Axiomatic Analysis of the Semi-Fuzzy Poverty Indices MIf and PGf

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Abstract: Every poverty index can be classified into one of the two major classes; classical indices and fuzzy indices; except for the semi-fuzzy poverty indices such as PGf and MIf which hybridize between the theory of classical sets and that of fuzzy sets, which makes their axiomatic analysis very special since it uses both classical and fuzzy mathematical tools. In order to better exploit and characterize the PGf and MIf indices, we propose in this paper an axiomatic analysis by mathematically demonstrating, on the one hand, the satisfaction of these two indices of a set of axioms most desirable by economists, which shows their performance in describing poverty. On the other hand, we discuss their limits according to three axioms that we demonstrate in order to improve the formula of these semi-fuzzy indices of poverty.

Keywords: Poverty measure; fuzzy set theory; confidence intervals; semi fuzzy poverty indices PG_f and MI_f ; axiomatic analysis

JEL Classification: I32

1. The Poverty Measure: A Scientific Challenge to the Development of More Effective Measures

Poverty is a socio-economic phenomenon faced by all nations of the world, starting from the marginalization and social exclusion in developed countries and arriving at hunger and death in very poor countries. It is a plague causing itself other terrible problems such as crime, prostitution, selling drugs, migration, terrorism, which aggravate increasingly health status, levels of economy, social, education and therefore deepen the poverty of these populations. It is a circle continuously extended to include more and more poor and worsens worse and worst living conditions.

Thus, the fight against poverty is a priority for all countries of the world, seen that the poverty of undeveloped countries has consequences that reach even indirectly developed countries, migration, the spread of disease and deadly viruses, terrorism,

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Or fight against poverty requires the implementation of a set of policies to improve the living standards of the poor, what remains a difficult task if we do not determine up front the real need of this target population.

For this, the researchers company also has contributed to the fight against poverty for many years, by developing several poverty indexes as quantitative analytical instruments that reflect the reality accordance with conditions of the poor, to optimize time and resources invested and establish best results.

2. Evolution of Poverty Indexes: From Basic Indices to Multidimensional and Complex Indices

The first of poverty indices that have been proposed is the Headcount ratio, denoted H, which represents the proportion of poor compared to the total population (Notes techniques, 2002), then the index Income gap ratio, denoted I, which is defined as the mean distance separating the poor from the poverty line (Notes techniques, 2002). These two indexes are the simplest and easiest to evaluate, and also remain the most used by several governments and international organizations as first poverty assessment tools of a given population. But after formalizing the study of aggregation of poverty by economists, several criticisms of both indexes were evaluated (Sen, 1976). By following, several indices and poverty measures have been proposed that we can assign them into two classes, the first is classic and the second is fuzzy.

- Class of classical approaches:

These are all based on the following hypothesis:

"it is possible to delimit poverty and thus to identify the poor by determining a poverty line" (Deaton, 2005; Hagenaars, 1986; Meyer & Sullivan, 2003; McKinnish, 2005).

Using the classical mathematics logic, the concept of these approaches is to declare that a person is poor compared to an attribute if the realization of this attribute is below a fixed threshold, said *line* or *poverty threshold*. Mathematically this is reflected by the definition of a deprivation function $\varphi(x_{ij}, z_j)$, (Delhausse, 2002, p. 55) (Bertin, 2007) such as:

$$\varphi(x_{ij}, z_j) = \begin{cases} 1 \;\; si\; x_{ij} \geq z_j \;\rightarrow\; non\; privation \\ 0 \;\; si\; x_{ij} < z_j \;\rightarrow\; privation \end{cases}$$

Where x_{ij} is the level of functioning carried out by the individual i for the attribute j, and z_j is the deprivation threshold for the attribute j.

As an example, there are several indices such as index H and I cited above, as well as Sen index, Thon index, FGT index, Clark, Hemming et Ulph index, Kakwani index which is among the generalized poverty indicators, since it is a generalization of the FGT, Sen, Tsui indexes, the human poverty index IPH… and the list is still open to new indexes more performing.

- Class of fuzzy approaches:

This class of measures refuses hypothesis seen above that there is not a threshold or line of poverty unanimously adapted by the various classical approaches, also it is difficult to accept that the passage of a state of poor to non-poor is brutal, because of some differences milimes in income for example. Thus, a fuzzy approach models poverty as a state of an individual who has a depth (level of poverty) and not a characteristic that an individual has.

Fuzzy approaches include fuzzy mathematical logic, or the fuzzy sets theory, to address these deficiencies cited in the first class of approach. Indeed, it consists in the adaptation of a membership function μ such that:

$$\mu_B\big(X_j(a_i)\big) = \begin{cases} 1 \quad \textit{does not possess the attribute } j\ (a_i \in B \textit{ certainly}) \\ x_{ij}\ ; 0 < x_{ij} < 1\ \textit{possesses partially the attribute } j\ (a_i \in B \textit{ partially}) \\ 0\ \textit{possesses the attribute } j\ (a_i \textit{ non poor}:\ a_i \notin B \textit{ certainly}) \end{cases}$$

- $X_j(a_i)$ represents the realization of a poor individual a_i in terms of the attribute j (or also the indicator j).

In other words, the value of the membership function μ to the fuzzy subset B of the i[th] individual (i = 1,2, ... n) relative to the j[th] attribute (j = 1,2, ... m) is defined as next:

$$x_{ij} = \mu\left(X_j(a_i)\right)\ ;\ 0 \le x_{ij} \le 1$$

Where:

- $x_{ij} = 1$ if the i[th] individual does not possess the j[th] attribute;
- $x_{ij} = 0$ if the i[th] individual possess the j[th] attribute;
- $0 < x_{ij} < 1$ if the i[th] individual possess the j[th] attribute with an intensity between 0 and 1.

In this context, several indices have been developed such that the index of Cerioli and Zani 90 followed by Cheli and Lemmi 95, Belhadj B. in 2005 and the list of these indices is still more enriched by new ones.

As part of the two approaches of poverty, the indexes have evolved in the growing sense of performance and credibility of indexes. In fact, the construction of these indexes has passed through two main phases that have contributed to this development:

- The first phase: it was designed to provide a picture of the proportion or distribution of the poor compared to the overall studied population through global indexes (indices H and I).

- The second phase: through reproaches and critical analysis of the imperfections of the existing indexes, we could make improvements and modifications to some of these indexes to exceed their deficiencies. This prompted the researchers to establish axiomatic approaches, each of which rests on one or more axioms that we find essential in a poverty index. These axioms will be subsequently as standards for the qualification or not of a poverty index. Thereby we continue to construct a general axiomatic framework of poverty indexes that does not cease to include new axioms until now.

3. Axioms: A Means of Characterizing Poverty Indexes

The axiomatic approach was first founded by Sen. Indeed, to construct his measure, Sen proposes to satisfy a set of ethical and moral principles characterizing the population of the poor, that he translated into axioms that a good index must satisfy (Sen, 1976). Then, several researchers have adapted the same principle to construct more efficient indices, introducing new axioms, thus good indicators satisfy most of axioms and especially those most desirable by economists.

Among all the axioms that a poverty index must satisfy, we find the following list, the two first are those proposed by Sen:

- ***Monotony axiom:*** All things being equal, a reduction in the income of a person who is below the poverty line should increase the poverty measure.

This axiom has been created on the basis of a critique of the H index that does not satisfy this axiom despite its obviousness.

- ***Transfer axiom:*** All things being equal, a transfer of income between a person who is below poverty line and someone who is richer must increase the poverty measure.

- ***Axiom of continuity***: the poverty measure should not be very sensitive to a marginal variation of the quantity of an attribute.

- ***Symmetry axiom or anonymity:*** it characteristics other than the attributes used to define poverty does not affect the measurement of poverty.

- ***Transfer sensitivity axiom:*** All things being equal, a regressive transfer of an amount *w* of the i^{th} to the j^{th} poor cause a greater increase in the poverty measure than a regressive transfer of the same amount from the k^{th} to the l^{th} poor if:

$$y_j - y_i = y_l - y_k > 0 \quad and \quad y_k > y_l$$

Such as y_i is the income of individual i.

This axiom established that aggregate poverty increases with a regressive transfer, and that more people involved in this transfer are poorer, more increasing the poverty level will be high. It therefore gives greater importance to transfers made between the poorest people.

- ***Decomposition axiom:*** Let be a population consisting of m groups, each group containing n_j individuals $\left(\mathrm{j} = 1,2.., \mathrm{m} \text{ and} \sum_{\mathrm{j}=1}^{\mathrm{m}} \mathrm{n_j} = \mathrm{n} \right)$

If we note *P* aggregate measure poverty calculated on the entire population and P_j which is calculated on the j^{th} group, then:

$$P = \sum_{j=1}^{m} \frac{n_j}{n} P_j$$

In other words, the aggregate poverty of the entire population is a sum of the aggregate poverty for all groups weighted by the share of each group $\left(\frac{n_j}{n}\right)$ in the total population.

The impact of poverty's variation of a group on total poverty increases with the number of persons forming this group.

- ***Axiom of the population's principle:*** If an attribute matrix is replicated several times, then overall poverty remains unchanged.

- ***Axiom of the invariance to the scale's variations***: The poverty measure is homogeneous with a degree 0 with respect to X and Z, where Z is the threshold vector.

- ***Axiom of concentration:*** The poverty measure is unchanged if an attribute j increases for an individual i characterized by $x_{i,j} \geq Z_j$. ($x_{i,j}$ is the value of attribute j for individual i).

- ***Axiom of monotonicity:*** The measure of poverty decreases, or does not increase following an improvement in one of the attributes of a poor.

In the following of this work, we recall first the semi fuzzy index and semi fuzzy vector of poverty, and then we present an axiomatic analysis showing the advantages and limitations of these indices.

4. Semi Fuzzy Index PG_f and Semi Fuzzy Vector MI_f of Poverty

We recall in this section the construction and the general formula of PG_f and MI_f semi fuzzy indices. To do this:

Let μ_B be a membership function chosen by the decision maker to integrate different criteria, that he finds necessary to measure poverty in a given population Ω.

Let Y_{qf} the total income of all the poor in population determined by the membership function μ_B, where:

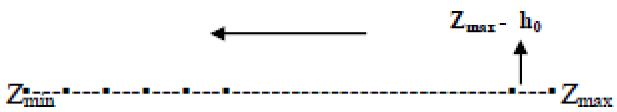
$$Y_{qf} = \sum_{i=1}^{qf} y_i \quad : \quad y_i \; is\ the\ income\ of\ the\ individual\ i$$

With:

$$q_f = Cardinal(B) \;\; where \;\; B = \{i \in \Omega : \mu_B(i) > 0\}$$

Let $[\mathrm{Zmin}, \mathrm{Zmax}]$ a confidence interval (Belhadj & Matoussi, 2007), as Z_{min} is the minimum value that is desired to take the poverty line, and Z_{max} is its maximum value (Ravallion, 1994; Ravallion, 2003).

Consider $n \in \mathbb{N}^*$ the order of the discretization of the confidence interval [Zmin, Zmax], and $(h_1, h_2, h_3, h_4, \ldots, h_n) \in \mathrm{IR}_+^{n*}$ steps of this discretization. These steps h_i express the differences that the expert considers reasonable, to measure income degradation, as is known to the evaluation and devaluation of wages.



A first step in our index construction process consists in a Euclidean division of Yqf by Zmax, which gives us:

$$Y_{qf} = a_0(Z_{max} - h_0) + r_0 \;\; where \; 0 \le r_0 < Z_{max}$$

If $Z_{max} - h_0 < r_0$, we still perform the following division:

$r_0 = a_1(Z_{max} - h_0) + r_1 \;\; where \;\; 0 \le r_1 < Z_{max} - h_0$

Furthermore, if $Z_{max} - h_1 < r_1$, we can write:

$$r_1 = a_2(Z_{max} - h_1) + r_2 \;\; where \;\; 0 \le r_2 < Z_{max} - h_1$$

If $Z_{max} - h_m < r_m$, we can write:

$r_{m-2} = a_{m-1}(Z_{max} - h_{m-2}) + r_{m-1}$

Until last division we can perform if $Z_{min} < r_{m-1}$

$r_{m-1} = a_m Z_{min} + r_m$

From the first division, we have a_0 persons supposed to live with an income Zmax. Similarly, according to the second Euclidean division, there is a_1 persons assumed to have an income ($Z\max - h0$), so on until the last equality that explains the existence of a_m 9 persons supposed to live on an income Zmin, the rest of the population of q_f poor is $\left(q_f - (a_0 + a_1 + .. + a_m)\right)$ persons supposed to live with an income near to zero, and we note that the set B^*. So we get the construction of $m + 1$ subpopulations of poor forming a disjointed recovery of the poor population B, where each requires special treatment. Consequently, the class B of the poor is decomposed into disjoint union of the following sets:

$$B = \dot{\bigcup_{i=0}^{m}} B_{ai} \dot{\bigcup} B^*$$

The choice of steps and the order of the discretization depends on the extent of the interval $[Zmin, Zmax]$ selected at the beginning, as it also depends on the description and the meaning associated with each terminal Zi such that:

$$Z_i = Z_{max} - h_i \;\; : \;\; i \in \{1,2,3, \dots m\}$$

If we choose a fixed discretization's step:

$$h_i = i.h \;\; where \;\; i \in \{1,2,3, \dots m\}$$

Classes will be equidistant, but with different cardinals according to data from the studied population. Therefore, we obtain a vector MI_f defined by:

$$MI_f = \begin{pmatrix} I_1 \\ I_2 \\ \dots \\ I_m \end{pmatrix}$$

Where each component Ij $(j = 1,2 \dots m)$ is determined by:

$$I_j = \frac{q_f - \sum_{k=1}^{j} a_k}{n}$$

With a_k $(\mathrm{k} = 1, \dots \mathrm{j})$ the values obtained by the above process.

Note that $PG_f = I_m$ is the last component of the vector MI_f.

By construction, indices Ij $(j = 1,2 \dots m)$ are decreasing in the sense that we pass from the calculation of Ij to I_{j+1} by:

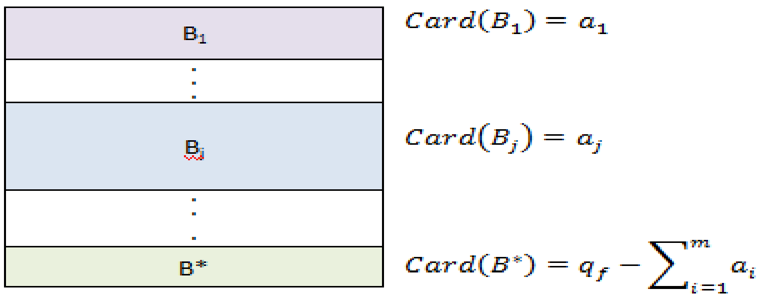
$$I_j - I_{j+1} = \frac{a_{j+1}}{n} \geq 0$$

which represents the weight of the $(j+1)^{th}$ set B_{j+1}relative to the entire population, thus, we have built a system of weights giving the thickness of each subset of poor.

The last class B^* is a particular class as it represents the misery in the studied society, characterized by:

$$PG_f = \frac{q_f - \sum_{i=1}^{m} a_i}{n}$$

This index reflects the weight of people living misery in the studied population.



Poverty classes of a population Ω

An example of a case of four classes is detailed in (Fikri, El Hilali Alaoui & El Khomssi, 2012).

5. Axiomatic Analysis of Semi Fuzzy Index and Semi Fuzzy Vector of Poverty

The introduction of axioms allowed to characterize poverty indicators through the validation of properties clearly explained. Indeed, this approach represents an indicator verification tool for a number of social and economic properties of the poor. Thus, the more an indicator verifies more axioms, the more this indicator is reliable. Consequently, researchers tend to build new indicators based on the maximum satisfaction of regarded axioms.

In this section, we will demonstrate the validation of a set of axioms by the semi fuzzy vector MI_f, by restricting demonstrations to four classes, because the general case is a simple extension of the case of four classes.

In a first axiomatic analysis of the indices "PG_f" and vector "MI_f" semi fuzzy of poverty has allowed us to confirm the satisfaction of the following axioms:

1) **Focus axiom**; (Fikri, El Khomssi & Saoud, 2011)

2) **Axiom of monotony**; (Fikri, El Khomssi & Saoud, 2011)

3) **Transfer axiom**; (Fikri, El Khomssi & Saoud, 2011)

In our following axiomatic analysis, we consider the following data:

Ω is a study population, containing n individuals.

We consider that an individual *i* has an income noted $xi \in$ D such as $1 \leq x2 \leq \cdots \leq xn$; and D the set of values that can take the income, with $D \subset IR^+$.

Income distribution of all individuals is denoted $x = (x1, x2, \ldots, xn)$.

In the following, we consider μ_p the membership function selected[1] and defined on Ω, and **B** the subset of poor defined by:

$B = \{ i \in \Omega : \mu_p(i) > 0 \}$

B is also said support of the membership function μ_p.

We note q_f the number of poor in the distribution x (also of Ω) such as:

q_f = Cardinal(B)

Let $Y_{qf} = \sum_{i \in B} x_i$ total of poor incomes Ω,

and $Z_B = (Z_{max} + Z_{min})/2$ with $[Z_{max}; Z_{min}]$ confidence interval considered.

We note [m] the whole part of positive real m.

Let a, b and c natural integers, and r1,r2 and r3 in $\mathbf{IR}^+$ such as:

$$Y_{qf} = a.Z_{max} + r1 \; with \;\; 0 \leq r1 < Z_{max} \; (1)$$
$$r1 = b.Z_B + r2 \;\; with \;\; 0 \leq r2 < Z_B \quad (2)$$
$$r2 = c.Z_{min} + r3 \;\; with \;\; 0 \leq r3 < Z_{min} \; (3)$$

4) **Axiom of symmetry**: permutation between the incomes of two individuals does not influence the measurement of poverty.

Indeed, given a distribution $x = (x1, x2, \ldots, xn)$ income of all individuals. Permutation between two elements of x does not impact the values of the

[1] For the choice of the membership function specialists can make their choice according to the dimensions they want to integrate (income, illiteracy, wellness ...).

membership function considered[1] , seeing that this one depends on the values *xi* and not their round.

Consequently, the value of $Y_{qf} = \sum_{i \in B} x_i$ it does not change, and also the values of the components of the vector MI_f and PG_f indices do not change.

5) **Axiom of homogeneity**: a multiplication by a positive constant for all incomes of x and for the poverty line z, does not impact $P(z; x)$.

To justify this property, we consider a distribution $x = (x_1, x_2, . ., , x_n)$ of the population Ω.

Let *x'* be the distribution obtained by multiplying the elements of *x* by a positive number *k* nonzero.

The same for the confidence interval $[Z_{min}, Z_{max}]$ substituted by the interval $[kZ_{min}, kZ_{max}]$.

Before verifying the sensitivity of our semi-fuzzy indexes to the multiplication, we note that the classification in poor and non-poor with the first distribution is the same for the second distribution. Indeed, we are left with two possibilities:

- **First case**, if the membership function is not based in its formula only on income, then the problem is simple because the degree of membership of x_i in the interval$[Z_{min}, Z_{max}]$, is the same as that of membership of kx_i to $[kZ_{min}, kZ_{max}]$ seeing that all the function values are included between 0 and 1. For example, Belhadj in (Belhadj, 2005) proposed the following membership function based on x_i the income or expenses of the i^{th} household as a dimension of poverty:

$$\mu_Q(i) = \begin{cases} 1 & if \quad 0 < x_i < Z_{imin} \\ \dfrac{-4}{2Z_{imax} - Z_{imin}} x_i + \dfrac{4Z_{imax}}{2Z_{imax} - Z_{imin}} & if \quad Z_{imin} \le x_i < Z_{imax} \\ 0 & if \quad x_i \ge Z_{imax} \end{cases}$$

By multiplying all the elements of the distribution *x* with a positive *k* and considering the interval $[kZ_{min}, kZ_{max}]$ we will have:

a) If $0 < kx_i < kZ_{imin}$ *then* $0 < x_i < Z_{imin}$ *thus* $\mu_Q(i) = 1$;

b) If $kZ_{imin} \le kx_i < kZ_{imax}$ then $Z_{imin} \le x_i \le Z_{imax}$ therefore;

[1] The choice is free for the membership function.

$$\mu_Q(i) = \frac{-4}{2kZ_{imax} - kZ_{imin}} kx_i + \frac{4kZ_{imax}}{2kZ_{imax} - kZ_{imin}}$$
$$= \frac{-4}{2Z_{imax} - Z_{imin}} x_i + \frac{4Z_{imax}}{2Z_{imax} - Z_{imin}}$$

c) If $kx_i \geq kZ_{imax}$ then, $x_i \geq Z_{imax}$ which implies $\mu_Q(i) = 0$.

Thus $\mu_Q(i)$ the degree of membership of an individual *i* in the sub-population of the poor remains unchanged if we multiply the income of all individuals and the thresholds of the confidence interval by the same positive number.

- Second case, when the selected membership function includes several attributes when calculating the degree of membership (Multidimensional Poverty), such as income, health, education. In this case if there is a scale that allows the homogenization of new incomes with other dimensions, then the fuzzy set of the poor does not change. If not, this set of poor can be changed according to the weight of each of the dimensions considered in the formula of the membership function.

In cases where the sub fuzzy B of the poor remains invariant with respect to the new distribution *x'*, the calculation of our semi fuzzy indices for this new distribution gives:

Total income of the poor is $Y'_{qf} = \sum_{i \in B} x'_i$

That is to say: $Y'_{qf} = \sum_{i \in B} kx_i = k.\sum_{i \in B} x_i = k.Y_{qf}$

Thus: $Y'_{qf} = k.Y_{qf}$

Subsequently equations (1), (2) and (3) obtained for the distribution *x* become for the new distribution *x'* as follows:

$$Y'_{qf} = k.Y_{qf}$$
$$Y'_{qf} = k.(a.Z_{max} + r_1)$$
$$Y'_{qf} = a.(kZ_{max}) + r'_1 \qquad (*)$$
$$with\, r'_1 = k.r_1 \;\; and \;\; 0 \leq r'_1 \leq kZ_{max}$$

Euclidean division of r'_1 by kZ_B gives:

$$r'_1 = k.r_1$$
$$r'_1 = k.(b.Z_B + r_2)$$
$$r'_1 = b.(k.Z_B) + r'_2 \quad (**)$$
$$with\, r'_2 = k.r_2 \;\; and\; 0 \leq r'_2 \leq kZ_B$$

A new euclidean division of r'_2 by kZ_{min} gives us:

$$r'_2 = k.r_2$$
$$r'_2 = k.(c.Z_{min} + r_3)$$
$$r'_2 = c.(k.Z_{min}) + r'_3 \quad (***)$$
$$with\ r'_3 = k.r_3 \quad and\ 0 \le r'_3 \le kZ_{min}$$

According to equations (*), (**) and (***), we remark that the results of the Euclidean divisions of the new values $Y'_{qf};\ r'_1\ and\ r'_2$ using the new values $kZ_{min}; kZ_B\ \ and\ kZ_{max}$ of the new corresponding confidence interval, are exactly « a », « b » and « c » the results of Euclidean division in equations (1) , (2) and (3) corresponding to the distribution *x*.

We therefore conclude that the components of the MI_f vectors and the PG_f index well respect the homogeneity property if the appropriate membership function considers income as a single attribute, where if the membership function measures multidimensional poverty with a formula invariant with respect to the multiplication of revenue by a positive non-zero.

6) **Axiom for Standardisation**: *A measure is "normalized" when it takes a special value to indicate that there is no poverty.*

Generally, it said that a measure is normalized when:

If no one live if no one lives below the poverty threshold for a given threshold z then the measure is null: P(x; z) = 0.

Indeed, in cases where all individuals in the population Ω are above Z_{max}, then the fuzzy set B is empty, as a result:

q_f=Card(B) = 0 and $Y_{qf} = \sum_{i=1}^{qf} x_i = 0$

Considering the equations (1), (2) and (3),

Since $Y_{qf} = 0$ then all the numbers a, b and c are zero.

Consequently: $I_1 = \frac{q-a}{n} = 0$ and similarly for I2, I3 and PG_f found that they are all null.

Thus all components of our vector MI_f and the semi-fuzzy index PG_f respect the normalization axiom.

Reciprocally: if the MI_f vector is null i.e. that I_1=I_2=I_3=0

From the expression of I_1 : If I_1=0 then $q = a$,

But « a » is defined as the number of poor people supposed to live with an income Z_{max}

In this case, we have « q » poor people supposed to live with an income Z_{max}

In other words, all the poor are supposed to live with an income Z_{max} ,

i.e $x_i \geq Z_{max} \; \forall \; i \; \in B$

Absurd. Hence, the set B of poor is empty.

Note: Since all the other I_2 , I_3 and PG_f indices are always lower than I_1 (Fikri, El Khomssi & Saoud, 2011), so just to have I_1=0 so that to such indices are zero.

6. Limits of the PG_f Index and the Semi-Fuzzy Vector MI_f

In the remainder of this section, we demonstrate a set of axioms not validate by the semi fuzzy indices PG_f and MI_f, this deficiency will be the first step towards improving the formulation of these two semi fuzzy indices.

6.1. Axiom of Independence

Consider two distributions *x and y* presenting the same poverty level in the sense of the indicator *P* for a given poverty line *z*. If the two distributions in question have a common part so the poverty level within the meaning of *P* for the threshold *z*, is equal to the distributions x and y without their common part.

The PG_f index and components of MI_f vector do not validate this axiom.

Indeed, let be:

Z_{max}=8 the poverty line

a distribution $x = (x_1, x_2, . ., x_{k-1}, x_k, x_{k+1}, \ldots . x_{k+l}, x_{k+l+1}, . . , x_n)$

a distribution $y = (y_1, y_2, .., y_{k-1}, x_k, x_{k+1}, \ldots . x_{k+l}, y_{k+l+1}, \ldots, y_n)$.

Such as :

$Y_x = \sum_{i \in B_x} x_i = 53 \quad and \quad Y_y = \sum_{i \in B_y} y_i = 162$

With B_x (resp. B_y) is the fuzzy set of the poor of the distribution *x* (resp.*y*) whose Cardinal qx=10 (resp. qy=24).

Suppose E=(14 ; 6) is the common part between the two distributions x and y.

So :

$F = \sum_{i \in E} x_i = 20$ the total income of poor individuals belonging to the common part between the two distributions.

The total income of the poor distribution is: $Y_x = \sum_{i \in B_x} x_i = 53$

So we have the following calculation:

$$Y_x = 6 \times Z_{max} + 5$$
$$therefore \quad a_x = 6 \;\; et \; r_x^1 = 5$$

Hence

$$I_1^x = \frac{q_x - a_x}{n} = \frac{10-6}{n} = \frac{4}{n}$$

For distribution *y*, we proceed in the same way and we find:

$$I_1^y = \frac{q_y - a_y}{n} = \frac{24-20}{n} = \frac{4}{n}$$

Hence we have: $I_1^x = I_1^y$

However, let x*'* (resp. *y'*) the distribution obtained from *x* (resp. *y*) with extraction of the common part.

The new value of the total of incomes of the poor of the distribution *x'* is :

$$Y_{x\prime} = \sum_{i \in B_{x\prime}} x_i = 53 - 20 = 33$$

Euclidean division by $Z_{max} = 8$ give:

$$Y_{x\prime} = 4 \times Z_{max} + 1$$
$$hence \quad a_{x\prime} = 4 \;\; and \;\; r_{x\prime}^1 = 1$$

Hence,

$$I_1^{x\prime} = \frac{q_{x\prime} - a_{x\prime}}{n-2} = \frac{(10-2)-4}{n-2} = \frac{4}{n-2}$$

Similarly for distribution *y'*, we find:

$$I_1^{y\prime} = \frac{q_{y\prime} - a_{y\prime}}{n-2} = \frac{(24-2)-17}{n-2} = \frac{5}{n-2}$$

Therefore : $I_1^{x\prime} \neq I_1^{y\prime}$

Through this against-example, we can conclude that the component I_1 of the vector MI_f does not respect the independence axiom.

Similarly we can prove that the other components as well as the index PG_f does not meet this axiom.

6.2. Invariance Axiom by Replication

An index of poverty P respects this axiom if:

Given a distribution $x=(x_1,x_2,\ldots,x_n)$, For any replication y having an order k of x (i.e $y = \underbrace{(x, x, x, \ldots \ldots x)}_{\text{k times}}$ with $k \in IN^* - \{1\}$ and for a fixed threshold z we have : P(x,z)=P(y,z).

Considering the hypotheses of the axiom, and by noting:

$Y_y = \sum_{i \in B'} x_i$ the total of incomes of the poor of the distribution y.

Euclidean division of this number by the threshold $Z_{\max}$ gives:

$$Y_y = \sum_{i \in B'} x_i = \sum_{i \in B} x_i + \sum_{i \in B} x_i \ldots \ldots \sum_{i \in B} x_i \quad (\, k\ times)$$

Hence the following calculation :

$$Y_y = k. \sum_{i \in B} x_i = k. Y_x$$

$$Y_y = k.(a. Z_{max} + r_1) \quad with\, r_1 < Z_{max}$$

$$Y_y = (k.a). Z_{max} + k. r_1 \quad with\, r_1 < Z_{max}$$

Since $r_1 < Z_{max}$ and $k \in IN^* - \{1\}$ so there are two possible cases:

$k. r_1 < Z_{max} \quad else \quad kr_1 \geq Z_{max}$

In the case where $kr_1 \geq Z_{max}$ we can write:

$$kr_1 = \alpha. Z_{max} + \beta \quad with\, \alpha \in IN^*\, and\, 0 \leq \beta < Z_{max}$$

Therefore

$$Y_y = (k.a + \alpha). Z_{max} + \beta$$

Hence the expression of the first component of the vector MI_f is as following:

$$I_1^y = \frac{k.q - (k.a + \alpha)}{n.k} \quad : \quad \alpha \in IN^* \ and\ k \in IN^* - \{1\}$$

But $I_1^x = \frac{q-a}{n}$

So $I_1^x \neq I_1^y$, and as a result, the vector MI_f does not respect the property of invariance by replication.

The same reasoning for the other components of MI_f and the PG_f index.

6.3. Axiom of Decomposability

Let n(x) the number of individuals in the distribution x, and z a poverty line

Given a distribution *x=(x',x'')* such as *n(x)=n(x')+n(x'').*

A poverty measure *P* is called decomposable if and only if:

$$P(x,z) = \frac{n(x')}{n(x)} P(x',z) + \frac{n(x'')}{n(x)} P(x'',z)$$

In other words :

$$P(x,z) = \frac{1}{n(x)} \sum_{i=1}^{n} P(x_i,z) \; where \;\; P(x_i,z) = 0 \; for \; all \; x_i \; non \; poor$$

Proof:

Suppose that:

n is the number of individuals in the distribution *x*.

n' (resp.*n''*) is the number of individuals in the distribution *x'* (resp. *x''*).

q the number of poor in the distribution *x*,

q' (resp.*q''* the number of poor in the distribution *x'* (resp. *x''*).

so we have *n=n'+n''* and *q=q'+q''*

let's remember that [m] denotes the integer part of the real m.

Given $Y'_q = \sum_{\substack{i \in B \\ x_i \in x'}} x_i$ the total incomes of the poor in the sub-distribution x'.

$Y''_q = \sum_{\substack{i \in B \\ x_i \in x''}} x_i$ the total incomes of the poor in the sub-distribution x''

As a result, the total of incomes of the poor of the distribution *x* is given by

$$Y_q = \sum_{i \in B} x_i = Y'_q + Y''_q = \sum_{\substack{i \in B \\ x_i \in x'}} x_i + \sum_{\substack{i \in B \\ x_i \in x''}} x_i$$

By performing a Euclidean division of the previous totals Z_{max}, we find:

$$Y_q = a.Z_{max} + r$$

$$Y'_q = a'.Z_{max} + r'$$

$$Y''_q = a''.Z_{max} + r''$$

The first components of the vector MI_f corresponding to each of the distributions are given by:

$$I_1^{x} = \frac{1}{n}(q - a) = \frac{1}{n}\left(q - \left[\frac{Y_q}{Z_{max}}\right]\right)$$
$$I_1^{x\prime} = \frac{1}{n'}(q' - a') = \frac{1}{n'}\left(q' - \left[\frac{Y'_q}{Z_{max}}\right]\right)$$
$$I_1^{x\prime\prime} = \frac{1}{n"}(q" - a'') = \frac{1}{n"}\left(q" - \left[\frac{Y''_q}{Z_{max}}\right]\right)$$

Hence:

$$\frac{n'}{n} I_1^{x\prime} + \frac{n''}{n} I_1^{x\prime} = \frac{1}{n}\left(q' + q" - \left(\left[\frac{Y'_q}{Z_{max}}\right] + \left[\frac{Y"_q}{Z_{max}}\right]\right)\right)$$
$$= \frac{1}{n}\left(q - \left(\left[\frac{Y'_q}{Z_{max}}\right] + \left[\frac{Y"_q}{Z_{max}}\right]\right)\right)$$

But for all positive real numbers α and β we have $[\alpha] + [\beta] \leq [\alpha + \beta]$ is a property of the integer part function. Therefore,

$$\frac{n'}{n} I_1^{x\prime} + \frac{n''}{n} I_1^{x\prime} \leq I_1^{x}$$

Hence the result.

7. Conclusion

The classification of the PG_f index and the MI_f vector as semi fuzzy poverty indices puts at the crossroads of traditional approaches and fuzzy approaches of poverty. In fact, they call on the one hand, tools of fuzzy logic (a membership function and a confidence interval...), and on the other hand, calculations from a classic cardinal of a set of poor. This positioning between classical and fuzzy made the axiomatic characterization and analysis of these two indices itself semi fuzzy, thus, the verification of a set of axioms is original in the sense that every axiomatic analyzes are either in the fuzzy frame, or in the classic but not in a frame combining the two.

In this article we demonstrated a set of axioms that the PG_f index and the MI_f vector semi fuzzy poverty validate, reflecting their relevance in describing poverty. We have also shown the limits of these two semi fuzzy measures through three axioms which do not satisfy in order to improve future writing these two measurements, or find conditions under which these semi fuzzy measures exceed their limits, and thus improving performance and relevance.

8. References

Notes techniques (2002). Techniques principales et questions interdisciplinaires. *Mesure et analyse de la pauvreté/ Poverty measurement and analysis*. Volume 1.

Sen, A.K. (1976). Poverty: An Ordinal Approach to Measurement. *Econometrica*, Vol. 44, pp. 219-231.

Deaton, A.S. (2005). Measuring Povrety in a Growing World. *Review of Economic Statistics*, 87(1), pp. 1-19.

Hagenaars, A.J.M. (1986). *The Perception of Poverty*. Amsterdam: North – Holland.

Meyer, B.D. & Sullivan, J.X. (2003). Measuring the Well-Being of the Poor Using Income and Consumption. *Journal of Human Research,* 38(5), pp. 1180-1220.

McKinnish, T. (2005). Importing the Poor: Welfare Magnetism and Cross-Border Welfare Migration. *Journal of Human Research*, 40(1), pp. 57-76.

Delhausse, B. (2002). Le Noyau Dur de Pauvreté en Wallonie: une Actualisation/ The Poverty Core in Wallonia: an Update. *Reflets et perspectives de la vie économique/Reflections and perspectives of economic life*, tome XLI, 4, pp. 55-63.

Bertin, A. (2007). Pauvreté Monétaire, Pauvreté Non Monétaire Une Analyse Des Interactions Appliquée à La Guinée/Monetary Poverty, Non-Monetary Poverty An Analysis of Interactions Applied to Guinea. *Thèse Pour Le Doctorat En Sciences Economiques- Université Montesquieu-BORDEAUX IV/ Doctoral Thesis in Economics - Montesquieu University -BORDEAUX IV.*

Belhadj, B. & Matoussi, M. (2007). Proposition d'un indice flou de pauvreté en utilisant une fonction d'information/Proposition of a poverty-fuzzy index using an information function. *International conference: Sciences of Electronic, Technologies of Information and Telecommunications*- March 25-29, – TUNISIA.

Ravallion, M. (1994). *Poverty Comparisons*. The Word Bank, Washington, DC, USA. A Volume in the distribution section. Edited by: Atkinson, A.B, Londons School of Economics Harwood Academic Publishers.

Ravallion, M. (2003). Transferts ciblés dans les pays pauvres: Reconsidérer les choix et les options de politiques/ Targeted Transfers in Poor Countries: Revisiting policy choices and options. *Groupe de Recherche de développement/ Development Research Group.* Banque Mondiale.

Fikri, M.; El Hilali Alaoui A. & El Khomssi, M. (2012). *Planification dans les Multiprojects & Mesure Semi Floue de Pauvreté: Planification du personnel: Modélisation & Résolution par les Métaheuristiques*. Paris: Éditions Universitaires Europeennes EUE.

Fikri, M.; El Khomssi, M. & Saoud, S. (2011). Proposal of a Semi Fuzzy Poverty Index. *EuroEconomica*, Issue 2(28).

Belhadj, B. Pauvretés persistante. (2005). Chronique et transitoire Construction des indices flous/Chronic and transitory Construction of fuzzy indices. *3rd International Conference: Sciences of Electronic, Technologies of Information and Telecommunications* March 27-31, TUNISIA.

Using Quantitative and Mixed Research Methods in Marketing: A Meta-Analytic Approach

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Abstract: The problem of the inconsistent results of empirical studies is a reality in any research field. Literature provides the meta-analysis approach as a solution that responds to the challenge of evaluating, combining, comparing and synthesizing the accumulation of results to a typical, common and representative value of a particular research topic. In this paper, through meta-analysis, we aim to respond to a double challenge within the marketing scientific research field. We analyze the mixed method applicability level in relation to quantitative methods by evaluating the differences among the empirical results of the studies whose aim concerns the same research topic, namely *customer behavior*. Based on a set of well-defined criteria, we have selected 20 studies published in two journals from the American Marketing Association database. The search has been limited to a number of keywords included in the title of these papers: *consumer, behavior* and *customer*. The results obtained following the quantitative review of the specialized literature specific to consumer behavior analysis suggest that the *type of method* is a significant determinant of the existing differences among the primary studies' empirical results.

Keywords: Literature review; empirical results; consumer behavior

JEL Classification: A10; M30

1. Introduction

Methodological studies may have several objectives, such as the assessment of methods used in a particular field or a particular science, the development of new research methods, testing new methodological instruments etc. In such research, the approach is generally a theoretical one, but there are also empirical studies.

When it comes to empirical studies, the qualitative methods are more appropriate and easier to apply. In order to perform a quantitative study from a methodological perspective, the specialized literature suggests at least two possibilities. The first one requires a strict approach, following some methodological steps that lead to a well defined result type. In this case, we are speaking of meta-analysis. A second possibility involves a multidimensional statistical study on a set of variables defined on the basis of a sample of studies published in scientific journals, in a particular

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research field. The variables are developed according to a set of methodological aims, such as: the identification of method types used in these studies, the existing correlations between methods and research topics, the types of assumptions and the obtained results etc. Bass (1995) suggests four approaches in order to develop an empirical generalization: traditional literature review, meta-analysis, content analysis and clustering, seeking out irregularities by examining different data sets.

In order to perform this study, we have chosen to conduct a *meta-analysis*. In time, the meta-analysis has become a dominant method for the review of scientific literature (Aguinis et al., 2011), allowing the examination of a research field and determining the degree at which a particular outcome has been replicated successfully in various studies (Eden, 2002). Despite the fact that the development of this method has not been without criticism, presenting certain limits (for example, it only refers to the results of studies on a particular research topic), meta-analysis has become the quantitative analysis technique of reviewing the empirical results obtained from studies carried out in a specific research field.

The fundamental aim of our study is to verify if the empirical results of economic studies differ significantly in terms of the type of method applied, namely quantitative or mixed (qualitative and quantitative). However, according to the meta-analysis methodology, we have considered the domain of *marketing* as research field and the *consumer (customer) behavior* as research topic. The customer behavior is one of the key insights of marketing scientific study, which is always evolving and characterized by constant change. Thus, understanding how consumers think and make decisions can provide researchers with the knowledge they need to develop effective marketing models of communication that influence people to purchase goods and services (for more details about the main models of marketing communication, see the study of Oancea (2015)).

The paper is structured as it follows. The ensuing section deals with the fundamental aspects of meta-analysis, enabling a clear understanding of the concept, as well as its applicability in Economics. The third section deals with a brief presentation of methodological steps of meta-analysis. The fourth section of this paper presents an empirical study that analyzes a relatively reduced sample of studies published in two AMA (American Marketing Association) journals. The paper ends with concluding remarks, directions for future research and references.

2. Meta-Analysis in Economics

Meta-analysis is a concept coined by Gene Glass in order to define „the analysis of analyses”. The author states that meta-analysis refers *to the statistical analysis of a large collection of results from individual studies, for the purpose of integrating their findings. It connotes a rigorous alternative to the casual, narrative discussions of* *research studies which typify our attempts to make sense of the rapidly expanding research literature* (Glass, 1976, p. 3).

In comparison to other reviewing methods (narrative analysis, for example), meta-analysis can be distinguished by imposing the assessment of the association level between the studies features and their results by means of effect size indices. Thus, from a methodological point of view, meta-analysis can be defined as a quantitative statistical procedure (Glass, 1976), which involves estimating the global effect size of a set of primary studies on the basis of their individual effect sizes (Field, 2001).

Throughout the history of scientific research, various forms of meta-analyses can be distinguished. Starting from the comparison of different astronomy results, in the 18th and the 19th centuries (Gauss and Laplace), followed by a quantitative analysis of the results selected from a series of planned studies in medical research (Pearson, 1904; Fisher, 1935; Cochran, 1937) and then in social sciences (Glass, 1976; Rosenthal, 1984), finally reaching the formalized quantitative synthesized technique of a large amount of results from almost any scientific research field. Over the past two decades, in economics there have been many „crisis” proclamations (Blaug, 1980, pp. 253-264). The Keynesian followers, monetarists and classical economists are not able to engage themselves in a constructive dialogue (Klamer, 1985). Moreover, the methodology and the “orthodox” language of micro-economists make the communication with the behavioral economists impossible (Frantz, 1985; Leibenstein, 1985; Stanley, 1986). In this context, the current literature, no matter how well performed, raises the question of whether it is reasonable to establish a consensus or to identify a clear and uncontroversial pattern of developing economic knowledge.

Literature reviews are essential instruments in summarizing economic theories and identifying unsolved research problems. However, they are dominated by a high level of subjectivity. Researchers often make unjustified choices regarding the reviewed studies, the importance given to certain results of these studies, their interpretation and the selection of determinants explaining the differences between these results. In this context, the questions about the legitimacy of the conclusions formulated on the basis of economic literature review are inevitable. Why is there a so high variation level in the empirical results of economic research? Why do economic researchers obtain different results when analyzing the same phenomenon? Does the reason lie in the choice of statistical methods or is the result of a pattern specification error?

The aim of approaching meta-analysis does not intend to limit the examination of specialized literature to mere speculations, concluded on the basis of economic empirical studies. By using meta-analysis, these assumptions may be tested in the same manner in which any economic phenomenon is empirically assessed. Although it is relatively new in the circle of economists, meta-analysis has developed quickly and continues to gain acceptance among research economists (for an overview of the state of meta-analysis in economics, see Figure 1 in the paper of Koning, 2002). In this matter, Stanley and Jarrell (1989) had an important contribution. Their remarks had a major impact in approaching the meta-analysis methodology (particularly, meta-regression) to assess economic empirical results. Meta-analysis has also become important in finance, marketing and management research. The important place reserved to meta-analytical studies in scientific journals shows an increased interest for this method in marketing, especially in strategic or behavioral marketing topics, such as consumer or customer behavior (Zablah et al., 2012; Chang & Taylor, 2016; Pick & Eisend, 2016; Purmehdi et al., 2017), but also in methodological issues (Franke, 2001; Laroche &Soulez, 2012; Eisend, 2015). In fact, nowadays, it is probably difficult to find a research field in which meta-analysis cannot be applied.

3. Meta-analysis Methodology

In time, meta-analysis has known many methodological approaches, but the most comprehensive are those proposed by Glass et al. (1981), Hunter and Schmidt (1981) and Hunter et al. (1982). Starting from these approaches, the emphasis further falls on developing a meta-analysis methodological scheme (Figure 1) that can respond to the aim of our research. According to specialized literature, the main methodological steps are planning and conducting the meta-analysis, with corresponding sub-steps for each of them.

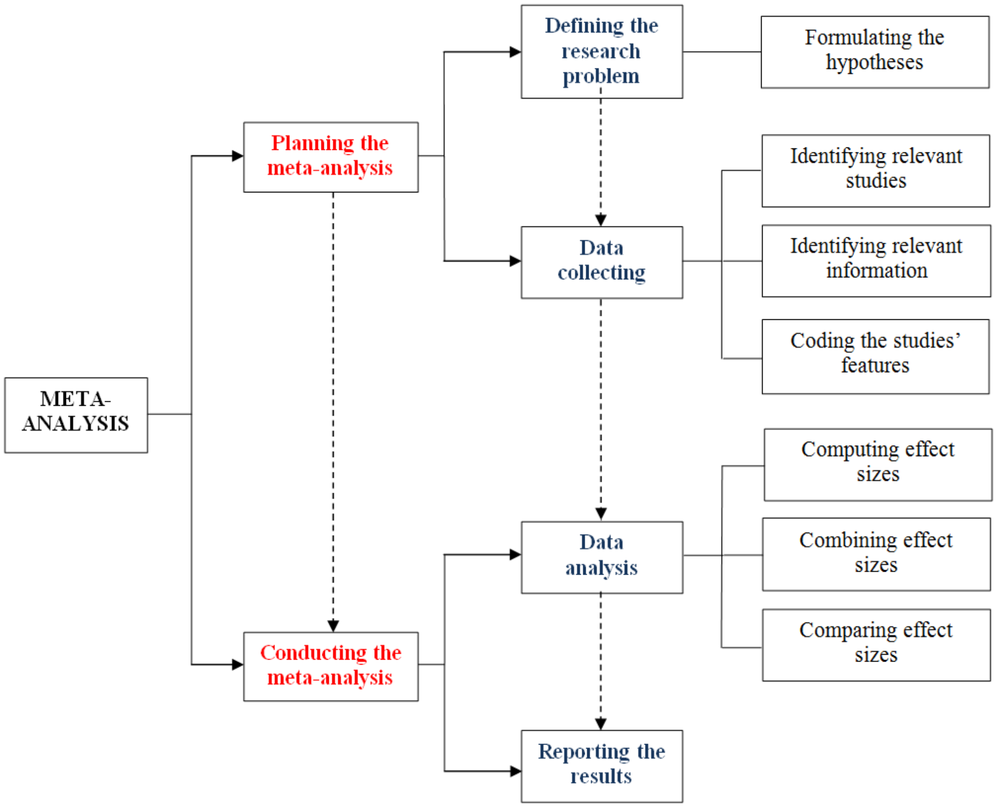

Figure 1. The methodological steps of meta-analysis

3.1. Planning the Meta-analysis

The first step involves the establishing of the meta-analysis main objective and formulating the research hypotheses, obtaining the sample of analyzed primary studies, identifying the relevant information and coding the features of these studies.

1st Step: Defining the research problem

As any other scientific approach, the meta-analysis starts with the defining of the research problem step[1]. This step requires identifying the research topic, precisely defining the objective, formulating the research questions that should be answered, and the a priori hypotheses, choosing the meta-analytical approach and defining the features of the primary studies.

[1] see (Hedges et al., 1989; Mullen, 1989; Cooper, 1998; Card, 2012).

2nd Step: Data collecting

The next step is the actual collection of data in order to obtain the sample. In this regard, the study identification methods and the relevant information from these studies, as well as the possible ways of coding the information should be considered.

To *identify the studies* that respond to the former research questions, an important step in meta-analysis is to determine which type of studies will be included, specifying a series of inclusion or exclusion criteria (Card, 2012). These criteria refer to features that define the statistical population, study design, type of publication etc. The selection of primary studies can be performed either by considering certain keywords that identify the research topic or by searching for relevant references cited in some of these studies. The collection of relevant information for meta-analysis is conducted according to the research goals and the design of the analyzed studies (descriptive studies, experimental studies etc.). In this stage, it is important to define the research problem from at least four points of view: variables used in study; sampling procedure; used statistical methods; obtained statistical results.

Coding the information is very important in establishing and computing the effect sizes, based on which the quantitative analysis of primary studies' results will be possible. Card (2012) provides some examples that require coding the information based on the type of meta-analysis.

3.2. Conducting the Meta-analysis

The second fundamental phase of meta-analysis methodology consists in conducting the actual quantitative analysis, which implies two other steps. The first one targets the analysis of data presented in papers from the primary analysis. The second one refers to the actual reporting results of the meta-analysis that is treated in this paper not as a separate section, but as a part of the other steps.

3rd Step: Data analysis

On one hand, data analysis involves computing the effect size for each primary study included in meta-analysis and, on the other hand, the analysis of these effect sizes by means of some specific models.

a. Computing the effect sizes

The effect size is the most important information extracted from the studies included in a meta-analysis. Therefore, computing this indicator from the data resulting from the studies' original analysis requires special attention. The most commonly used indices for representing the effect sizes are: *r* (*Pearson correlation coefficient*), *g* (an indicator of *standardized mean difference*), and *o* (*odds ratio*). In this regard, several important aspects have to be considered. First, there are different ways of computing the effect sizes, depending on the available information or data reported in primary studies: inferential statistics, descriptive statistics, and information regarding the level of statistical significance. Secondly, if necessary, the comparison and transformation methods among these three indices should be considered.

Pearson' correlation coefficient measures the association between two continuous variables (symbolized by r), between a dichotomous variable and a contionuous one (symbolized by r_{pb}) or between two dichotomous variables (symbolized by ϕ). Pearson's coefficient is considered a useful and easily interpretable indicator of effect size. However, in many meta-analysis, r is converted before the effect sizes should be combined or compared among studies[1] The most common transformation of r is the one developed by Fisher, which is obtained based on the relationship:

$$Z_r = \frac{1}{2} ln\left(\frac{1+r}{1-r}\right),$$

where: r is the correlation coefficient between the two variables; Z_r represents the Fisher transformation of r indicator.

Knowing the sample size (n) within each primary study, the estimated standard error corresponding to Z_r has the following expression:

$$s_{Z_r} = 1/\sqrt{n-3},$$

which shows that, as the sample size increases, the error standard decreases.

The indices family of standardized mean difference represents the difference magnitude between the means of two groups as a function of groups' standard deviations. Therefore, it can be considered that these effect sizes express the association of a dichotomous variable (as grouping factor) and a continuous variable. The specialized literature presents three standardized mean difference indices (Rosenthal, 1994; Grissom and Kim, 2005): *Hedges' coefficient (g)*, *Cohen's coefficient (d)* and *Glass's coefficient (g_{Glass})*. The most widely used is the coefficient of Hedges, which is computed via the following formula:

$$g = \bar{x}_1 - \bar{x}_2/s'_{pooled},$$

where $\bar{x}_1$ and $\bar{x}_2$ are the means of the first and the second group, respectively; s'_{pooled} is the pooled estimate of the population standard deviation.

Odds ratio, denoted as o or symbolized often by OR, represents a useful indicator of the effect size in the case of association between two dichotomous variables. The formula of computing o within a primary data set is:

$$o = n_{00}n_{11}/n_{01}n_{10},$$

[1] for details, see (Hedges & Olkin, 1985, pp. 226-228; Schulze, 2004, pp. 21-28).

where n_{00}, n_{01}, n_{10}, n_{11} are the number of observations corresponding to the association between each two categories of the two variables.

The equations for computing the three indices of effect sizes, based on the data reported in primary studies, are presented in Table 1 (Annex 1). It should be also noted that the value of one coefficient can be obtained from the other two indices (Card, 2012).

b. Combining the effect sizes

Following the meta-analytical process, the stage of effect size analysis by means of several types of models is considered. In this context, the specialized literature clearly differentiates between fixed-effect and random-effect models. The fixed-effects model takes into account the estimation error of each effect size in relation to an overall effect, considered unique for all studies. Unlike the former model, the random-effects model considers the estimation error of each study in relation to the other ones.

Considering the aim of this paper, we discuss only the fixed-effects model. The estimation of this model requires several steps, described in the table below.

Table 2. Steps of fixed-effects model

Steps	Observation
Computing the standard error of the effect size estimate from study *i*	denoted as s_{ES_i}, it differs depending on the effect size indices
Evaluating the precision of effect size estimate	$w_i = 1/s^2_{ES_i}$
Computing the weighted mean effect size	$\overline{ES} = \frac{\sum_{i=1}^{m}(w_i ES_i)}{\sum_{i=1}^{m} w_i}$
Computing the standard error of the mean effect size	$s_{\overline{ES}} = \sqrt{1/\sum w_i}$
Testing the statistical significance of the mean effect size	$Z = \overline{ES}/s_{\overline{ES}}$
Computing the lower- and upper-bound effect sizes for confidence intervals	lower limit: $ES_{LI} = \overline{ES} - z_{\alpha/2} s_{\overline{ES}}$ upper limit: $ES_{LS} = \overline{ES} + z_{\alpha/2} s_{\overline{ES}}$
Testing the heterogeneity among effect sizes	$Q = \sum (w_i (ES_i - \overline{ES})^2) \Leftrightarrow$ $Q = \sum (w_i ES_i^2) - \frac{(\sum (w_i ES_i))^2}{\sum w_i}$

The Q statistics is distributed as χ^2 with $(m-1)$ degrees of freedom, and the decision concerning the null hypothesis is taken based on the comparison of the calculated value with the theoretical one. Thus, if Q statistics exceeds the critical value of χ^2, the null hypothesis of homogeneity is rejected. In other words, the effect sizes are heterogeneous, meaning that there are significant differences among the analyzed primary studies.

c. Comparing the effect sizes

If the meta-analysis shows a significant heterogeneity of effect sizes among studies, the analysis is continued with determining the source of heterogeneity step, by means of the moderator analysis (Baron & Kenny, 1986; Little et al., 2007). This type of analysis aims to explain the variation in effect sizes using the studies' coded features as independent variables. More specifically, the moderator analysis within a meta-analysis determines whether the association between two variables (represented by the effect size) varies significantly depending on a potential moderator (defined by the characteristics of primary studies).

Within a meta-analysis, the moderators of effect sizes can be either categorical variables (for example, the type of method used in primary studies) or continuous variables (for example, the average age). A simultaneous analysis of these moderators is also possible by using the meta-regression procedure.

The logic of assessing the impact of a categorical moderator in meta-analysis is similar to the procedure used for the analysis of variance (ANOVA) in primary studies. While the ANOVA procedure allows dividing the total variability between groups and within groups (defined by a certain group factor), the moderator analysis partitions the overall heterogeneity among effect sizes of studies into between- and within-groups of studies' heterogeneity (Hedges, 1982; Lipsey & Wilson, 2001, pp. 120-121). In other words, approaching the ANOVA procedure in a meta-analysis involves testing the influence of a categorical moderator that acts at two or more levels on the effect size.

In table 3 are listed the steps of evaluating a categorical moderator in meta-analysis.

Table 3. Steps of moderator analysis

Steps	Statistics	Degrees of freedom
Rule of partitioning the total heterogeneity	$Q_{total} = Q_{between} + Q_{within}$	
Total heterogeneity	$Q_{total} = \sum(w_i ES_i^2) - \frac{(\sum(w_i ES_i))^2}{\sum w_i}$	$df_{total} = m - 1$
Group heterogeneity	$Q_{group} = \sum(w_i(ES_i - \overline{ES}_k)^2)$	$df_k = m_k - 1$
Within group heterogeneity	$Q_{within} = \sum_{k=1}^{m} Q_{group}$	$df_{within} = m - k$
Between groups heterogeneity	$Q_{between} = Q_{total} - Q_{within}$	$df_{between} = k - 1$

The statistical significance of between groups heterogeneity is evaluated by comparing the calculated test value ($Q_{between}$) with a critical value (χ^2) relative to $(k-1)$ degrees of freedom and a chosen level of statistical significance (α).

4. Meta-analysis of Effect Sizes in Consumer Behavior

The empirical study of this paper involves illustrating the steps of the meta-analysis process using a set of primary studies, which have as research topic the consumer behavior.

The research approach follows closely the methodological scheme discussed in the previous section (see Figure 1).

4.1. Formulating the Problem

The main research objective is to identify the factors that explain the differences between the empirical results of studies on a specific marketing research topic. Starting with the question "Is the heterogeneity of the results explained by the characteristics of primary studies?", we formulated the principal research hypothesis: the result heterogeneity of primary studies is explained by several categorical moderators.

In order to test this assumption, we have used meta-analysis on a set of primary studies, analyzing the association between a dependent variable that defines the consumer behavior and an independent variable that indicates a consumer behavior determinant. At this point of the analysis, it is important to mention that the approach of a meta-analysis requires the evaluation of the effect size within each primary study included in the sample. In our research, the effect size is the correlation between the two type of variable mentioned above.

Based on the main research hypothesis, we formulate a secondary one: the study groups defined by the categorical moderator – the type of method used, namely the quantitative or mixed analysis methods – differ significantly with respect to the effect sizes.

4.2. Collecting and Coding the Information

Identifying the relevant studies

The criteria considered for the selection of the relevant primary studies were: database (AMA - American Marketing Association); journals with the highest impact factor (two journals, Journal of Marketing, with an impact factor of 3.3, and Journal of International Marketing, having an impact factor of 3,9); year of publication (2015-2016); several keywords (*consumer*, *behavior*, *customer*).

Based on these criteria, we have selected 24 papers by identifying in their title at least one of the mentioned keywords, but we have been able to include only 20 in our meta-analysis. The reason for two of them is related to the inaccessibility of the entire paper, and the other two did not provide sufficient data for computing the corresponding effect size. The arguments for defining these criteria used in evaluating and selecting the studies refer to: marketing research field for choosing AMA database, journals with the highest impact factor within the AMA database, study publication year (2015-2016) to highlight a more current state of research, and keywords that reflect the topic of interest for our research, namely the *consumer behavior*.

Selecting the relevant information

The relevant pieces of information have been selected from primary studies so that the data reported are closely related to the main research goal. To achieve their aims, we have observed that some analyses have been conducted in several stages represented either by different studies, or resulting from one another.

For computing the effect size of each study, we have defined the two variables based on each study research aim and hypotheses (dependent variable reflecting the consumer behavior and independent variable(s) of interest for the respective study), relevant empirical results for computing the effect sizes, other relevant information (for example, sample size).

Coding the studies

The included studies have been coded according to a number of characteristics: sample size (continuous variable); categorical moderator - type of method (dichotomous variable: quantitative and mixed); dependent variable defining the consumer behavior; independent variables reflecting the determinants of consumer behavior.

4.3. Data Analysis

In data analysis stage, the emphasis is on choosing the most adequate ways of computing the effect sizes and the methods of testing the influence of categorical moderator on effect sizes.

Computing the effect sizes

In order to compute the effect sizes we have considered a number of criteria discussed theoretically in the methodology section and described below, in the context of our meta-analysis.

a. Type of effect size

According to the chosen research topic, the effect size is defined by the correlation between the dependent variable reflecting the consumer behavior and the independent variable, considered a determinant of the first one.

b. Indicator for representing the effect sizes

Among the three indices most frequently used in meta-analyses, we have chosen Pearson's correlation coefficient due to its high level of applicability in primary studies, but also because this coefficient can be computed based on a variety of data reported in these studies: results of descriptive statistics, results of statistical tests, frequencies (for identifying the size samples of the tested groups). Even though this coefficient is relatively easy to obtain, we have also considered that certain information from the studies requires computing other types of indices, such as the standardized mean differences or the odds ratio. For the studies that included control and experimental groups in their analysis, we have used the Hedges' coefficient, and for those studies analyzing two dichotomous variables, for which the available data allowed us to construct only a contingency table, we have chosen to compute the odds ratio.

c. Transformation between indices

Considering the latter remark, the results of computing the effect size by means of Hedges' coefficient and odds ratio were converted to Pearson's correlation coefficient.

d. Multiple effect sizes from a single primary study

In order to verify the independence assumption in the meta-analysis dataset, we have paid special attention to those studies providing multiple effect sizes. In this respect, we have taken into consideration two options for handling the non-independence in our dataset and obtaining a single effect size from each study. The first option was to identify the results that were more adequate for the main objective within each study. The alternative option required computing the mean of all effect sizes identified in the same study. Finally, based on those studies that included more analyses performed on different samples and whose results led to the achievement of the research goal, we have computed multiple effect sizes, meaning that these studies were included in our sample for two or more times. In this context, we consider that the independence among effect sizes is not violated.

In table 4 (Annex 2) are listed, in detail, more ways of computing the effect size for each primary study. With the evaluation of the correlation between the two type variables, our analysis continues with combining and comparing the effect sizes. These two steps allow us to test our research hypothesis.

Fixed-Effects Model

In order to facilitate the analysis approach of the fixed-effects model, the meta-analysis database (table 5) is recommended to include the following variables: sample size (n), effect size (r), Fisher's transformation of effect size (z_r), standard error $\left(s_{z_r}\right)$ and the weight corresponding to each effect size estimates (w), which is determined by means of standard error.

Table 5. Computation elements for fixed-effects model

No.	Size sample (n)	Effect size (r)	Fisher's transformation $\left(Z_r = \frac{1}{2} ln\left(\frac{1+r}{1-r}\right)\right)$	Standard error $\left(s_{Z_r} = \frac{1}{\sqrt{(n-3)}}\right)$	Weight $\left(w = 1/s^2_{Z_r}\right)$	Weight of effect size $(w * Z_r)$	$\left(w * Z_r^2\right)$
1	100	0.02	0.02	0.10	97	1.94	0.04
2	372	0.09	0.09	0.05	369	33.30	2.99
3	49	0.29	0.30	0.15	46	13.73	3.87
4	85	0.27	0.28	0.11	82	22.70	5.98
5	53729	0.11	0.11	0.00	53726	5933.87	650.08
6	5000	0.22	0.22	0.01	4997	1117.61	241.85
7	1213	0.52	0.58	0.03	1210	697.37	327.18
8	14384	0.02	0.02	0.01	14381	287.66	5.75
9	1346	-0.31	-0.32	0.03	1343	-430.49	129.06
10	3196	0.20	0.20	0.02	3193	647.33	127.72
11	803	-0.43	-0.46	0.04	800	-367.92	147.92
12	824	0.03	0.03	0.03	821	24.64	0.74
13	1180	0.29	0.30	0.03	1177	351.41	98.99
14	1703	0.31	0.32	0.02	1700	544.93	163.37
15	885	0.18	0.18	0.03	882	160.51	28.58
16	484	0.47	0.51	0.05	481	245.34	106.25
17	773262	0.31	0.32	0.00	773259	247864.6	74310.2
18	838	-0.01	-0.01	0.03	835	-8.35	0.08
19	1309	0.61	0.71	0.03	1306	925.85	485.96
20	411	0.37	0.39	0.05	408	158.48	55.86
21	405	0.26	0.27	0.05	402	106.98	27.18
22	5425	0.67	0.81	0.01	5422	4395.85	2433.94
23	204	0.18	0.18	0.07	201	36.58	6.51
24	30000	0.17	0.17	0.01	29997	5149.48	866.91
$\sum$	-	-	-	-	**897135**	**267913.4**	**80227**

Source: Author's computations of selected data from primary studies using Excel

Considering all these elements indispensable for the comparison of effect sizes among studies, our analysis continues with the steps imposed by the approach of fixed-effects model. The sequence of these steps and their corresponding results are listed in Table 6.

Summarizing the results from Table 6, it is found that the mean effect size defined by the correlation between the two variables (consumer behavior and its various determinants) is significantly different from zero, falling within the confidence interval (0.281; 0.282). Nevertheless, we have observed that the value of Q statistics exceeds the critical value of χ^2, leading us to reject the null hypothesis of homogeneity and to conclude that there is a significant heterogeneity among the studies around the mean effect size. Our findings highlight the importance of explaining this heterogeneity by means of moderator analysis.

Table 6. Fixed-effects model results

Steps	**Obtained results**
Weighted mean effect size	$\bar{Z}_r = \frac{267913.4}{897135} = 0.29 \Rightarrow \bar{r} = \frac{e^{\bar{Z}_r} - 1}{e^{\bar{Z}_r} + 1} = \frac{e^{0.29} - 1}{e^{0.29} + 1} = 0.28$
Standard error of the mean effect size	$s_{\overline{ES}} = \sqrt{1/897135} = 0.001$
Statistical significance test of the mean effect size	$Z = 0.29/0.001 = 290$
Confidence interval of effect size	$(LI)ES_{Z_r} = 0.288 \Rightarrow (LI)ES_r = \frac{e^{0.288} - 1}{e^{0.288} + 1} = 0.281$ $(LS)ES_{Z_r} = 0.290 \Rightarrow (LS)ES_r = \frac{e^{0.290} - 1}{e^{0.290} + 1} = 0.282$
Heterogeneity test of effect size	$Q = 86852.45 - \frac{(267913.4)^2}{897135} = 6844.86$ $Q = 6844.86 > \chi^2_{\alpha;m-1} = \chi^2_{0.05;23} = 35.17$

Source: Author's computations of selected data from primary studies using Excel

Moderator Analysis

In accordance with the paper main objective and the results obtained until this stage, the research hypothesis is justified in the context of our meta-analysis. Considering the type of used data, we have performed the ANOVA analysis that allows us to evaluate the impact of the potential moderator, type of method, on effect sizes, namely the correlation between consumer behavior and its determinants.

To illustrate the analysis of the variance procedure in our meta-analysis, we have covered each step of obtaining the necessary elements, insisting on the result interpretation. All of these data are listed in Tables 7 and 8.

Table 7. Computation elements for moderator analysis

No.	Size sample (n)	Effect size(r)	Fisher's transformation (Z_r)	Standard error (s_{Z_r})	Weight (w)	Weight of effect size $(w * Z_r)$	$(w * Z_r^2)$
Type of method: quantitative $(m_1 = 19)$							
1	100	0.02	0.02	0.1	97	1.94	0.04
2	49	0.29	0.3	0.15	46	13.73	3.87
3	53729	0.11	0.11	0	53726	5933.87	650.08
4	5000	0.22	0.22	0.01	4997	1117.61	241.85
5	1213	0.52	0.58	0.03	1210	697.37	327.18
6	14384	0.02	0.02	0.01	14381	287.66	5.75
7	1346	-0.31	-0.32	0.03	1343	-430.49	129.06
8	3196	0.2	0.2	0.02	3193	647.33	127.72
9	803	-0.43	-0.46	0.04	800	-367.92	147.92
10	1703	0.31	0.32	0.02	1700	544.93	163.37
11	885	0.18	0.18	0.03	882	160.51	28.58
12	773262	0.31	0.32	0	773259	247864.6	74310.2
13	411	0.37	0.39	0.05	408	158.48	55.86
14	405	0.26	0.27	0.05	402	106.98	27.18
15	5425	0.67	0.81	0.01	5422	4395.85	2433.94
16	204	0.18	0.18	0.07	201	36.58	6.51
17	30000	0.17	0.17	0.01	29997	5149.48	866.91
18	372	0.09	0.09	0.05	369	33.3	2.99
19	824	0.03	0.03	0.03	821	24.64	0.74
				Sum within group	**893254**	**266376.5**	**79529.75**
Type of method: mixed $(m_2 = 5)$							
20	85	0.27	0.28	0.11	82	22.7	5.98
21	1180	0.29	0.3	0.03	1177	351.41	98.99
22	484	0.47	0.51	0.05	481	245.34	106.25
23	838	-0.01	-0.01	0.03	835	-8.35	0.08
24	1309	0.61	0.71	0.03	1306	925.85	485.96
				Sum within group	**3881**	**1536.95**	**697.26**
				Total sum of the two groups	**897135**	**267913.4**	**80227**

Source: Author's computations of selected data from primary studies using Excel

Based on the results from Table 7, we obtain the computing elements of the within-group heterogeneity, thus being able to test if one source of this effect size heterogeneity might be due to the use of the quantitative or mixed methods. The steps are detailed in the table below.

Table 8. ANOVA results considering the type of method as categorical moderator

Steps	Statistics
Total heterogeneity	$Q_{total} = 219.42 > \chi^2_{\alpha;m-1} = \chi^2_{0.05;23} = 35.17$
Group heterogeneity	$Q_{group1} = 79529.7 - \frac{(266376.5)^2}{3881} = 93.8 > \chi^2_{\alpha;m_1-1} = \chi^2_{0.05;18} = 28.8$
	$Q_{group2} = 697.26 - \frac{(1536.95)^2}{3881} = 88.59 > \chi^2_{\alpha;m_2-1} = \chi^2_{0.05;4} = 9.49$
Within group heterogeneity	$Q_{within} = 93.86 + 88.59 = 182.45 > \chi^2_{\alpha;m-k} = \chi^2_{0.05;22} = 33.92$
Between groups heterogeneity	$Q_{between} = 219.48 - 182.45 = 36.97 > \chi^2_{\alpha;k-1} = \chi^2_{0.05;1} = 3.84$

Source: Author's computations of selected data from primary studies using Excel

The findings suggest that there is a significant heterogeneity within the set of primary studies. There is a significant heterogeneity among the studies from the quantitative group, among the studies included in the mixed group and within each of the two groups. Also, the value of $Q_{between}$ is high enough that we can reject the null hypothesis and accept the alternative hypothesis, according to which between the group of quantitative studies and the one of mixed studies, there are significant differences in terms of their effect sizes. In other words, the type of method moderates the association between the customer behavior and its determinants. Therefore, our research hypothesis is validated; meaning that one source of the heterogeneity among studies might be due to the use of a different type of method.

5. Concluding Remarks

The problem of inconsistent results of empirical studies is a reality in any scientific research field. Literature provides the meta-analysis approach as a solution because it responds to the challenge of evaluating, combining, comparing and synthesizing the accumulation of results to a typical, common and representative value of a particular research topic.

In this paper, we aimed to respond through meta-analysis to a double challenge within the marketing scientific research field. We analyzed the applicability level of the mixed methods in relation to the quantitative methods by means of evaluating the differences among empirical results obtained in studies with the same research topic. The results obtained from the quantitative review of literature specific to consumer behavior analysis suggest that the type of method is a significant factor explaining the presence of heterogeneity among effect sizes.

Given the complexity and rigors of the meta-analysis methodology, it is inevitable not to reveal certain limits and it is difficult to exceed them at this stage of the research. The major limitation is the small number of studies included in our meta-analysis, especially since we considered a fundamental and wide marketing research topic. Therefore, the research results can be negatively influenced by the small sample size of studies. Another weak point is the exclusion of some important factors in assessing the quality of the results reported in primary studies or other features of these studies. The third limitation concerns the fact that we restricted our analysis to the fixed-effects model. The meta-analysis methodology is very wide, including many other ways of comparing and combining the studies' effect sizes. We highlight, however, that the analysis proposed in this paper represents a basis for the development of our research.

In this regard, the mentioned limits outline at least two other further research directions. The first one is the attempt to identify other moderators explaining the heterogeneity among the empirical results of marketing studies. The second research direction refers to explaining the differences between studies by the simultaneous influence of the potential moderators. Finally, in order to assess the utility of the type of research methods, it is our intention to develop this meta-analytic study by considering several research marketing topics within the same analysis.

This research perspective highlights a possible contribution to the specialized literature by applying the meta-analysis methodology to a general research framework, taking into account that we aim to test a hypothesis regarding a research field, not only a specific research topic.

6. References

Aguinis, H.; Pierce, C.A.; Bosco, F.A.; Dalton, D.R. & Dalton, C.M. (2011). Debunking myths and urban legends about meta-analysis. *Organizational Research Methods*, 14, 2, pp. 306-331.

Bass, F. (1995). Empirical generalizations and marketing science: a personal view. *Marketing Science*, 14, 3, pp. 6-19.

Baron, R.M. & Kenny, D.A. (1986). The moderator-mediator variable distinction in social psychological research: Conceptual, strategic, and statistical considerations. *Journal of Personality and Social Psychology*, 51, pp. 1173-1182.

Blaug, M. (1980). *The Methodology of Economics: Or How Economists Explain*. Cambridge: Cambridge University Press.

Card, N. (2012). *Applied Meta-Analysis for Social Science Research*. New York: The Guilford Press.

Chang, W. & Taylor, S.A. (2016). The effectiveness of customer participation in new product development: A Meta-Analysis. *Journal of Marketing*, 80, 1, pp. 47-64.

Cochran, W.G. (1937). Problems arising in the analysis of a series of similar experiments. *Journal of the Royal Statistical Society, Supplement*, 4, pp. 102-118.

Cooper, H.M. (1998). *Synthesizing research: A guide for literature reviews*. Thousand Oaks, CA: Sage.

Eden, D. (2002). Replication, Meta-Analysis, Scientific Progress and AMJ's Publication Policy. *Academy of Management Journal*, 45, pp. 841-846.

Eisend, M. (2015). Have we progressed marketing knowledge? A meta-meta-analysis of effect sizes in Marketing research. *Journal of Marketing*, 79, 3, pp. 23-40.

Fisher R.A. (1935). *The Design of Experiments*. Edinburgh: Oliver and Boyd.

Field, A.P. (2001). Meta-analysis of correlation coefficients: A Monte Carlo comparison of fixed- and random-effects methods. *Psychological Methods*, 6, pp. 161-180.

Frantz, R. (1985). X-efficiency theory and its critics. *Quarterly Review of Economics and Business*, 25, pp. 38-58.

Franke, G.R. (2001). Applications of meta-analysis for marketing and public policy: a review. *Journal of Public Policy & Marketing*, 20, 2, pp. 186-200.

Glass, G.V. (1976). Primary, secondary and meta-analysis of research. *Educational Researcher*, 5, pp. 3-8.

Glass, G.V.; McGaw, B. & Smith, M. (1981). *Meta-Analysis in social research*. Beverly Hills: Sage.

Grissom, R.J. & Kim, J.J. (2005). *Effect sizes for research: A broad practical approach*. New Jersey: Lawrence Erlbaum Associates, Inc.

Hedges, L.V. (1982). Fitting categorical models to effect sizes from a series of experiments. *Journal of Educational Statistics*, 7, pp. 119-13.

Hedges, L.V. & Olkin, I. (1985). *Statistical methods for meta-analysis*. San Diego, CA: Academic Press.

Hedges, L.V.; Shymansky, J.A. & Woodworth, G. (1989). *A Practical Guide to Modern Methods of Meta-Analysis*. Washington, D.C.: National Science Teachers Association.

Hunter, J.E. & Schmidt, F.L. (1981). Cumulating results across studies: Correction for sampling error, a proposed moratorium on the significance test, and a critique of current multivariate reporting practices. *Unpublished manuscript*, Department of Psychology, Michigan State University.

Hunter, J.E.; Schmidt, F.L. & Jackson, G.B. (1982). *Meta-analysis: Cumulating research findings across studies*. Beverly Hills: CA: Sage.

Klamer, A. (1985). *Conversations with Economists*. Totawa: Rowman and Allanheld.

Koning, R.H. (2002). Meta-analysis in Economics. *Default journal*. Department of Econometrics, University of Groningen, 2, pp. 319-320.

Laroche, P. & Soulez, S. (2012). Meta-Analysis for Marketing Research. Recherche *et Applications en Marketing* (English Edition), 27, 1, pp. 79-104.

Leibenstein, H. (1985). On relaxing the maximization postulate. *Journal of Behavioral Economics*, 14, pp. 5-20.

Lipsey, M.W. & Wilson, D.B. (2001). *Practical meta-analysis*. Thousand Oaks, CA: Sage.

Little, T.D.; Card, N.A.; Bovaird, J.A.; Preacher, K.J. & Crandall, C.S. (2007). Structural equation modeling of mediation and moderation with contextual factors. In T.D. Little, J.A. Bovaird, N.A. Card (Eds.), *Modeling ecological and contextual effects in longitudinal studies*. New Jersey: Lawrence Erlbaum Associates, Inc., pp. 207–230.

Mullen, B. (1989). *Basic Advanced Meta-analysis*. New Jersey: Lawrence Erlbaum.

Oancea, O.E.M. (2015). The model of integrated marketing communication: Who has the role to influence consumer behavior. *Acta Universitatis Danubius. Œconomica*, 11, 1, pp. 22-31.

Pearson, K. (1904). Report on certain enteric fever inoculation statistics. *British Medical Journal*, 2, 2288, pp. 1243-1246.

Pick, D. & Eisend, M. (2016). Customer responses to switching costs: A meta-analytic investigation of the moderating influence of culture. *Journal of International Marketing*, 24, 4, pp. 39-60.

Purmehdi, M.; Legoux, R.; Carrillat, F. & Senecal, S. (2017). The effectiveness of warning labels for consumers: A meta-analytic investigation into their underlying process and contingencies. *Journal of Public Policy & Marketing*, 36, 1, pp. 36-53.

Rosenthal, R. (1984). *Meta-analytic procedures for social research.* Beverly Hills, CA: Sage.

Rosenthal, R. (1994). Parametric measures of effect size. In Cooper, H. & Hedges, L.V. (Eds.). *The handbook of research synthesis*. New York: Russell Sage Foundation, pp. 231-244.

Schulze, R. (2004). *Meta-analysis: A comparison of approaches*. Cambridge, MA: Hogrefe & Huber.

Stanley, T.D. (1986). Recursive economic knowledge: hierarchy, maximization and behavioral economics. *Journal of Behavioral Economics*, 15, pp. 85-99.

Stanley, T.D. & Jarrell, S.B. (1989). Meta-regression analysis: a quantitative method of literature surveys. *Journal of Economic Surveys*, 3, 2, pp. 161-170.

Zablah, A.R.; Franke, G.R.; Brown, T.J. & Bartholomew, D.E. (2012). How and when does customer orientation influence frontline employee job outcomes? A meta-analytic evaluation. *Journal of Marketing*, 76, 3, pp. 21-40.

Annex 1. Computing the effect sizes from commonly results reported in primary studies

Table 1. Computation formula of effect size represented by the three indices

	Pearson's correlation coefficient (r)	**Hedges's coefficient (g)**	**Odds ratio (o)**
Definitional formula	$\frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{(n-1)s_x s_y}$	$\frac{\bar{x}_1-\bar{x}_2}{s_{pooled}}$	$\frac{n_{00}n_{11}}{n_{01}n_{10}}$
Independent *t*-test with unequal group sizes	$\sqrt{\frac{t^2}{t^2+df}}$	$\frac{t\sqrt{n_1+n_2}}{\sqrt{n_1 n_2}}$	-
Independent *t*-test with equal group sizes	$\sqrt{\frac{t^2}{t^2+df}}$	$\frac{2t}{\sqrt{n}}$	-
Independent *F*-ratio with unequal group sizes	$\sqrt{\frac{F_{(1,df)}}{F_{(1,df)}+df}}$	$\sqrt{\frac{F_{(1,df)}+(n_1+n_2)}{n_1 n_2}}$	-

Independent F-ratio with equal group sizes	$\sqrt{\frac{F_{(1,df)}}{F_{(1,df)} + df}}$	$2\sqrt{\frac{F_{(1,df)}}{n}}$	-
Dependent (repeated-measures) t-test	$\sqrt{\frac{t^2_{dependent}}{t^2_{dependent} + df}}$	$\frac{t_{dependent}}{\sqrt{n}}$	-
Dependent (repeated-measures) F-ratio	$\sqrt{\frac{F_{repetat(1,df)}}{F_{repetat(1,df)} + df}}$	$\sqrt{\frac{F_{repetat(1,df)}}{n}}$	-
2×2 (1 degree of freedom) contingency χ^2	$\sqrt{\frac{\chi^2_{(1)}}{n}}$	$2\sqrt{\frac{\chi^2_{(1)}}{n - \chi^2_{(1)}}}$	Reconstruct contingency table
Probability levels from significance tests	$\frac{Z}{\sqrt{n}}$	$\frac{2Z}{\sqrt{n}}$	Reconstruct contingency table

Source: (Card, 2012, p. 97)

Annex 2. Computing the effect sizes using the relevant data selected from each primary study

Table 4. Computing the effect sizes using Pearson's correlation coefficient

No.	**Primary study**	**Size sample (n)**	**Information available for computing r**	**Different ways of computing r**	**Effect size (r)**
1	Study 1	100	Correlation coefficients: $r_1 = 0.00$ $(p < 0.01)$ $r_2 = 0.04$ $(p < 0.01)$	$r = (r_1 + r_2)/2 = 0.04/2 = 0.02$	**0.02**
2	Study 2	372	Probability level from significance t test: $p < 0.001$	$p \approx 0.001 \Rightarrow Z = 2.58$ $r = Z/\sqrt{n} = 2.58/\sqrt{372} = 0.09$	**0.09**
3	Study 3	49	Calculated value of Chi-square test: $\chi^2_{(1)} = 4.18$ $(p < 0.05)$	$r = \sqrt{\frac{\chi^2_{(1)}}{n}} = \sqrt{\frac{4.18}{49}} = 0.29$	**0.29**
4	Study 4	85	Probability level from significance t test: $p = 0.013$	$p = 0.013 \Rightarrow Z = 2.48$ $r = Z/\sqrt{n} = 2.48/\sqrt{85} = 0.27$	**0.27**
5	Study 5	53729	Descriptive statistics indicators for: - regained customer group (control group: $n_1 = 39345$): $\bar{x}_1 = 0.27$; $s_1 = 0.44$	$g = \frac{\bar{x}_1 - \bar{x}_2}{s_{pooled}} = \frac{\bar{x}_1 - \bar{x}_2}{(n_1 s_1 + n_2 s_2)/(n_1 + n_2)}$ $g = \frac{0.11}{0.451} = 0.24$	**0.11**

<table>
<tr><td></td><td></td><td></td><td>- lost customer group ($n_2 = 14384$):
$\bar{x}_2 = 0.38;\ s_2 = 0.49$</td><td>$r = \sqrt{\frac{g^2 n_1 n_2}{g^2 n_1 n_2 + (n_1 + n_2)df}} = 0.11$</td><td></td></tr>
<tr><td>6</td><td>Study 5</td><td>5000</td><td>Frequencies table for the association controlled by the offer of regaining (price and service) between variables regaining probability and reason for leaving:
- customers who left because of the price: $n_{\cdot 1} = 2330$
- customers who left because of the service: $n_{\cdot 2} = 1666$
- customers who left because of the price and service: $n_{\cdot 3} = 1004$
- regained customers who left because of the price: $n_{11} = 1213$
- regained customers who left because of the service: $n_{21} = 711$
- regained customers who left because of the price and service: $n_{21} = 228$</td><td>Table of observed frequencies ($n_{observed}$)
<table>
<tr><td rowspan="2">reason of leaving</td><td colspan="2">regaining probability</td><td rowspan="2">Total</td></tr>
<tr><td>regained customers</td><td>lost customers</td></tr>
<tr><td>price</td><td>1213</td><td>1117</td><td>2330</td></tr>
<tr><td>service</td><td>711</td><td>955</td><td>1666</td></tr>
<tr><td>price and service</td><td>228</td><td>776</td><td>1004</td></tr>
<tr><td>Total</td><td>2152</td><td>2848</td><td>5000</td></tr>
</table>
Table of estimated frequencies ($n_{estimated}$)
<table>
<tr><td rowspan="2">reason of leaving</td><td colspan="2">regaining probability</td><td rowspan="2">Total</td></tr>
<tr><td>regained customers</td><td>lost customers</td></tr>
<tr><td>price</td><td>1002.83</td><td>1327.17</td><td>2330</td></tr>
<tr><td>service</td><td>717.05</td><td>948.95</td><td>1666</td></tr>
<tr><td>price and service</td><td>432.12</td><td>571.88</td><td>1004</td></tr>
<tr><td>Total</td><td>2152</td><td>2848</td><td>5000</td></tr>
</table>
$X^2 = \sum \frac{(n_{observed} - n_{estimated})^2}{n_{estimated}} = 246.69$
$r = \sqrt{\frac{X^2}{n}} = \sqrt{\frac{246.69}{5000}} = 0.22$</td><td>0.22</td></tr>
<tr><td>7</td><td>Study 5</td><td>1213</td><td>Independent t-test with unequal group sizes:
$t = 21.04$</td><td>$r = \sqrt{\frac{t^2}{t^2 + df}} = \sqrt{\frac{21.04^2}{21.04^2 + (1213 - 1)}} = 0.52$</td><td>0.52</td></tr>
<tr><td>8</td><td>Study 5</td><td>14384</td><td>Probability level from significance t test:
$p < 0.05$</td><td>$p \approx 0.05 \Rightarrow Z = 1.96$
$r = Z/\sqrt{n} = 1.96/\sqrt{14384} = 0.02$</td><td>0.02</td></tr>
<tr><td>9</td><td>Study 6</td><td>1346</td><td>Correlation coefficient:
$r = -0.31$ ($p < 0.01$)</td><td>-</td><td>-0.31</td></tr>
</table>

10	Study 7	3196	Independent *t*-test with equal group sizes: $t_1 = 2.87$ $t_2 = 21.41$	$r = \sqrt{\frac{t^2}{t^2 + (n-2)}} \Rightarrow \begin{cases} r_1 = \sqrt{\frac{2.87^2}{2.87^2 + (3196-2)}} = 0.05 \\ r_2 = \sqrt{\frac{21.41^2}{21.41^2 + (3196-2)}} = 0.35 \end{cases}$ $r = (r_1 + r_2)/2 = 0.4/2 = 0.20$	**0.20**
11	Study 8	803	Correlation coefficient: $r = -0.43$ ($p < 0.01$)	-	**-0.43**
12	Study 9	824	Independent *t*-test with equal group sizes: $t_1 = 1.26$ ($p < 0.01$) $t_2 = 0.46$ ($p < 0.01$) $t_3 = 1.02$ ($p < 0.01$)	$g = \frac{2t}{\sqrt{n}} \Rightarrow \begin{cases} g_1 = 2\frac{1.26}{\sqrt{824}} = 0.09 \\ g_2 = 2\frac{0.46}{\sqrt{824}} = 0.03 \\ g_3 = 2\frac{1.02}{\sqrt{824}} = 0.07 \end{cases} \Rightarrow g = \frac{g_1 + g_2 + g_3}{3} = 0.06$ $r = \sqrt{\frac{g^2 n_1 n_2}{g^2 n_1 n_2 + (n_1 + n_2)df}} \Rightarrow r = 0.03$ $r = \sqrt{\frac{0.06^2 \cdot 412 \cdot 412}{0.06^2 \cdot 412 \cdot 412 + 824 \cdot (824-2)}}$	**0.03**
13	Study 10	1180	Independent *F*-test with equal group sizes: $F_{(1,df)} = 4.77$ ($p < 0.05$) Correlation coefficient: $r_2 = 0.51$ ($p < 0.001$)	$g = 2\sqrt{\frac{F_{(1,df)}}{n}} = 2\sqrt{\frac{4.77}{1180}} = 0.13$ $r_1 = \sqrt{\frac{g^2 n_1 n_2}{g^2 n_1 n_2 + (n_1 + n_2)df}} \Rightarrow$ $r_1 = \sqrt{\frac{0.13^2 \cdot 590^2}{0.13^2 \cdot 590^2 + 1180 \cdot 1178}} = 0.07$ $r = (r_1 + r_2)/2 = 0.58/2 = 0.29$	**0.29**
14	Study 10	1703	Correlation coefficient: $r = 0.31$ ($p < 0.01$)	-	**0.31**
15	Study 10	885	Correlation coefficient: $r = 0.18$ ($p < 0.01$)	-	**0.18**
16	Study 11	484	Correlation coefficients: $r_1 = 0.47$ ($p < 0.01$) $r_2 = 0.46$ ($p < 0.01$)	$r = (r_1 + r_2)/2 = 0.93/2 = 0.47$	**0.47**
17	Study 12	773262	Correlation coefficient: $r = 0.31$ ($p < 0.01$)	-	**0.31**
18	Study 13	838	Frequencies table for the association between two dichotomous variables:	Table of observed frequencies ($n_{observed}$) **value-in-use** | **customer perception of *value-in-use*** | **Total**	**-0.01**

<table>
<tr><td></td><td></td><td></td><td>- customer perception concerning the value-in-use solution:
$p_1 = 0.07$ (direct)
$p_2 = 0.08$ (indirect)
- value-in-use:
$p_1 = 0.12$ (direct)
$p_2 = 0.23$ (indirect)</td><td><table><tr><td></td><td>direct</td><td>indirect</td><td></td></tr><tr><td>direct</td><td>0.19</td><td>0.20</td><td>0.39</td></tr><tr><td>indirect</td><td>0.30</td><td>0.31</td><td>0.61</td></tr><tr><td>Total</td><td>0.49</td><td>0.51</td><td>1</td></tr></table>
$o = \frac{p_{00}p_{11}}{p_{01}p_{10}} = \frac{153 \cdot 266}{164 \cdot 255} = 0.97$
$r = cos\left(\frac{\pi}{(1 + o^{1/2})}\right)$
$= cos\left(\frac{3.14}{(1 + 0.97^{1/2})}\right)$
$= -0.01$</td><td></td></tr>
<tr><td>19</td><td>Study 14</td><td>1309</td><td>Independent F-test with unequal group sizes:
$F = 782.64$ ($p < 0.01$)</td><td>$r = \sqrt{\frac{F_{(1,df)}}{F_{(1,df)} + df}}$
$= \sqrt{\frac{782.64}{782.64 + (1309 - 2)}} = 0.61$</td><td>0.61</td></tr>
<tr><td>20</td><td>Study 15</td><td>411</td><td>Correlation coefficient:
$r = 0.37$ ($p < 0.05$)</td><td>-</td><td>0.37</td></tr>
<tr><td>21</td><td>Study 15</td><td>405</td><td>Correlation coefficient:
$r = 0.26$ ($p < 0.05$)</td><td>-</td><td>0.26</td></tr>
<tr><td>22</td><td>Study 16</td><td>5425</td><td>Correlation coefficient:
$r = 0.67$ ($p < 0.05$)</td><td>-</td><td>0.67</td></tr>
<tr><td>23</td><td>Study 17</td><td>204</td><td>Probability level from significance t test:
$p < 0.01$</td><td>$p \approx 0.01 \Rightarrow Z = 2.58$
$r = Z/\sqrt{n} = 2.58/\sqrt{204} = 0.18$</td><td>0.18</td></tr>
<tr><td>24</td><td>Study 18</td><td>30000</td><td>Correlation coefficients:
$r_1 = 0.176$ ($p < 0.05$)
$r_2 = 0.162$ ($p < 0.05$)</td><td>$r = (r_1 + r_2)/2 = 0.338/2 = 0.17$</td><td>0.17</td></tr>
</table>

Source: Author's computations based on selected data from primary studies

New Approach for Monetary Valuation of the Statistical Life

Frantz Daniel Fistung[1]

Abstract: This work aim at proposing a new method for monetary valuation of Statistical life. This approach is different from the existing ones in that moment because propose to link the Value of Statistical Life with to major economic indicators: Gross Domestic Product per capita and the Life Expectancy. Comparing the results obtained using the new formulas proposed in this work, with some other analysis made, at the international level, on the same purpose, the differences are not significant. The proposed method is more relevant and creates the possibility of adopting a unique value for the Value of Statistical Life at world level.

Keykords: Life expectancy; monetary valuation method; value of statistical life comparisons

JEL Classification: A13; B41; C22

1. Introduction

I believe in God and I think that the human life is priceless. Even I have these believes I must accept that science needs, often, some instruments, techniques or models that are outside of our intimate convictions. This is for a simple reason. Many times, we must justify or dimension our activities in order to survive and develop ourselves.

In this respect, certain approaches such as monetary evaluation of some aspects, such as human life, represent always, a real problem, both in terms of methodological approaches, and in some cases, of moral reasons. However, it turned out that such approaches are need in order to dimension the changes made on human health status and relate this with some other important life aspects such morbidity or mortality.

Therefore, the Value of Statistical Life (VSL), in monetary terms, is only a theoretic tool needed in human society planning, developing and management. Between the areas where VSL is used it can be mentioned the insurances industry and transportation. In this last domain, VSL is use in order to proper evaluation of the

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total social cost of the transportation activities and especially for the monetary valuation of the external costs due to transportation.

The evaluation of the total social cost of transportation represent one of the most important activity goals, in the last decades, for many researchers in transportation economics. Between them, I can mention J.M. Beauvais (1977)[1], G. Bouladon (1979), A. Kanafi, who have done some special studies for OECD, A.J. Harrison (1983) who investigate the situation in EU and recently E. Quinet (1990) or D. Maddison and D. Pearce (1996).

Even so, some insuperable difficulties still exist in monetary valuation of the total social costs of transportation due, mainly, by the lack of some generally accepted method for the external costs monetization. However, in the last 10-15 years some big steps forward has been done in that area. For instance, for the air pollution and noise especially the scientific researchers made some promising results from EU and USA.

Interesting is that in all the recent researches, make at international level, it aimed to evaluate the effects on the environment taking into consideration:

- The evaluation of the impact on the environment;
- The technological evaluation;
- The elaboration of the ecological balance.

Obviously, the analysis concerning the impact on the natural environment cannot be separate from those of some other important aspects such as social and economic indicators such as: demographic evolution and effects produced on migration/immigration, population repartition and the evolution of the employment, jobs repartition, space utilization, urban planning and so one. For these analyses, the VSL is extremely important.

In economic terms, the Value of a Statistical Life (VSL) is the amount of money a person (or society) is willing to spend to save a life[2]. The act of placing a monetary value on human life is bound to stir up ethical, religious and philosophical questions. Even if one can be pass these deeper issues, there is still much debate on the correct way to dimension, indirectly, the VSL.

Nils Axel Braathen at the OECD[3], who collected all the published values for statistical life calculated by contingent valuation methods, therefore carried out a meta-analysis, regarding the models for evaluation of VSL. According to this

[1] See (Fistung, 1999).
[2] Definition taken after Maxwell School of Syracuse University, at http://sites.maxwell.syr.edu/vsl/.
[3] The Value of Statistical life: a meta-analysis, (2012), Working Party on National Environmental Policies, OECD, ENV/EPOC/WPNEP(2010)9/FINAL.

analysis, we can divide into three main categories the methods that were used to determine the value of statistical life. The first category based on the compensation paid to accident victims by insurance companies and accounts for the fact that these benefits only cover insured losses. The second category, referred to the human capital, estimates the prejudice caused to society by the death or injury of an individual. The third category is referring to the willingness to pay principle.

My new approach is relate to the second category, even if the third one has received the most attention in recent years.

2. Methodology

The new model postulate that for each individual it is necessary to attach an expected utility function related to the living conditions at national level, very well determined by the value of GDP/capita (note with GDP in further formula) and the expectancy of life (E[1]):

$$VSL = f\,(E, GDP) \qquad /1/$$

These two variables includes, in my opinion, the most important characteristics that could influence the dimension of the Statistical Value of life. GDP/capita reflects most accurately the annual value of the country economic performance related to each individual, and I underline that, in my opinion, this is the only monetary valuable indicator in this approach. In addition, the expectancy of life reflects the period, between borne and death moments, that individuals may gather the value of the annual country economic performance. Therefore, in this respect, VSL of the individuals of each country of the world differs because of these two variables.

According to this premise, our formula will be:

$$VSL_i = E_i \text{ x } GDP_i \quad \text{(monetary units),} \qquad /2/$$

Moreover, for a period the formula became:

$$\overline{VSLi} = \frac{\sum_{i=1}^{n} E_i x GDP_i}{n} \quad \text{(monetary units),} \qquad /2'/$$

With: i = the year of the VSL evaluation;

n = number of years of the period that is taking into analysis;

E_i = the expectancy of life in the “i” year;

GDP_i = GDP/capita in the “i” year.

[1] Life expectancy at birth is defined as the mean number of years still to be lived by a person at birth - if subjected throughout the rest of his or her life to the current mortality conditions (http://ec.europa.eu/eurostat/tgm/web/table/description.jsp).

Some of the most important methods for the evaluation of VSL show us the variety of VSL dimensions (Table 1)

Table 1. VSL estimate in different countries on the based of different methods[1]

PAPER (AUTHORS)	No.obs	Publication year	Country	Average VSL (USD)	Range (million USD)
Alberini et al.	2	2004	United States	1,421,025	1.1-1.7
Alberini et al.	3	2007	Italy	3,598,485	1.4-1.6
Alberini et al.	2	2006	Canada – United States	1,036,062	0.8-1.2
Chestnut et al.	12	2009	Canada – United States	5,142,629	2.5-9.4
Desaigues et al.	6	2004-2007	Denmark	2,651,682	1.1-4.9
Gibson et al.	1	2007	Thailand	659,955	----
Giergiczny	3	2006	Poland	795,082	0.2-1.7
Hakes & Viscusi	2	2004	United States	6,247,816	6.1-6.4
Hammit & Zhou	12	2006	China	115,515	0.02-0.4
Itaoka et al.	19	2007	Japan	1,280,220	0.5-2.8
Johannesson, Johansson & O'Conor	4	1996	Sweden	4,652,973	2.0-7.1
Jones-Lee, Hammerton & Philips	4	1985	United Kingdom	5,226,967	3.9-7.2
Krupnick et al.	8	2002	Canada	1,758,343	1.1-3.6
Krupnick et al.	110	2006	China	562,225	0.1-1.7
Leiter & Pruckner	24	2008-2009	Austria	3,021,948	1.9-5.2
Leiter & Pruckner	4	2008	Austria	2,445,736	2.1-2.8
Mahmud	4	2006	Bangladesh	5,248	0.04-0.07
Leung et al.	8	2009	New Zealand	2,870,491	1.8-4.4
Rheinberger	2	2009	Switzerland	4,362,827	4.2-4.5
Schwab Christe & Soguel	6	1995	Denmark	13,600,000	9.0-17.5

[1] Source: The Value of Statistical life: a meta-analysis,(2012), Working Party on National Environmental Policies, OECD, ENV/EPOC/WPNEP(2010)9/FINAL

Svensson	14	2009	Sweden	7,693,884	3.0-9.6
Vassanadumrondgee & Matusoka	4	2005	Thailand	1,555,256	1.3-1.8

3. Data and Results

In the Table 2, I present some comparisons between VSL levels presented in Table 1 and those obtained using the formula /2/.

Table 2. Comparisons between the sizes of the VSL calculated within the formula /2/ and other authors' methods

Country	Year	E[1] (years)	GDP/capita (USD)**	VSL after /2/ formula (USD)	VSL after other authors (USD)	Ratio between columns (5) and (6)	Rate *	Author
1	2	3	4	5	6	7	8	9
USA	2004	81.1	41,921.80	3,399,857.98	1,421,025	2.39[2]	-	Alberini
Italy	2007	84.2	40,005.20	3,368,437.84	3,598,485	0.94	1.460	Alberini
Poland	2006	79.7	9,500.70	757,205.79	795,082	0.95	1.319	Giergiczny
USA	2004	81.1	41,921.80	3,399,857.98	6,247,818	0.54	-	Hakes & Viscusi
China	2006	75.5	2,082.20	157,206.1	115,515	1.36	-	Hammitt & Zhou
Japan	2007	86.0	34,033.70	2,926,898.2	1,280,220	2.29[2]	-	Itaoka
Sweden	1996	81.7	32,587.30	2,662,382.41	4,652,973	0.57	-	Johannesson & others
UK	1985	77.6	8,652.20	671,410.72	5,226,967	0.13[2]	-	Jones-Lee & others
Canada	2002	82.0	23,995.00	1,967,590.00	1,758,343	1.12	-	Krupnick
China	2006	75.5	2,082.20	157,206.10	562,225	0.28[2]	-	Krupnick
Austria	2008	83.3	48,860.74	4,070,099.64	2,445,763	1.66	1.392	Leiter & Pruckner
New Zealand	2009	82.6	27,998.60	2,312,684.36	2,870,491	0.81	-	Leung
Switzerland	2009	84.6	71,678.30	6,063,984.18	4,362,827	1.39	1.433	Rheinberger
Sweden	2009	83.5	47,737.75	3,986,102.12	7,693,884	0.52	1.433	Svensson

Italy***	2007	84.2	37,716.40	3,175,720.88	3,598,485	0.88	-	Alberini
Poland***	2006	79.7	8,999.70	717,276.09	795,082	0.90	-	Giergiczny
Austria***	2008	83.3	51,386.40	4,280,487.12	2,445,763	1.75	-	Leiter & Pruckner
Switzerland ***	2009	84.6	69,672.00	5,894,251.20	4,362,827	1.35	-	Rheinberger
Sweden***	2009	83.5	46,207.10	3,858,292.85	7,693,884	0.50	-	Svensson

Notes:

1 – I take the maximal values of E and for this reason the table present the values specific to females because they are, in general, greater than for males

2 - Values considered by me to be extremes and not taken into consideration in the analysis

* - Rate = the exchange rate between USD and EURO for 2006-2009 period is based on X-RATES at http://www.x-rates.com/historical/?from=USD&amount=1&date=2016-03-30.

** - For Europe, GDP/capita data is from EUROSTAT at http://ec.europa.eu/geninfo/legal_notices_en.htm, for other countries from OECD database at http://data.worldbank.org/indicator/NY.GDP.PCAP.CD.

**** - GDP/capita data is from OECD database at http://data.worldbank.org/indicator/NY.GDP.PCAP.CD.*

In this example, I use data collected from OECD (expectancy of life and GDP/capita)[1] and EUROSTAT databases (GDP/capita)[2].

Comparing data from columns 5 and 6 of the Table 2 (ratio between them is presented in column 7) we could easy see that if we cut the extremes values, the VSL levels obtained with formula /2/ differ between - 48.2% and + 66.4%, comparing with the medium values calculated according to other methods. That is not a very big margin of differentiation and the values calculated are in the range of the data obtained by the various researchers presented in the same Table 2.

Moreover, in my opinion, formula/2/is more appropriate for VSL estimation than other methods. The explication is very simple. If we compare the columns 5 and 6 of the Table 2 and agree that the VSL is a function related to E and GDP, the results obtained for VSL using some other authors' methods are not realistic. For example, in Table 2 the ratio, between VSL and GDP show us, for example, some values around 161 for Sweden (Svensson, 2009) 270 for China (Krupnick, 2006) and 150 for USA (Hakes & Viscusi, 2004). Taking into consideration formula /2/ that values

[1] Source: http://stats.oecd.org/(Health Status: Life expectancy).
[2] EUROSTAT at: http://ec.europa.eu/eurostat/tgm/table.do?tab=table&init=1&plugin=1&language=en&pcode=tec00001.

must indicate the years related to the life expectancy for the countries previous mentioned. That is not the reality, obviously. Therefore, formula/2/is more accurate than the other methods used for VSL calculation.

The function presented in /1/assumes the, each individual of a country have the same VSL, in the same period of analysis. This theory is different to that of Jones-Lee who underline that the VSL level and individual age are related (Jones-Lee et. alli., 1993). In that theory Jones-Lee consider that, the reference value (maximal value) of VSL is for an individual of 40 years old. In his opinion the lowest VSL are for youngest (under 18 years) and oldest (over 65 years) peoples (see Table 3).

Table 3. The variation statistical value of life depending on age

Age	Report to the VSL reference (40 years)
18	0,67
20	0,77
30	0,89
40	1,00
50	0,79
60	0,70

Source: Jones-Lee (1993)

I totally disagree with this idea. In Jones-Lee, theory it appears that a little children life is less "valuable" that of some mature individual. That is a mass. In my opinion, each individual must be, analytically and statistically, considered equal with others without taking into consideration the age. It is obvious that the people community is composed by kids, mature people and older. They are different off course but, economically speaking, the GDP of a nation is do for all the country inhabitants, without taking account of their age. So, why should we adopt different levels of VSL, based on the age of individuals? Moreover, the calculation of GDP/capita never take into account the age of the countries inhabitants.

However, is necessary to use the age of each individual for evaluate this VSL? For that response, I propose to take an example. I will use the /2'/formula for calculate the VSL for two different aged people both from the same country. I will make this, in two ways. Firstly, I will calculate the VSL in the year of investigation with formula /2/. Secondly I will calculate the medium value of VSL for the periods determinate by the born data of the individuals of analysis and the actual year of evaluation (with formula /2'/).

For instance, I take the situation of two Romanian born one in 2006 and another one in 2012. The year of investigation is 2014. Generally VSL for Romania in 2014 is (using /2/ formula) 714,806.33 USD (Table 4).

Table 4. Annually VSL for Romania according to /2/ correlation, 2006-2014

YEAR	E (years)**	GDP/capita (EURO)	EURO/USD (exchange rate)*	GDP/capita (USD)	VSL_{RO} (USD)
2006	76.1	4600	1.319548	6,023.92	458,420.31
2007	76.8	6000	1.460044	8,760.26	672,787.97
2008	77.5	6900	1.392044	9,605.10	744,395.25
2009	77.7	5900	1.433566	8,458.04	657,189.71
2010	77.7	6300	1.340191	8,443.20	656,036.64
2011	78.2	6600	1.295900	8,552.94	668,839.91
2012	78.1	6700	1.318464	8,833.71	689,912.75
2013	78.7	7200	1.377614	9,918.82	780,611.13
2014	78.7	7500	1.211023	9,082.67	714,806.33

NOTES:

* - The exchange rate between USD and EURO for 2006-2009 period is based on X-RATES at http://www.x-rates.com/historical/?from=USD&amount=1&date=2016-04-12 (at 31 December of each year)

** - Source: http://ec.europa.eu/eurostat/tgm/refreshTableAction.do?tab=table&plugin=1&pcode=tsdph100&language=en

According to formula /2'/, and the Table 4 data, the medium VSL for Romania in the period 2006-2014 ($\overline{VSL_{RO(2006\text{-}2014)}}$) is:

$$\overline{VSL_{RO(2006\text{-}2014)}} = (VSL_{RO\text{-}2006} + VSL_{RO\text{-}2007} + VSL_{RO\text{-}2008} + VSL_{RO\text{-}2009} + VSL_{RO\text{-}2010} + VSL_{RO\text{-}2011} + VSL_{RO\text{-}2012} + VSL_{RO\text{-}2013} + VSL_{RO\text{-}2014})/ 9 = 671{,}444.44 \text{ USD}$$

The same formula gives us the:

$$\overline{VSL_{RO(2012\text{-}2014)}} = (VSL_{RO\text{-}2012} + VSL_{RO\text{-}2013} + VSL_{RO\text{-}2014})/ 3 = 728{,}443 \text{ USD}$$

Therefore, using medium values, for the Romanian child born in 2006 the VSL is equal with $\overline{VSL_{RO(2006\text{-}2014)}}$ and have the value of 671,444 USD. For the other child, borne in 2012 the VSL is equal with $\overline{VSL_{RO(2012\text{-}2014)}}$ which value is 728,443 USD. Nevertheless, annual value, for the year of interest (2014) is 714,806.33 USD. In this case, a life insurance policy will offer, in the same year 2014, terms far more generous for the child borne in 2012 than that one borne in 2006. This is not a normal point of view and is opposite with the Jones-Lee theory which assume that the youngest individual have a lover VSL. So, is much proper to use, in that case, for both kids, the annual VSL in order to calculate the value on a life insurance policy.

In the same time, in my example for the period analyzed, cutting out the extreme values, we can see that the medium value of VSL is, mostly, closely to each annual VSL figures (see Fig. 1).

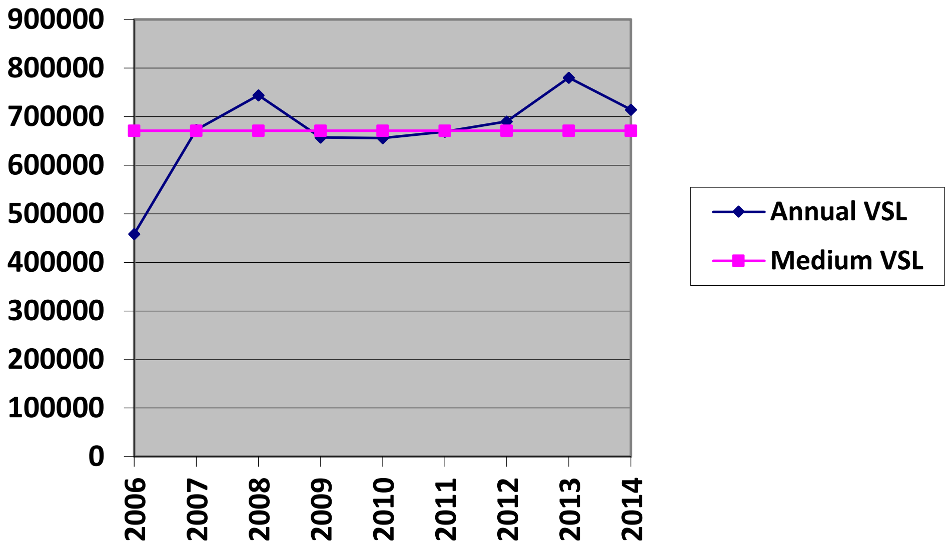

Figure 1. Annual and medium values for VSL in Romania, in 2006-2014 period

Despite this is obviously that if we calculate the medium VSL the age of individuals influence the values obtained. That confirms somehow the Jones-Lee theory but, in my opinion, it is very unjust.

In that respect, I recommend to use only the /2/formula when we want to make some investigations to evaluate some specific socio-economic aspects on specific conditions. For example, when we try to evaluate the level of damages produced on human health by the air pollution due to cars. The medium value of VSL is also OK, in my opinion, but in another context.

In that idea, we must introduce, into analysis, a new dimension: the space. In my attempt, the space will be define by the nations.

A controversial issue is if the VSL is the same for each individual, no matter where he lives. Certainly, in my opinion, the life value of each individual, in this world, must be the same. However, unfortunately, here appears many distortions related to economic activities, age, place of living and others. It is obviously that, taking into consideration the formula /2/described before, each individual VSL, differs from country to country. This is because of the GDP/capita variations. Very developed countries will have inhabitants much „valuable”, in statistical matter of speaking, comparing with the inhabitants of the poor countries. Nevertheless, in my opinion, the life expectancy corresponds quite accurately with the level of GDP/capita of the countries where we make our investigations. According to this idea, if we want to make a uniformed VSL for the entire world, or a country union (such OECD or UE, for instance) we can use arithmetic media of the /2/and /2'/formulas for the all-union countries (Formula /3/). For example, we can calculate the VSL of the OECD countries, in a desired year (Table 5) or a medium VSL for a period. In my opinion, this assumption will attract a lot of complaints and contradictory debates so, in that working paper I want only to launch this debate.

Table 5. General VSL for OECD countries in 2011-2013 period

Country	2011			2012			2013		
	E[1]	GDP[2] /capita (USD)	VSL (USD)	E[1]	GDP[2] /capita (USD)	VSL (USD)	E[1]	GDP[2] /capita (USD)	VSL (USD)
Australia	84.2	43,702	3,679,708.4	84.3	43,081	3,631,728.3	84.3	46,826	3,947,431.8
Austria	83.8	44,039	3,690,468.2	83.6	45,878	3,835,400.8	83.8	47,428	3,974,466.4
Belgium	83.3	41,118	3,425,129.4	83.1	42,209	3,507,567.9	83.2	43,362	3,607,718.4
Canada	83.6	41,565	3,474,834.0	83.6	42,144	3,523,238.4	83.6	44,281	3,701,891.6
Chile	81.4	20,189	1643384.6	81.3	21,295	1,731,283.5	81.4	21,366	1,739,192.4
Czech Republic	81.1	28,603	2,319,703.3	81.2	28,732	2,333,038.4	81.3	30,054	2,443,390.2
Denmark	81.9	43,319	3,547,826.1	82.1	44,251	3,633,007.1	82.4	45,697	3,765,432.8
Estonia	81.3	23,914	1,944,208.2	81.5	25,872	2,108,568	81.7	27,124	2,216,030.8
Finland	83.8	40,251	3,373,033.8	83.7	40,437	3,384,576.9	84.1	40,951	3,443,979.1
France	85.7	37,353	3,201,152.1	85.4	37,499	3,202,414.6	85.6	39,236	3,358,601.6
Germany	83.2	42,942	3,572,774.4	83.3	43,600	3,631,880	83.2	44,999	3,743,916.8
Greece	83.6	26,626	2,225,933.6	83.4	25,980	2,166,732	84.0	26,753	2,247,252.0
Hungary	78.7	22,603	1,778,856.1	78.7	22,701	1,786,568.7	79.1	24,037	1,901,326.7
Iceland	84.1	39,558	3,326,827.8	84.3	40,278	3,395,435.4	83.7	42,715	3,575,245.5
Ireland	83.0	45,670	3,790,610	83.2	46,030	3,829,696	83.1	47,563	3,952,485.3
Israel	83.5	30,585	2,553,847.5	83.6	32,007	2,675,785.2	83.9	33,397	2,802,008.3
Italy	84.8	35,464	3,007,347.2	84.8	35,424	3,003,955.2	85.2	35,465	3,021,618.0
Japan	85.9	34,332	2,949,118.8	86.4	35,738	3,087,763.2	86.6	36,620	3,171,292.0
Korea	84.5	31,327	2,647,131.5	84.6	32,223	2,726,065.8	85.1	32,664	2,779,706.4
Luxembourg	83.6	90,889	7,598,320.4	83.8	90,694	7,600,157.2	83.9	95,587	8,019,749.3
Mexico	77.2	16,366	1,263,455.2	77.3	16,959	1,310,930.7	77.4	16,947	1,311,697.8
Netherlands	83.1	46,389	3,854,925.9	83.0	46,457	3,855,931.0	83.2	47,967	3,990,854.4
New Zealand	82.9	32667	2,708,094.3	83.0	32,991	2,738,253.0	83.2	36,947	3,073,990.4
Norway	83.6	62,738	5,244,896.8	83.5	65,394	5,460,399	83.8	66,812	5,598,845.6
Poland	81.1	22,250	1,804,475.0	81.1	23,310	1,890,441	81.2	24200	1,965,040.0
Portugal	83.8	26,932	2,256,901.6	83.6	27,125	2,267,650	84	27,930	2,346,120.0
Slovak Republic	79.8	25,169	2,008,486.2	79.9	26,098	2,085,230.2	80.1	27,416	2,196,021.6
Slovenia	83.3	28,513	2,375,132.9	83.3	28,487	2,372,967.1	83.6	29,103	2,433,010.8
Spain	85.6	32,535	2,784,996.0	85.5	32,240	2,756,520.0	86.1	32,861	2,829,332.1
Sweden	83.8	43,709	3,662,814.2	83.6	44,434	3,714,682.4	83.8	45,067	3,776,614.6
Switzerland	85.0	54,551	4,636,835.0	84.9	57,205	4,856,704.5	85.0	59,351	5,044,835.0

Turkey	77.1	17,692	1,364,053.2	77.2	18,437	1,423,336.4	79.4	19,156	1,520,986.4
United Kingdom	83.0	36,575	3,035,725	82.8	37,567	3,110,547.6	82.9	39,125	3,243,462.5
United States	81.1	49,710	4,031,481	81.2	51,368	4,171,081.6	81.2	52,592	4,270,470.4
VSL OECD	**3,081,837.874**			**3,141,456.974**			**3,265,118.147**		

Source: 1 - http://stats.oecd.org/(Health Status : Life expectancy)

2 - http://stats.oecd.org/ (National accounts; Gross domestic product (GDP: GDP per head, USD, current prices, current PPPs)

Medium level for VSL, in a period and for a countries union will be based on formula:

$$\overline{VSL_{CU}} = \{\sum_{j=1}^{m} \sum_{i=1}^{n} (\frac{Eij\ x\ GDPij}{n})\}/m \quad \text{(monetary units)} \quad /3/$$

With (in addition to those previously defined):

$\overline{VSL_{CU}}$ = Value of Statistical Life for a "countries union"

j = the country "j"

m = number of the countries in the union

E_{ij} = the expectancy of life for the country "j" in the year "i"

GDP_{ij} = GDP/capita for the country "j" in the year "i"

Using the data presented in the Table 5, and the /3/ formula, we can calculate, for the 2011-2013 period, the medium value of $VSL_{OECD:}$

$\overline{VSL_{OECD(2011-2013)}}$ = ($VSL_{OECD\ (2011)}$ + $VSL_{OECD\ (2012)}$ + $VSL_{OECD\ (2013)}$)/3 = 3,162,804.331 **USD**.

Therefore, for each individual of OECD countries shown in Table 5 we can assume that his VSL is around 3.2 million USD. This assumption seems to be OK if we consider that, for example, the UK VSL_{UK} used in evaluation of external costs due to transportation is around 3 million of British Pounds (D.Maddison, D.Pearce, coord (1996)).

Interesting is if we make the same analysis for the VSL evolution in EU. After the EUROSTAT databases, the medium annual VSL for EU (with 28 members) in 2010 was 2,094,840 EURO and increase with 8.9% in 2014 (see Table 6).

Table 6. Annual and medium VSL evolution for EU and EU countries in 2010-2014 period

YEAR	2010			2012			2014			
COUNTRY	GDP /capita (EURO)	E	VSL (EURO)	GDP /capita (EURO)	E	VSL (EURO)	GDP /capita (EURO)	E	VSL (EURO)	R
EU (28 countries)	25,300	82.8	**2,094,840**	26,400	83.1	**2,193,840**	27,300	83.6	**2,282,280**	***2,190,320***
Belgium	33,600	83.0	2,788,800	35,100	83.1	2,916,810	36,000	83.9	3,020,400	
Bulgaria	4,900	77.4	379,260	5,600	77.9	436,240	5,800	78.0	452,400	
Czech Republic	14,900	80.9	1,205,410	15,300	81.2	1,242,360	14,700	82.0	1,205,400	
Denmark	43,500	81.4	3,540,900	44,900	82.1	3,686,290	45,600	82.8	3,775,680	
Germany	31,600	83.0	2,622,800	33,600	83.3	2,798,880	35,400	83.6	2,959,440	
Estonia	11,000	80.8	888,800	13,300	81.5	1,083,950	14,800	81.9	1,212,120	
Ireland	36,200	83.1	3,008,220	37,600	83.2	3,128,320	40,200	83.5	3,356,700	
Greece	20,300	83.3	1,690,990	17,500	83.4	1,459,500	16,300	84.1	1,370,830	
Spain	23,200	85.5	1,983,600	22,600	85.5	1,932,300	22,800	86.2	1,965,360	
France	30,800	85.3	2,627,240	31,800	85.4	2,715,720	32,200	86.0	2,769,200	
Croatia	10,500	79.9	838,950	10,300	80.6	830,180	10,200	81.0	826,200	
Italy	26,800	84.7	2,269,960	26,800	84.8	2,272,640	26,600	85.6	2,276,960	
Cyprus	23,000	83.9	1,929,700	22,500	83.4	1,876,500	20,500	84.7	1,736,350	
Latvia	8,600	78.0	670,800	10,900	78.9	860,010	12,100	79.4	960,740	
Lithuania	9,000	78.9	710,100	11,200	79.6	891,520	12,400	80.1	993,240	
Luxembourg	77,900	83.5	6,504,650	8,2000	83.8	6,871,600	88,500	85.2	7,540,200	
Hungary	9,800	78.6	770,280	9,900	78.7	779,130	10,500	79.4	833,700	
Malta	15,900	83.6	1,329,240	17,200	83.0	1,427,600	18,500	84.2	1,557,700	
Netherlands	38,000	83.0	3,154,000	38,500	83.0	3,195,500	39,300	83.5	3,281,550	
Austria	35,200	83.5	,2939,200	37,600	83.6	3,143,360	38,500	84.0	3234,000	
Poland	9,300	80.7	750,510	10,000	81.1	811,000	10,700	81.7	874,190	
Portugal	17,000	83.2	1,414,400	16,000	83.6	1,337,600	16,600	84.4	1,401,040	
Romania	6,300	77.7	489,510	6,700	78.1	523,270	7,500	78.7	590,250	
Slovenia	17,700	83.1	1,470,870	17,500	83.3	1,457,750	18,100	84.1	1,522,210	
Slovakia	12,400	79.3	983,320	13,400	79.9	1,070,660	13,900	80.5	1,118,950	
Finland	34,900	83.5	2,914,150	36,900	83.7	3,088,530	37,600	84.1	3,162,160	
Sweden	39,400	83.6	3,293,840	44,500	83.6	3,720,200	44400	84.2	3,738,480	
United Kingdom	28,900	82.6	2,387,140	32,000	82.8	2,649,600	34,500	83.2	2,870,400	
EU Year Average			**1,984,165.7**			**2,078,822.0**			**2,164,495.0**	**2,075,827.5**

Source: EUROSTAT, at http://ec.europa.eu/eurostat/tgm/table.do?tab=table&init=1&plugin=1&language=en&pcode=tec00001 and http://ec.europa.eu/eurostat/tgm/table.do?tab=table&init=1&plugin=1&language=en&pcode=tsdph100.

NOTE: I take the maximal values of E and for this reason the table present the values specific to females because they are, in general, greater than for males.

$R = \overline{VSL_{EU\ 2010\text{-}2014}}$ (EURO)

Also, if we calculate the $\overline{VSL_{UE(2010\text{-}2014)}}$ using the EU (28 countries) with /2/ formula and data for 2010, 2012 and 2014 we obtain a value of 2,190,320 Euro. Making the arithmetic for the 2010, 2012 and 2014 VSL year average (shown in Table 6) we obtain a value of 2,075,827.5 Euro. Between these two values, the difference is about 5.2%, and that is more than acceptable.

Moreover, in the analyzed period the annual $\overline{VSL_{UE}}$ increase with 9%, but $\overline{VSL_{UE(2010\text{-}2014)}}$ differs only with 4.6% to -4.1% according to the annual values of the period (see also Fig. 2).

Therefore, similar to the formula /2/, formula /3/ could be used at annual or medium values. In my opinion, because we speak of a large space dimension (regions, countries union or even the entire world) an economic strategy that is focused to evaluate the possibility to increase the human life conditions and need to use VSL must take into consideration the annual value of VSL (formula /3/ for the year of the analysis).

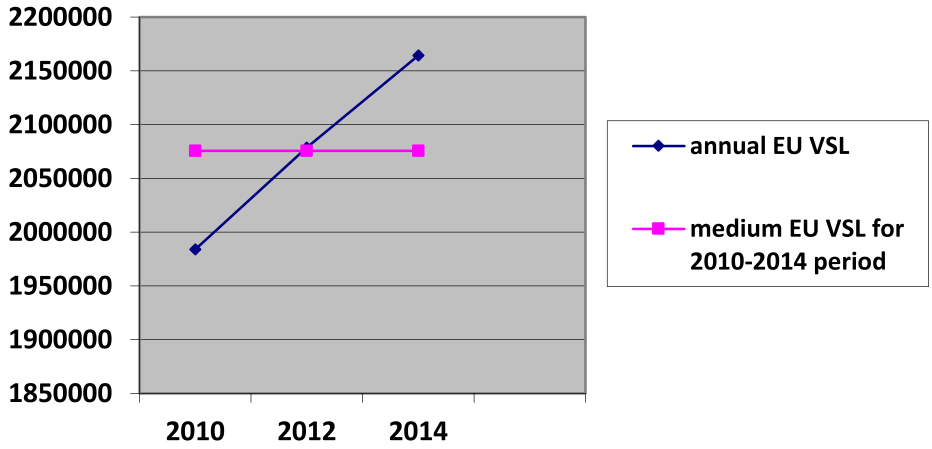

Figure 2. Differences between yearly medium and period VSL for EU, in 2010-2014 period

In addition, if we want to make some comparisons between some regions VSL in different time periods we better use the medium VSL values for those periods and in the same countries union.

In conclusion, I think that, generally speaking, is more proper to use the annual VSL calculate with /2/formula or /3/formula for only one year. In some specific conditions and for bigger spaces (countries unions) we can use, also, medium VSL calculate with /3/ formula.

4. Conclusions

Analyzing the data resulted by using this new approach, I can underline the most important conclusions:

- Despite other points of view, I consider that VSL is not relate to the individual age.
- The VSL level is the same for each individual of an area of analysis (country, region) and in the same period where the main variables of the /2/ formula (GDP/capita and expectancy of life) are identically.
- The annual and medium values of VSL are different but both could be useful for economic analyses.
- The annual VSL is better to be use in transportation dimension of external costs and in life insurance activities.
- The medium value of VSL is properly to be use in the economic analyses that are make at world or regional levels for time period comparisons.
- It would be great if we could adopt a unique VSL (using /3/ formula) for each individual of the earth.

5. Bibliography

Braathen, N.A. (2012). *The Value of Statistical life: a meta-analysis*. *Working Party on National Environmental Policies*, OECD, ENV/EPOC/WPNEP(2010)9/FINAL.

EUROSTAT (2016). Gross domestic product at market prices, EUROSTAT at: http://ec.europa.eu/eurostat/tgm/table.do?tab=table&init=1&plugin=1&language=en&pcode=tec00001.

Fistung, D. (1999). Transporturi. Teorie economica, ecologie, legislatie (only in romanian language). Bucharest: All Beck Publishing House.

Jones-Lee, M.W. & Loames, G. et.al. (1993). The value of preventing non-fatal injuries: findings of a willingness to pay national sample survey. *Working Paper SRC/2*, Transport and Road Research Laboratory, Crowthorne.

Maddison, D. & Pearce, D. coord. (1996). *The true costs of road transport*. London: Earthscan Publications Ltd.

OECD (2016). *Health Status: Life expectancy*, OECD statistics at: http://stats.oecd.org/Index.aspx?DatasetCode=HEALTH_STAT

WORLD BANK (2016). *GDP per capita*, WORLD BANK at: http://data.worldbank.org/indicator/NY.GDP.PCAP.CD.

*** (2016). *Historic Lookup: US DOLLAR rates table*, The XE Currency Data API, at: http://www.x-rates.com/historical/?from=USD&amount=1&date=2016-03-30.