to start their own business 'during their studies' also agreed to the statement 'if I believe in an idea, no obstacle will prevent me from making it happen' (Mann-Whitney, p=0.039). Respondents who agreed to start their own business 'just after graduation' also agreed to the statement 'I can spot a good opportunity long before others can' (Mann-Whitney, p=0.049). Also, respondents who agreed to start their own business 'a long time after graduation' agreed to the statement 'I can spot a good opportunity long before others can' (Mann-Whitney, p=0.044).

**Table 7. Respondents' reflection on entrepreneurial thinking and enablers (n=115)**

| **Statements** | **Yes** | **No** | |
|---|---|---|---|
| I am in general, creative, full of ideas and open to change | 87.7 | 12.3 | |
| In the creation of a business. I appreciate the independence and self-confidence | | | |
| Do you impose upon yourself difficult and ambitious tasks? | 93.0 | 7.0 | |
| I am a born entrepreneur | 22.3 | 77.7 | |
| | **Yes** | **No** | **Difficult to say** |
| Would you be willing to take some risk (personal, financial) to increase your social and professional status? | 70.2 | 6.1 | 23.7 |
| Could certain academic activities encourage the development of entrepreneurship amongst students (e.g. projects, initiatives, competitions, placements, simulations, etc.)? | 88.7 | 0.9 | 10.4 |
| Do you think you are an entrepreneurial individual? | 57.0 | 13.2 | 29.8 |
| Do you think that the modules (courses) offered by the University motivate the students to become entrepreneurs? | 56.5 | 22.6 | 20.9 |

Respondents expecting to graduate in the year 2016-2017 agreed more to the statement 'do you think you are an entrepreneurial individual?' (Chi-square, p=0.026). Black South Africans are more inclined to think that they are entrepreneurial individuals than the other cultural groups (Chi-square, p=0.046). Black South African students were also more inclined to think that the modules (courses) offered by the university motivate the students to become entrepreneurs, than the other cultural groups' (Chi-square, p=0.033). This finding somewhat contradicts a previous study by Reuben and Bobat (2014), who, on the pervasiveness of the negativity that surrounds Affirmative Action, characterised Black South Africans as lazy. While this seem not to be directly related to the aim of this study, interestingly, the necessary deduction to make here is that perceptions of the Black South African as lazy and uninspired may no longer be the case as this study has shown.

## 4. Conclusion

South Africa's entrepreneurial level has been described by this study as very low. This study further reveals no statistically significant relationships between students' entrepreneurial intention and socio-demographic variables. Most students perceive entrepreneurship as creating an own business, and half the student sample view entrepreneurs as inventors and the practice of entrepreneurship as a risk-laden process. Largely, the participants suggested that their main motivations for desiring to turn to entrepreneurship were linked to the factors of 'personal fulfillment', 'becoming one's own boss, 'making money', and 'taking up a challenge'. The study has also revealed that the main barriers to entrepreneurship quest from a student's perspective include 'a lack of financial resources', 'a lack of support and assistance', and 'an unfavourable economic climate'. However, the results of this study show statistically significant relationships between some of the students' perceived entrepreneurial inclinations and 'enablers' on the one hand, and clear entrepreneurial intentions on the other hand. In any case, a portion of the findings of the study points to the fact that entrepreneurship education acts a significant stimulant for students' decision to become entrepreneurs.

We are aware that obtaining data from 150 students of a single university poses a challenge with respect to generalizing the findings. However, we believe that the results of this study have strong implications not only for South Africa but also for developing nations. For instance, the results show that students are willing to become entrepreneurs after their studies. This is therefore a call for developing nations to focus their attention on improving the economic situation of their countries as well as enable job creation, by:

- Placing emphasis on entrepreneurship education and practical entrepreneurship schemes(such as mentorship programmes) to help foster the desire of students to become entrepreneurs and increase students'/graduates' business management capabilities and job creation propensity; and
- Enlarging financial support for the youth who are willing to become entrepreneurs, especially university graduates with entrepreneurship education, to help improve access to financial resources, support and assistance perceived by students as a barrier to becoming entrepreneurs.

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# Relationship between Major Developed Equity Markets and Major Frontier Equity Markets of World

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**Abstract:** The core aim of this study is to compute the long run relationship between frontier equity markets Pakistan (KSE 100 Index), Argentina (MERVAL BUENOS AIRES) stock Exchange, NSE.20 (Kenya), MSM 30 (MSI) Oman and equity markets of developed world (OMXS30) Sweden, SMI (Switzerland), SSE Composite Index (China) and STI index (Singapore) by taking weekly values from stock return prices for the period 1st week of January-2000 to last week of January/2014. Descriptive statistic, Correlation, Augmented dickey fuller (ADF), Phillips Perron test, Johanson and Jelseluis test of co-integration, Granger causality test, Variance Decomposition Test and Impulse Response are used to find the relationship among frontier and developed markets. The results of this study reveal that frontier markets have no long run relationship with equity markets of developed world. Furthermore, this study is helpful for investors to enhance the returns by diversifying the unsystematic risk at given level of profit because results of this study confirm that markets are no co-integrated.

**Key words:** Diversification; portfolio; frontier markets; unit root test; Co-integration test

**JEL Classification:** G10; G20

## 1. Introduction

There are different types of investment institutions available almost all over the world which offers investment opportunities for investors to make investment in them. Frontier equity markets are also part of investment institution for investors defined as the markets at early stage of growth as compared to other markets, while emerging markets defined as a country having or possessing some of the qualities to reach the level of those developed market which have already occupied their position in the world.

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The word frontier equity market was first used by international finance corporation in 1996, represent a small number of liquid securities and offer excellent diversification benefits to investors. The word frontier defined as the small markets which impose restrictions on foreign ownership. The frontier equity markets are launched to achieve economic development and growth by diversifying risk. Before investing in frontier equity markets all shareholders, investors and portfolio managers make assure either their investment funds utilized efficiently or not, also they analyze that any sign of prosperity is visible or not and to how much extent their funds will give benefit to them. Further investors become more aware about safety of their funds saved and they already learn about amount of their risk and return, which may lead them for saving in frontier equity markets. Frontier markets are becoming important source of strong earnings in the form of return, so investors focus on these markets on the basis of following benefits which are offered to their policy owners, there is no ownership in frontier equity markets, creating potential earnings economy for all investors and shareholders in the form of return. No doubt, frontier markets are less liquid but trend of investments does not decrease. (Schroders)

To understand the relationship between frontier equity market and equity market of developed country, selected some major frontier equity market (Pakistan, Argentina, Kenya and Oman) with developed equity stock markets of Sweden, Switzerland China, Singapore for the period 1st week of January-2000 to last week of March/2014. If the markets of regional countries move together to invest in different equity markets would not gain any profit. Regional diversification suggests investing in those stock markets which are less correlated. To gain the benefit of diversifying, it is necessary that your portfolio assets should be invested in those markets which are negatively correlated as compared to developed markets which offer higher return to investors (Markowitz). Now a day's all investors are investing in frontier equity markets and developed equity markets. So individual, foreign and institutional investor began to diversify their risk by investing in different frontier and developed equity markets.

The terrorist’s activities are the major obstacles in the growth of frontier markets so there is huge amount of risk involved in frontier markets, but no doubt the investors are more interested to get higher return as compared to other markets. Effective liberalization encourages the investors to make their investments in domestic and foreign equity markets but unfortunately there is absence of effective liberalization due to market integration, so on these reasons investors get back from investments (Bekaert et all 2003). The deregulation and liberalization affect directly investors behavior and consequently investment trend declines day by day, so investors feel hesitant in making investments mansoor at al (2014).

All business private organizations have a primary objective to maximize the shareholder wealth in a good way. The investor or portfolio managers can enhance the returns by diversifying the unsystematic risk at given level of profit. The stock Investor by making investment in different stock of domestic country are unable to achieve optimum diversification (Mansoor et al.). This may be due to companies' face the same economic or political situation. So the Frontier equity markets have different economic environment as compared to developed equity market. This study will suggest the investors or portfolio managers to invest across the border in those equity markets which are different to each other economically and politically. In this way, the portfolio managers may be able to attain fully diversified portfolio and minimize the country risk.

The study has objectives to recognize a long run relationship between developed equity markets and frontier equity market and secondly there exists lead lag relationship or not.

## 2. Literature Review

Shezad et al (2014), examined the relationship between co-integration of Pakistani stock markets whose selected Asian stock market for the period 2001 to 2013 by taking monthly values of stock market return. This study used descriptive statistics, correlation analysis, unit root test, VAR, Co-integration test and VECM test. Result shows that KSE is not co-integrated with Japan, Malaysia, Taiwan and China. All these tests and their results show that there is correlation between Chines markets and KSE 100. This study also concluded that for the Chinese investors have opportunities to make investment in these markets.

Khan & Aslam (2014), explored the study on co-integration of Karachi Stock Exchange index 100 with major Asian stock exchange markets Bombay Stock Exchange (BSE Index 30), Malaysian Stock Exchange (FTSE) and Japan Stock Exchange for the period 2007 to 2013 by selecting monthly values of stock markets. This study use data description and Augmented Fuller test (ADF) result shows that there is no co-integration of KSE 100 index with developed countries such as China and Japan. But Pakistani KSE 100 index co-integrated with India and Malaysia stock markets.

Prakhar Porwal (2014), explored the concept of diversification that how diversification will be achieved by focusing on frontier markets as well as developed markets. For this purpose, data was collected by MSCI and S&P Sri Lanka of the frontier and emerging markets. The data was analyzed by correlation and volatility of MSCI indices. The result shows that in frontier markets there is more risk involved but higher return will be gained with low volatility as compared to other emerging market.

Narayan et al (2004) examined the dynamic linkage between the stock markets of developing countries such as Bangladesh, India, Pakistan and Sri Lanka by binding the relationship among the stock prices indices within a multivariate co integration framework for the period 1995-2001 by taking daily values of stock markets return. This study use co integration, causality testing, unit root test. Result shows that there exists a long run relationship between the Sri Lanka stock prices with Pakistan. It further used impulse response which concludes that Sri Lanka market has small impact on Pakistani market.

Aslam et al (2012) investigated the relationship between Karachi stock exchange with major developed equity market for the period 1999-212 by taking weekly values of stock prices. The stock data was analyzed by using VAR statistic, unit root test, unrestricted co-integration rank test (trace), unrestricted co-integration rank test (maximum Eigen value) granger causality. The result and finding shows that Karachi stock exchange is less or weakly correlated with developed equity markets and there is no co-integration exists among the stock markets.

Mansoor et al (2012) investigated a study on relationship between major Asian markets (kse 100,india BSE 500,srilanka CSE) with developed equity markets (cac40, ftse100, nikkie 225, s&p 500). The weekly data was collected for the period 2000-2012.the data was analyzed by applying descriptive statistic, augmented dickey fuller test, Phillips test, granger causality test, Johansen co-integration test, vector error correction model and variance decomposition test. The result shows that there is no long run relationship exists between south Asian equity markets while short run significant relationship exists. Further study help the investor or portfolio managers can enhance the returns by diversifying the unsystematic risk at given level of profit. The stock Investor by making investment in different stock of domestic country unable to achieve optimum diversification.

Khalil Jebran (2014) investigated a study on dynamic linkage between selected south Asian equity markets(India, Indonesia, China, Malaysia And Sri Lanka) with Pakistani stock market by using monthly data of stock prices was taken for the period 2003 to 2013. The correlation matrix, unit root test, Johansen and juselius co-integration, Granger Causality test and variance decomposition were applied to analyze data. The result shows that Indonesia stock market shows highest return among the selected Asian equity markets. India and Indonesia equity markets show high level of correlation and Johansen and Juselius result shows that long run relationship exist between selected stock markets. These all results show that there exists no confirmation of selected equity markets with Karachi stock exchange.

## 3. Hypothesis

**H1**: There is long run relationship exists between frontier equity markets and equity markets of Developed world.

**H01**: There is no long run relationship exists between frontier equity markets and equity markets of Developed world.

**H2**: There is Lead Lag relationship exists between the frontier equity markets and equity markets of Developed world.

**H02:** There is no Lead Lag relationship exists between the frontier equity markets and equity markets of Developed world.

## 4. Methodology

In this study weekly data of frontier equity markets and developed markets was collected by using Investing.com and Yahoo finance for the period 1st week of January-2000 to last week of January/2014. To explore the relationship, we selected some frontier equity market such as KSE 100 Index (Pakistan), Argentina (MERVAL BUENOS AIRES) stock Exchange, NSE.20 (Kenya), MSM 30 (MSI) Oman and major developed equity stock markets of (OMXS30) Sweden, SMI (Switzerland), SSE Composite Index (China), and STI index (Singapore). This study assists the portfolio manager and decision makers to calculate the return rate by applying the equation of Rtn=logn ( Prt./Prt-1)

Where Rtn =shows the return in a given period t

Prt =shows the price at the time of closing

Prt-1=shows the price at the time of opening

Logn=represent the natural logarithm

In this study the techniques of Correlation, unit root test, co- integration, variance decomposition, granger causality and impulse response are used to measure the nature of relationship.

## 5. Results

**Table 5.1. Descriptive statistics**

| | Argentina | Pakistan | Oman | Kenya | China | Singapore | Sweden | Switzerland |
|---|---|---|---|---|---|---|---|---|
| Mean | 0.003995 | 0.004248 | -0.00179 | -0.00129 | 8.04E-05 | 0.000697 | 0.000327 | 5.56E-05 |
| Median | 0.006076 | 0.007797 | -0.00174 | -0.00094 | 0 | 0.00209 | 0.002864 | 0.002456 |
| Maximum | 0.228494 | 0.109173 | 0.196173 | 0.146802 | 0.139447 | 0.153205 | 0.122749 | 0.162885 |
| Minimum | -0.31181 | -0.20098 | -0.1139 | -0.1481 | -0.14898 | -0.164684 | -0.22528 | -0.252017 |
| Std. Dev. | 0.048886 | 0.033678 | 0.024911 | 0.026935 | 0.033586 | 0.026978 | 0.031494 | 0.027724 |
| Skewness | -0.38899 | -1.21761 | 1.464611 | -0.39738 | 0.071572 | -0.516395 | -0.83174 | -1.033043 |
| Kurtosis | 7.705482 | 7.925848 | 15.51188 | 8.990935 | 5.088118 | 9.334665 | 7.843319 | 16.88758 |
| Jarque-Bera | 655.8666 | 870.6017 | 4761.176 | 1053.078 | 126.3109 | 1187.779 | 756.1505 | 5684.02 |
| Probability | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

The table 5.1 shows the description of markets. The table represents the value of mean, median, maximum, minimum Standard deviation, Skewness and kurtosis. The results reveal that Pakistan stock exchange 100 and Argentina show high return while Sweden and Singapore show the positive return. The stock markets of Oman and Kenya represent the negative values of return. On the other hand, in terms of standard deviation Argentina stock markets shows the highest value of standard deviation (0.04) which differentiate it from all other equity markets at given period of time.SO we can conclude that Argentina stock market is one of the riskier or higher return stock market because it gives the highest value of return in a given time period.

**Table 5.2. Correlation technique**

| | Argentina | Pakistan | Oman | Kenya | China | Singapore | Sweden | Switzerland |
|---|---|---|---|---|---|---|---|---|
| Argentina | 1 | | | | | | | |
| Pakistan | -0.05403 | 1 | | | | | | |
| OMAN | -0.01873 | 0.002242 | 1 | | | | | |
| Kenya | -0.0368 | -0.01364 | 0.114115 | 1 | | | | |
| China | 0.042664 | 0.003137 | 0.019924 | 0.117559 | 1 | | | |
| Singapore | 0.079592 | 0.042175 | 0.012116 | -0.01806 | -0.00205 | 1 | | |
| Sweden | -0.02248 | 0.005737 | -0.03101 | 0.014288 | -0.01266 | 0.622465 | 1 | |
| Switzerland | -0.01282 | -0.00328 | -0.03398 | -0.01858 | -0.02412 | 0.581179 | 0.760497 | 1 |

Table (5.2) explores the correlation among the different stock markets. It indicates that the frontier equity markets are negatively correlated to each other. Argentina frontier stock exchange is negatively correlated with Sweden and Switzerland stock markets. KSE is weekly correlated with china, Singapore and Sweden, while negatively correlated with Kenya and Switzerland. The frontier markets of OMAN and Kenya are also negatively correlated with Switzerland market.

**Table 5.3 Unit root test**

| | ADF LEVEL | ADF 1st DIF | PP LEVEL | PP 1st DIF |
|---|---|---|---|---|
| **Argentina** | -0.63543 | -16.9202 | -0.64664 | -25.608 |
| **Kenya** | -0.86179 | -16.4465 | -0.8063 | -23.1552 |
| **Oman** | -0.06037 | -17.6506 | -0.0431 | -25.0565 |
| **Pakistan** | -1.03391 | -16.0384 | -0.99302 | -22.2643 |
| **China** | -1.27974 | -16.925 | -1.24598 | -24.7775 |
| **Singapore** | -1.17255 | -17.097 | -1.10826 | -24.8885 |
| **Sweden** | -1.14818 | -18.1455 | -1.20293 | -27.7898 |
| **Switzerland** | -1.57687 | -18.5342 | -1.75573 | -30.9652 |
| **Critical values** | | | | |
| **1%** | -3.43959 | -3.4396 | -3.43957 | -3.43959 |
| **5%** | -2.86551 | -2.86551 | -2.8655 | -2.8655 |
| **10%** | -2.56894 | -2.56894 | -2.56894 | -2.56894 |

The table 5.3 shows both augmented and Philips- Perron test confirmed that data is not stationary at level but it is stationary at first difference.

**Table 5.4. Multivariate co integration**

| | | Eigen value | Trace statistic | Critical value 5% | Remarks |
|---|---|---|---|---|---|
| **Argentina** | **None*** | 0.079856 | 205.0772 | 159.5297 | Co-integrated |
| **Kenya** | **At most 1** | 0.067405 | 147.5686 | 125.6154 | Co-integrated |
| **KSE** | **At most 2** | 0.055726 | 99.34768 | 95.75366 | Co-integrated |
| **Oman** | **At most 3** | 0.035023 | 59.72683 | 69.81889 | No cointegration |
| **China** | **At most 4** | 0.024779 | 35.09179 | 47.85613 | No cointegration |
| **Singapore** | **At most 5** | 0.014847 | 17.75394 | 29.79707 | No cointegration |
| **Sweden** | **At most 6** | 0.010363 | 7.417996 | 15.49471 | No cointegration |
| **Switzerland** | **At most 7** | 0.000318 | 0.220076 | 3.841466 | No cointegration |

Table 5.4 shows the values of multivariate co integration. Result indicates that there exist three co-integration equations at the 0.05 level.

**Table 5.5. Bivariate co-integration Argentina**

| | Eigenvalue | Statistic | Critical Value | Prob.** | Remarks |
|---|---|---|---|---|---|
| **Argentina-Sweden** | 0.019866 | 13.86697 | 15.49471 | 0.0867 | NO-Cointegration |
| | 0.00000226 | 0.001563 | 3.841466 | 0.9664 | |
| **Argentina-Switzerland** | 0.012679 | 8.962591 | 15.49471 | 0.3688 | NO-Cointegration |
| | 0.00021 | 0.145117 | 3.841466 | 0.7032 | |
| **Argentina-China** | 0.007237 | 6.121436 | 15.49471 | 0.6812 | NO-Cointegration |
| | 0.001594 | 1.102339 | 3.841466 | 0.2938 | |
| **Argentina-Singapore** | 0.014223 | 10.20236 | 15.49471 | 0.2655 | NO-Cointegration |
| | 0.00044 | 0.303822 | 3.841466 | 0.5815 | |

The results of above table reveal that Argentina stock exchange are not co-integrated with Sweden, Switzerland, china and Singapore, which encourage all shareholders, portfolio managers and investors to get the benefit of diversification.

**Table 5.6. Bivariate co-integration KSE**

| | Eigenvalue | Statistic | Critical Value | Prob.** | Remarks |
|---|---|---|---|---|---|
| **KSE-SWEDEN** | 0.018355 | 13.09568 | 15.49471 | 0.1113 | NO-COINTEGRATION |
| | 0.000426 | 0.294604 | 3.841466 | 0.5873 | |
| **KSE-Switzerland** | 0.012848 | 9.589598 | 15.49471 | 0.3136 | NO-COINTEGRATION |
| | 0.000946 | 0.653812 | 3.841466 | 0.4188 | |
| **KSE-China** | 0.005785 | 5.389523 | 15.49471 | 0.7661 | NO-COINTEGRATION |
| | 0.001995 | 1.38024 | 3.841466 | 0.2401 | |
| **KSE-Singapore** | 0.014754 | 10.92561 | 15.49471 | 0.2161 | NO-COINTEGRATION |
| | 0.000947 | 0.654901 | 3.841466 | 0.4184 | |

The results of above table reveal that Karachi stock exchange are not co-integrated with Sweden, Switzerland, china and Singapore, which encourage all shareholders, portfolio managers and investors to get the benefit of diversification.

**Table 5.7. Bivariate co-integration Oman stock exchange**

| | Eigenvalue | Statistic | Critical Value | Prob.** | Explanation |
|---|---|---|---|---|---|
| **Oman-Sweden** | 0.005728 | 4.014098 | 15.49471 | 0.9024 | NO-cointegration |
| | 0.0000647 | 0.044739 | 3.841466 | 0.8325 | |
| **Oman - Switzerland** | 0.004745 | 3.306717 | 15.49471 | 0.9512 | NO-cointegration |
| | 0.0000293 | 0.020223 | 3.841466 | 0.8868 | |
| **Oman - china** | 0.020036 | 16.88333 | 15.49471 | 0.0307 | NO-cointegration |
| | 0.004185 | 2.897798 | 3.841466 | 0.0887 | |
| **Oman - Singapore** | 0.005934 | 4.214785 | 15.49471 | 0.8855 | NO-cointegration |
| | 0.000148 | 0.102079 | 3.841466 | 0.7493 | |

Above table represents the bivariate co-integration relationship of OMAN (MSM 30) with selected major developed market. The result shows that OMAN (MSM 30) is not co-integrated with Sweden, Switzerland, china and Singapore. So investors have potential to make investment in OMAN (MSM 30) to take the advantage of diversification.

**Table 5.8. Bivariate co-integration Kenya stock exchange**

| | Eigenvalue | Statistic | Critical Value | Prob.** | Explanations |
|---|---|---|---|---|---|
| **Kenya-Sweden** | 0.005576 | 4.748923 | 15.49471 | 0.8349 | NO-cointegration |
| | 0.00128 | 0.884919 | 3.841466 | 0.3469 | |
| **Kenya – Switzerland** | 0.00874 | 9.526947 | 15.49471 | 0.3189 | NO-cointegration |
| | 0.004997 | 3.461238 | 3.841466 | 0.0628 | |
| **Kenya – china** | 0.009734 | 9.905461 | 15.49471 | 0.2881 | NO-cointegration |
| | 0.004543 | 3.146245 | 3.841466 | 0.0761 | |
| **Kenya – Singapore** | 0.002869 | 2.645854 | 15.49471 | 0.9806 | NO-cointegration |
| | 0.000956 | 0.660824 | 3.841466 | 0.4163 | |

Above table represent the bivariate co-integration relationship between Kenya (NSE 20) with selected major developed markets. The result reveals that NSE 20 not co-integrated with Sweden, Switzerland, china and Singapore.

## Granger causality:

| Null Hypothesis: | F-Statistic | Prob. |
|---|---|---|
| CHINA does not Granger Cause ARGENTINA | 0.78103 | 0.6196 |
| ARGENTINA does not Granger Cause CHINA | 2.09873 | 0.0339 |
| KENYA does not Granger Cause ARGENTINA | 0.56165 | 0.8096 |
| ARGENTINA does not Granger Cause KENYA | 1.43952 | 0.1765 |
| KSE_100 does not Granger Cause ARGENTINA | 2.42754 | 0.0137 |
| ARGENTINA does not Granger Cause KSE_100 | 4.30704 | 5.E-05 |
| OMAN does not Granger Cause ARGENTINA | 0.50506 | 0.8529 |
| ARGENTINA does not Granger Cause OMAN | 0.91241 | 0.5055 |
| SINGAPUR does not Granger Cause ARGENTINA | 21.7933 | 1.E-29 |
| ARGENTINA does not Granger Cause SINGAPUR | 1.14324 | 0.3319 |
| SWEDEN does not Granger Cause ARGENTINA | 19.2906 | 3.E-26 |
| ARGENTINA does not Granger Cause SWEDEN | 1.55105 | 0.1363 |
| SWITZERLAND does not Granger Cause ARGENTINA | 15.6387 | 3.E-21 |
| ARGENTINA does not Granger Cause SWITZERLAND | 1.77595 | 0.0787 |
| KENYA does not Granger Cause CHINA | 0.75250 | 0.6450 |
| CHINA does not Granger Cause KENYA | 1.86265 | 0.0631 |
| KSE_100 does not Granger Cause CHINA | 2.48316 | 0.0117 |
| CHINA does not Granger Cause KSE_100 | 2.94565 | 0.0030 |
| OMAN does not Granger Cause CHINA | 0.73718 | 0.6587 |
| CHINA does not Granger Cause OMAN | 1.36321 | 0.2094 |
| SINGAPUR does not Granger Cause CHINA | 2.57337 | 0.0090 |
| CHINA does not Granger Cause SINGAPUR | 0.59373 | 0.7835 |
| SWEDEN does not Granger Cause CHINA | 1.94984 | 0.0503 |
| CHINA does not Granger Cause SWEDEN | 1.49569 | 0.1551 |
| SWITZERLAND does not Granger Cause CHINA | 1.51078 | 0.1498 |
| CHINA does not Granger Cause SWITZERLAND | 1.81077 | 0.0720 |
| KSE_100 does not Granger Cause KENYA | 1.41036 | 0.1885 |
| KENYA does not Granger Cause KSE_100 | 1.36271 | 0.2096 |
| OMAN does not Granger Cause KENYA | 4.43440 | 3.E-05 |
| KENYA does not Granger Cause OMAN | 1.73623 | 0.0869 |
| SINGAPUR does not Granger Cause KENYA | 1.56386 | 0.1322 |
| KENYA does not Granger Cause SINGAPUR | 0.47153 | 0.8765 |
| SWEDEN does not Granger Cause KENYA | 0.27483 | 0.9741 |
| KENYA does not Granger Cause SWEDEN | 0.58314 | 0.7922 |
| SWITZERLAND does not Granger Cause KENYA | 0.64928 | 0.7363 |
| KENYA does not Granger Cause SWITZERLAND | 0.96985 | 0.4584 |
| OMAN does not Granger Cause KSE_100 | 1.29593 | 0.2424 |
| KSE_100 does not Granger Cause OMAN | 0.62276 | 0.7591 |
| SINGAPUR does not Granger Cause KSE_100 | 1.98812 | 0.0455 |
| KSE_100 does not Granger Cause SINGAPUR | 2.03545 | 0.0401 |

| | | |
|---|---|---|
| SWEDEN does not Granger Cause KSE_100 | 1.78962 | 0.0760 |
| KSE_100 does not Granger Cause SWEDEN | 2.16044 | 0.0287 |
| SWITZERLAND does not Granger Cause KSE_100 | 2.33972 | 0.0175 |
| KSE_100 does not Granger Cause SWITZERLAND | 1.68682 | 0.0982 |
| SINGAPUR does not Granger Cause OMAN | 0.81179 | 0.5923 |
| OMAN does not Granger Cause SINGAPUR | 0.52281 | 0.8398 |
| SWEDEN does not Granger Cause OMAN | 0.53984 | 0.8268 |
| OMAN does not Granger Cause SWEDEN | 0.37690 | 0.9330 |
| SWITZERLAND does not Granger Cause OMAN | 0.39623 | 0.9228 |
| OMAN does not Granger Cause SWITZERLAND | 0.21419 | 0.9884 |
| SWEDEN does not Granger Cause SINGAPUR | 3.90892 | 0.0002 |
| SINGAPUR does not Granger Cause SWEDEN | 1.77492 | 0.0789 |
| SWITZERLAND does not Granger Cause SINGAPUR | 3.66881 | 0.0003 |
| SINGAPUR does not Granger Cause SWITZERLAND | 1.38331 | 0.2003 |
| SWITZERLAND does not Granger Cause SWEDEN | 2.30097 | 0.0195 |
| SWEDEN does not Granger Cause SWITZERLAND | 3.38883 | 0.0008 |

The above table shows the result of Granger causality technique, which explore that frontier equity market of Argentina does not granger cause the stock return in other equity markets excepting China, which clearly conclude that just unidirectional causality exists when we move Argentina to China. On the other hand, frontier market of KSE does not granger cause the stock return in Argentina, china, Switzerland and Singapore. SWITZERLAND stock market does not granger cause the stock return in Singapore and Sweden. While SWEDEN does not Granger Cause in Switzerland.

**Table 5.9 Variance Decomposition of Argentina:**

| Period | S.E. | 0man | Argentina | Kenya | Kse100 | China | Singapore | Sweden | Switzerland |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.048499 | 0.031996 | 99.968 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0.049135 | 0.032069 | 97.46985 | 0.014613 | 0.174115 | 0.001376 | 0.640953 | 1.291866 | 0.375155 |
| 3 | 0.04917 | 0.033014 | 97.33177 | 0.01965 | 0.176776 | 0.011867 | 0.678929 | 1.333615 | 0.41438 |
| 4 | 0.049171 | 0.033018 | 97.32713 | 0.019649 | 0.176916 | 0.011876 | 0.679265 | 1.334722 | 0.417426 |
| 5 | 0.049171 | 0.033019 | 97.32676 | 0.019653 | 0.176918 | 0.011878 | 0.679291 | 1.334771 | 0.417706 |
| 6 | 0.049171 | 0.033019 | 97.32674 | 0.019653 | 0.176919 | 0.011878 | 0.679294 | 1.334773 | 0.417728 |
| 7 | 0.049171 | 0.03302 | 97.32673 | 0.019653 | 0.176919 | 0.011878 | 0.679294 | 1.334773 | 0.41773 |
| 8 | 0.049171 | 0.03302 | 97.32673 | 0.019653 | 0.176919 | 0.011878 | 0.679294 | 1.334773 | 0.41773 |
| 9 | 0.049171 | 0.03302 | 97.32673 | 0.019653 | 0.176919 | 0.011878 | 0.679294 | 1.334773 | 0.41773 |
| 10 | 0.049171 | 0.03302 | 97.32673 | 0.019653 | 0.176919 | 0.011878 | 0.679294 | 1.334773 | 0.41773 |

Above table show change in Argentina stock exchange explained by due to its own innovation and also tells that other frontier & developed stock exchanges have no effect on it if any change or fluctuation occurs in these markets.

**Table 5.10. Variance Decomposition of Kenya**

| Period | S.E. | 0MAN | ARGENTINA | KENYA | KSE_100 | CHINA | Singapore | SWEDEN | Switzerland |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.026831 | 0.931497 | 0.089805 | 98.9787 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0.027086 | 1.100944 | 0.088908 | 98.54293 | 0.002707 | 0.00031 | 0.245298 | 0.005661 | 0.013246 |
| 3 | 0.027092 | 1.106061 | 0.089736 | 98.52877 | 0.003128 | 0.00118 | 0.250264 | 0.007477 | 0.01338 |
| 4 | 0.027092 | 1.106247 | 0.089747 | 98.52844 | 0.003129 | 0.001184 | 0.250355 | 0.007477 | 0.01342 |
| 5 | 0.027092 | 1.10625 | 0.089748 | 98.52843 | 0.003129 | 0.001185 | 0.250356 | 0.007478 | 0.013421 |
| 6 | 0.027092 | 1.10625 | 0.089748 | 98.52843 | 0.003129 | 0.001185 | 0.250356 | 0.007478 | 0.013421 |
| 7 | 0.027092 | 1.10625 | 0.089748 | 98.52843 | 0.003129 | 0.001185 | 0.250356 | 0.007478 | 0.013421 |
| 8 | 0.027092 | 1.10625 | 0.089748 | 98.52843 | 0.003129 | 0.001185 | 0.250356 | 0.007478 | 0.013421 |
| 9 | 0.027092 | 1.10625 | 0.089748 | 98.52843 | 0.003129 | 0.001185 | 0.250356 | 0.007478 | 0.013421 |
| 10 | 0.027092 | 1.10625 | 0.089748 | 98.52843 | 0.003129 | 0.001185 | 0.250356 | 0.007478 | 0.013421 |

Above Table shows change in Kenya stock exchange explained by due to its own innovation and also tells that other developed & developing stock exchanges have no effect on it if any change or fluctuation occurs in these markets.

**Table 5.11. Variance decomposition of KSE100**

| Period | S.E. | 0MAN | ARGENTINA | KENYA | KSE_100 | CHINA | Singapore | SWEDEN | Switzerland |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.033021 | 0.000338 | 0.201954 | 0.011011 | 99.7867 | 0 | 0 | 0 | 0 |
| 2 | 0.033836 | 0.051523 | 1.244051 | 0.253927 | 97.64739 | 0.178894 | 0.024304 | 0.472875 | 0.127035 |
| 3 | 0.033875 | 0.059767 | 1.280689 | 0.306769 | 97.5072 | 0.204338 | 0.02777 | 0.482418 | 0.131046 |
| 4 | 0.033876 | 0.060281 | 1.281665 | 0.309471 | 97.50124 | 0.204482 | 0.028727 | 0.483087 | 0.131047 |
| 5 | 0.033876 | 0.060314 | 1.281675 | 0.309634 | 97.50099 | 0.204494 | 0.028745 | 0.48309 | 0.131057 |
| 6 | 0.033876 | 0.060316 | 1.281676 | 0.309641 | 97.50098 | 0.204494 | 0.028747 | 0.483091 | 0.131057 |
| 7 | 0.033876 | 0.060316 | 1.281676 | 0.309641 | 97.50098 | 0.204495 | 0.028747 | 0.483091 | 0.131057 |
| 8 | 0.033876 | 0.060316 | 1.281676 | 0.309641 | 97.50098 | 0.204495 | 0.028747 | 0.483091 | 0.131057 |
| 9 | 0.033876 | 0.060316 | 1.281676 | 0.309641 | 97.50098 | 0.204495 | 0.028747 | 0.483091 | 0.131057 |
| 10 | 0.033876 | 0.060316 | 1.281676 | 0.309641 | 97.50098 | 0.204495 | 0.028747 | 0.483091 | 0.131057 |

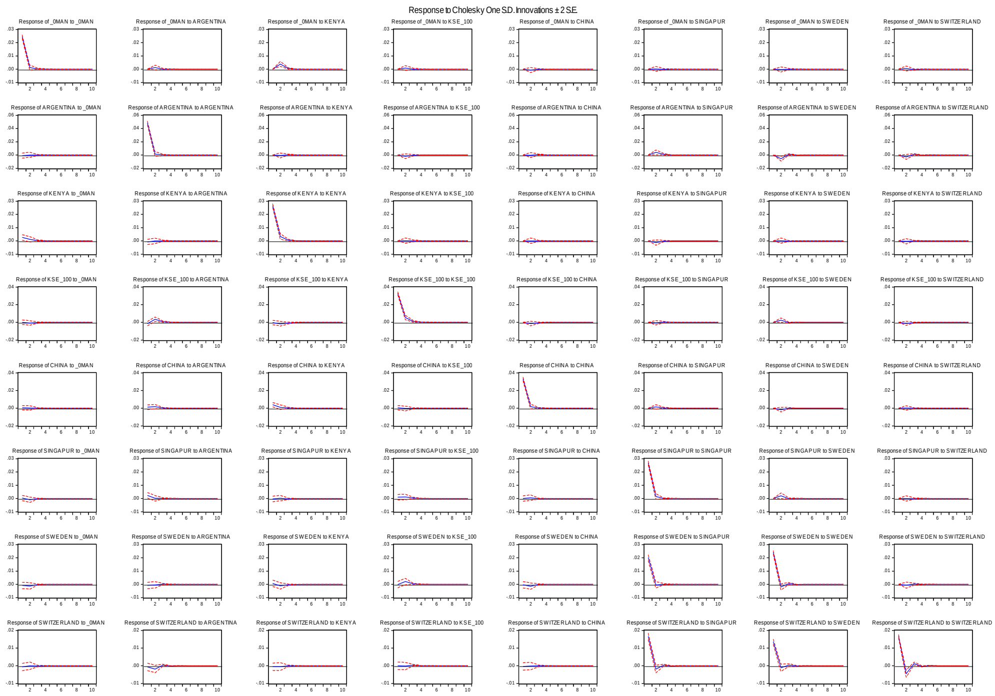
Above Table shows change in KSE stock exchange explained by due to its own innovation and also tells that other developed & developing stock exchanges have no effect on it if any change or fluctuation occurs in these markets.

**Table 5.12. Variance decomposition of OMAN (MSM 3O):**

| Period | S.E. | OMAN | ARGENTINA | KENYA | KSE_100 | CHINA | Singapore | SWEDEN | Switzerland |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.024666 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0.025047 | 97.17052 | 0.215842 | 2.371608 | 0.096049 | 0.088967 | 0.005227 | 0.011227 | 0.040558 |
| 3 | 0.025056 | 97.10904 | 0.216267 | 2.416374 | 0.096797 | 0.089359 | 0.009398 | 0.011219 | 0.051544 |
| 4 | 0.025056 | 97.10715 | 0.216501 | 2.417315 | 0.096999 | 0.089434 | 0.009398 | 0.011223 | 0.051982 |
| 5 | 0.025056 | 97.10707 | 0.216501 | 2.417329 | 0.096999 | 0.089435 | 0.009408 | 0.011223 | 0.052032 |
| 6 | 0.025056 | 97.10707 | 0.216501 | 2.41733 | 0.097 | 0.089435 | 0.009408 | 0.011223 | 0.052035 |
| 7 | 0.025056 | 97.10707 | 0.216501 | 2.41733 | 0.097 | 0.089435 | 0.009408 | 0.011224 | 0.052036 |
| 8 | 0.025056 | 97.10707 | 0.216501 | 2.41733 | 0.097 | 0.089435 | 0.009408 | 0.011224 | 0.052036 |
| 9 | 0.025056 | 97.10707 | 0.216501 | 2.41733 | 0.097 | 0.089435 | 0.009408 | 0.011224 | 0.052036 |
| 10 | 0.025056 | 97.10707 | 0.216501 | 2.41733 | 0.097 | 0.089435 | 0.009408 | 0.011224 | 0.052036 |

Table shows change in OMAN stock exchange explained by due to its own innovation and also tells that other developed & developing stock exchanges have no effect on it if any change or fluctuation occurs in these markets.

## Impulse Response:



Impulse response function explains the changes in standard deviation. Resutls shows the response of KSE to the changes in the developed equity markets. However, results of Impulse Response Function shows that Argentina returns are not influnced by the shocks in the other marekts.

## 6. Conclusion

The main objective of every study is to give direction to the readers. This study is conducted between frontier equity markets and developed equity markets. Both the types of stock markets have different economic, social and geographic conditions.so it may be possible that the economic environment for the investors of these countries is different and same is the case political conditions.

The purpose of this study to relationship among frontier equity markets of Pakistan, Argentina, Kenya, Oman, and developed equity markets including Sweden, Switzerland, China, Singapore for the period 1st week of January-2000 to last week of January/2014. The aim of this study is to investigate whether the co movement or integration exists among these stock markets or not because co movement is very important for the investors. The results of this study reveals that frontier market of Argentina is riskier and high return market, showing a behavior of more volatile market as compared to all other selected markets in the study, which is a best opportunity for local and foreign investors to minimize risk. The correlation analysis indicates that selected frontier markets (Pakistan, Oman, Argentina, Kenya) are weakly correlated with developed country stock markets. This study assists the investor or portfolio managers to enhance the returns by diversifying the unsystematic risk at given level of profit. For this purpose, augmented fuller (ADF) and Phillips-Perron techniques are used for stationary of data at similar order by applying on prices of stock return. Multivariate co integration is applied which indication of three equation of integration among stock markets. Later on bivariate co-integration results confirm that all frontier equity markets indicate no long run relationship with any developed markets. The finding of granger cause explore that frontier equity market of Argentina does not granger cause the stock return in other equity market of China, which clearly conclude that just unidirectional causality exists when we move Argentina to China. The results of vector decomposition designate that change in frontier markets (Argentina, Pakistan, Kenya, Oman) explained by due to its own innovation and other developed & developing stock exchanges have no effect on it if any change or fluctuation occurs in these markets.

This study will suggest the investors or portfolio managers to invest across the border in those equity markets which are different to each other economically and politically. In this way the portfolio managers may be able to attain optimum diversified portfolio and also minimize the country risk.

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