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Determination of the Fractal Character of the Romanian Capital Market by Using Hurst Exponent

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Abstract: Failure of classical statistical methods to model the behavior of stock market prices determined, in a domino effect, the failure of the classical paradigm of regarding markets as efficient systems. Alternative to this simple interpretation, markets should be regarded as far from equilibrum dynamical systems, complex evolving structures that encompass millions of participants, holding into their memory events that happened long time ago. This more realistic approach was developed by Fractal Market Hypothesis, as an alternative to Efficient Market Hypothesis. R/S Analysis is a robust tool for testing whether markets follow a Brownian motion or some memory effect is implied. The aim of the paper is to determine the Hurst Exponent, for company ALRO S.A., for the period of time since listing, until 16/07/2021. Results may generate indications about in the nature of the system represented by the prices of ALRO S.A. Conclusion may be that the Romanian capital market, as ALRO is one of the most representative companies listed at Bucharest Stock Exchange, has evolved from a very low stability market to a more stable investment environment.

Keywords: efficient market hypothesis; fractal market hypothesis; capital market; R/S analysis; Hurst Exponent; capital market

JEL Classification: L22

1. Introduction

The goal of the paper was to present determination of the Hurst Exponent on the historical time series of company Alro S.A., as R/S analysis (Hurst Exponent is the slope of the R/S discrete values) is the mathematical tool used for testing the Fractal Market Hypothesis. The

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results give substantial information, and, as a conclusion, the trend of ALRO daily closing prices indicates translation from an antipersistent trend, going to a random walk phase and reaching a persistent trend phase. The Hurst Exponent indicates the presence of a long-term memory effect, in the case H is different from 0.5. As the value of this metric is 0.5, it detects the independence of the studied series, but offers no indications about distribution.

Fractal market hypothesis (FMH) was developed by Peters (1989, 1992) as a replica of efficient market hypothesis. According to FMH, the emphasis is no longer on market efficiency, but on its stability. The market is considered stable when it is "liquid", in the sense of an unbalanced trading volume.

As Efficient Market Hypothesis failed to offer its validity, due to thousands of empirical studies, Fractal Market Hypothesis takes into consideration market liquidity, more than its efficiency.

2. Fractal Market Hypothesis

According to Efficient Market Hypothesis, prices reflect and incorporate instantly all available information (definiton of efficient market), so succesive price changes are attributable only to random process of generating new information (this is the true meaning of randomness, not information is random, but the process of penetration of new information is random). This means that the new information, and only it, can dictate future price changes, and as the process is random, future changes are random too. This means that predictibility is impossible, according to EMH. On the other hand, investors take decisions base on different time horizons, meaning that "significant information" for one investor may be categorised as "insignificant" to other, under different time horizons. This means that as new information appears into markets, some investors will not trade it, generating some degree of predictability.

Contrary to the theory of efficient markets, fractal market theory considers that information does not have a uniform impact on prices, information is assimilated differently, depending on different investment horizons. Different time horizons evaluate information differently. At any given time, the prices may not reflect all the information available, but only the information that is important for the investment horizon.

As a matter of fact, it is not information, but investment horizons of different types of investors are the crucial factor that determine liquidity, in other words, stability in the markets. Due to different approaches, a daily trader quantifies different information he receives regarding his daily trading strategy from an investment fund. For him, the impact of a financial news may lead to a "sell"order regarding a stock in his portfolio, at the end of the day, as for a trade fund manager may see as an opportunity to "buy" that same stock, generating an opposite position, and, at the end of the same day, the investment fund successfully "buys" and "closes" open positions of tens of daily traders. There is a hidden, single differenciating aspect that bonds all those together, which is different horizon of time used for taking action. It is time horizon, not information, that generates also stability, with other words, liquidity in the markets. (Peters, 1991)

In case of "turbulent" periods, investors do not have the same ability to uniformly process information. Liquidity effect over price is also not considered under EMH, but as liquidity decreases, and drops down, accordingly to an increasing trading volume, instability generates big price movements. Stability of the market is a matter of liquidity. And as expressed before in the paper, liquidity is available with many investors with different time horizons. For normal periods, liquidity generates market stability in this form. On the contrary, in periods when market loses its structure, all the investors have the same trading horizon. As a result, market becomes unstable, as liquidity fades away. This translates into losing the long-term investors, or their behaviour transformed into short term, meaning the whole market trades on the same information set, which may be technical or crowd phenomenon. For this, the market horizon becomes short term under uncertainty, as effect transforming the long-term information into unreliable or useless. Thus, largest crashes have occured when liquidity was low and trading volume is very high (trading volume means unbalanced supply and demand). EMH cannot explain price crashes, but when liquidity vanishes, getting the "fair" price does not seem so important as executing the trade at any cost. Thus, price is close to "fair" only correlated with liquidity, otherwise, investors are willing to take any price, whether "fair" or not. (Peters, 1994).

As regarding capital markets, their understanding as complex non-linear dynamical systems is far more close to their real identity. From this perspective, basic characteristics were derived:

a) feedback systems - past influences the present, and the present influences the future, in other words, P (t + 1) is a product of P (t).

b) existence of critical levels that allows more equilibrium points and stability bands.

c) self-similarity, feature characteristic for non-linear feedback processes. The complexity of such a system arises when the system is far from equilibrium.

d) the sensitive dependence on the initial conditions, in other words, P (t + n) is dependent, but still very different from P (t).

FMH allows understanding the behavior of market participants through the lens of complex dynamical non-linear processes. Benoit Mandelbrot considered time series of historical prices as derived from "fractional brownian motions", and labeled them as "fractal series". Basic characteristic is correlation between different events, and between time intervals. For stying those time series, the whole history of the price must be accounted. Also, according to FMH, each observation "carries" a memory of all the events that precede it. This is not a short-term "Markovian" memory, but a different, long-term one that should be "eternal". The importance of recent events upon actual price is higher than those of more distant ones, of a longer period of time ago, but there is still residual influence. On a larger scale, a system that displays Hurst statistics is the result of a long series of interconnected events. Basically, what is happening today influences the future, and what happened some time ago influences what is happening today.

From this perspective, FMH studies and applies along time series with large period. Reffering to capital market, the entire historical price time series must be applied to. This implies both time factor as maximal, despite EMH, and phycological behavioral impact of very distant events still discounting into actual price, in trading strategies.

3. Mathematical Tool for Testing FMH – R/S Analysis

Simplified mathematical tool used for testing FMH was developed by Hurst, and taken over by numerous empirical studies, for enlarging its sphere of application into economics and capital markets, is described below:

Let $X = (x_1, x_2, ..., x_n)$ be a sequence of observations at discrete time moments given by the subscript of the observation.
For each $m = 2^k < n$ let $d^{(m)} = \lfloor n/m \rfloor$ and for each b from 1 to $d^{(m)}$ and i from 1 to m let $x_i^{(m,b)} = x_{n-b\cdot m+i}$ and define

$$X^{(m,b)} = (x_1^{(m,b)}, ..., x_m^{(m,b)})$$

as the b-th subsequence of size m

$$E^{(m,b)} = \frac{1}{m}\sum_{i=1}^{m} x_i^{(m,b)}$$

as the mean of observations in $X^{(m,b)}$

$$\sigma^{(m,b)} = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(x_i^{(m,b)} - E^{(m,b)})^2}$$

as the standard deviation of observations in $X^{(m,b)}$

$$N^{(m,b)} = \left(y_i^{(m,b)}\right)_{i=\overline{1,m}} := \left(x_i^{(m,b)} - E^{(m,b)}\right)_{i=\overline{1,m}}$$

as the b-th normalized subsequence of size m

$$C^{(m,b)} = \left(c_i^{(m,b)}\right)_{i=\overline{1,m}} := \left(\sum_{j=1}^{i} y_j^{(m,b)}\right)_{i=\overline{1,m}}$$

as the cumulative sequence for the b-th normalized subsequence of size m

$$\rho^{(m,b)} = \max_i\{c_i^{(m,b)}\} - \min_i\{c_i^{(m,b)}\}$$

as the range of the cumulative sequence defined above

$$r^{(m,b)} = \frac{\rho^{(m,b)}}{\sigma^{(m,b)}}$$

as the rescaled range of the cumulative sequence defined above For each m as above, let

$$r^{(m)} = \frac{1}{d}\sum_{b=1}^{d^{(m)}} r^{(m,b)}$$

Finally, the hurst exponent H_X for the initial sequence of obervations X will be the slope of the regression line of the set of points $\{(\log(m), \log(r^{(m)}))\}_m$.

Note: To calculate the slope α of the regression line of a set of N points $\{(x_k, y_k)\}_k$ we use the following formula derived from the condition that the cummulated fitting error is minimal (i.e. the sum of squares of distances from each point to the regression line is minimal):

$$\alpha = \frac{\left(\sum_k x_k y_k\right) - \frac{\left(\sum_k x_k\right)\cdot\left(\sum_k y_k\right)}{N}}{\left(\sum_k x_k\right)^2 - \frac{\left(\sum_k x_k\right)^2}{N}}$$

The second alternative can be for the parameter k to vary in a linear manner, as taking values from 1 up to 512.

However, estimating the Hurst coefficient does not provide any clues about the time series distribution.

4. Interpretation of the Hurst Exponent

R/S Analyis is still considered a robust tool for determining the memory effect, or the brownian character of a time series.

Depending on the values obtained for the Hurst exponent, there can be identified three categories:

(1) H = 0.5 - pure brownian motion

(2) 0 ≤ H ≤0.5 – anti-persistent time series (probability of a different price change is 1-H)

(3) 0.5 <H <1 – persistent time series.

Most of the economic time series or time series generated by natural phenomenon are persistent time series. Persistent time series, defined by 0.5 <H <1, are fractal because they can be represented by fractional Brownian motions.

In a general manner of interpretation, Hurst exponent determines the probability that if the last price change was positive, the next price would be positive, too. The closest to 1, the strongest the probability the next move would be identical to the prior.

Hurst exponent is a good indicator over the memory effect over price during time. R/S analysis is a robust statistical measure of independent processes. A value of H=0.5 does not imply a random Gaussian process. It only assumes there is no memory effect, with other words, any independent system, either Gaussian or not, may generate a value for H equal to 0.5.

5. Application of R/S Analysis, Using Different Sets of Parameters, on the Company Alro S.A.

Testing FMH on the Romanian Capital market was conducted by applying the R/S analysis and implemented using an informatic script, based on mathematical formulation explained in Section 3, on the historical price time series of company ALRO S.A. In order to test the fractalic character of the time series, all the price observations, since first day of listing at BVB, were taken into consideration, generating a time period between listing and 16th of July, 2021. For the first version of parameters, we used the value k=10, as a predetermined parameter.

The results obtained for the Hurst Exponent are displayed in the below graphical illustrations (Graph 1, 2, 3), print-screens from FRACTAL – RISK platform, and developed using a graphical user interface:

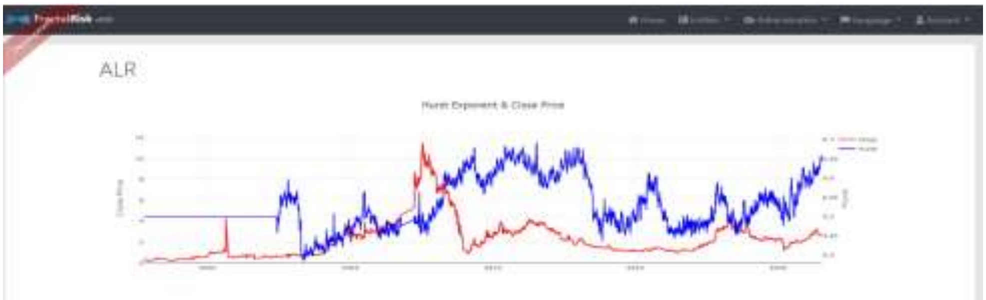

Figure 1. Print screen from FRACTAL-RISK: Estimation of the Hurst exponent for the closing daily prices of ALRO S.A., for the period since first trading day, 16/10/1997 until 16/07/2021 (full length of time)

As it may be noticed in the results obtained, values are placed within interval [0.4, 0.7].

For the first period, between launching and mid-year 2010, values are normally below 0.5, meaning the presence of an ergodic process. The trend is anti-persistent, meaning tendency to reverse. Basically, the value doesn't reach the neighborhood of zero, it is stabilised between 0.4 and 0.5. As a result, there is no stability, according to FMH, meaning no liquidity. One good explanation may be the under-development of the Romanian capital market, lack of experience and interest for potential and actual traders. As a result, we may divide the time span into two intervals for interpreting the results: since launching until mid 2010, and after 2010 until 16/07/2021.

For resolution purpose, the estimations for the Hurst exponent along with the closing daily prices are displayed in the figure below, corresponding the time interval 1997 – 2010 (zoom into data displayed):

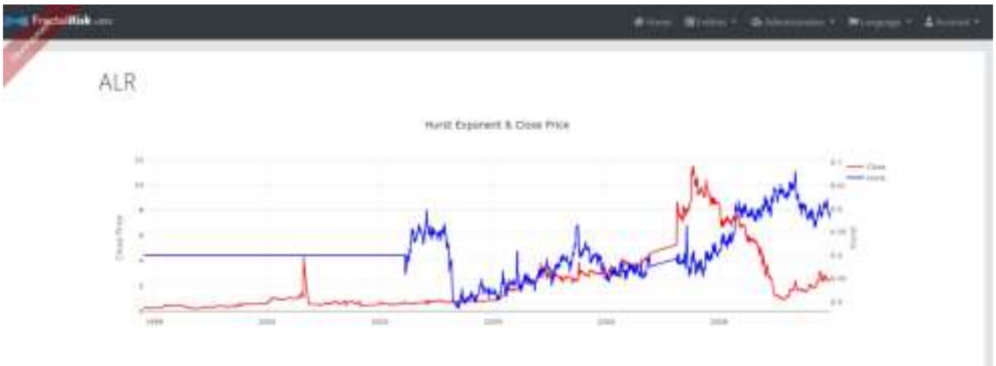

Figure 2. Print screen from FRACTAL-RISK: Estimation of the Hurst exponent for the closing daily prices of ALRO S.A., for the period since first trading day, 16/10/1997 until 2010 (zoom function)

The second group of values for the Hurst exponent display values above 0.5, which would be equal to the random process. Hurst exponent determines a persistent trend, meaning that there may be some memory effect presented. Overall, the memory effect is not very pronounced, as values do not exceed the 0.7 limit. The importance of distant past event is small, as for recent events. Probability that the trend modifies is smaller (1-H), meaning the "strength" of the persistent behavior is above 0.5, but still below 0.7. We may identify some memory effect, meaning the events are being correlated, and also there is some stability inside market.

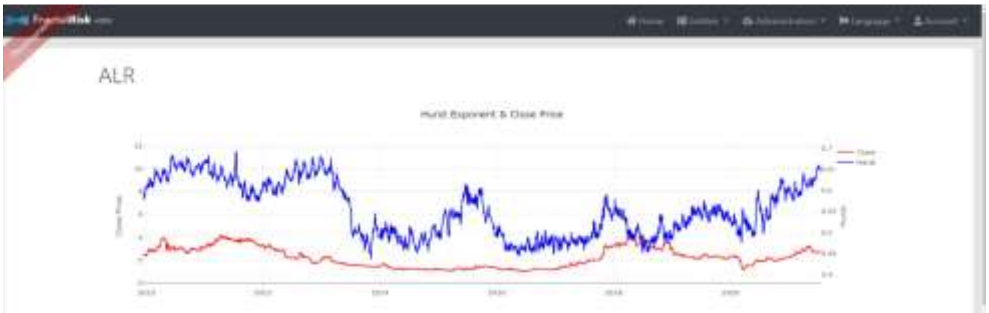

Figure 3. Print screen from FRACTAL-RISK: Estimation of the Hurst exponent for the closing daily prices of ALRO S.A., for the period since 2010 – 2021 (zoom function)

The values of the Hurst exponent reflect the presence of the memory effect, as an indicator for market stability, which translates into increasing liquidity, and a balanced trading volume. Causes for this phenomenon are multiple, among: the development of the Romanian capital market, increasing presence of more educated investors into the market, inflow of money by international funds investing in Romanian stocks. As postulated by FMH, the Hurst exponent simply quantifies the memory effect, in a market with increasing stabililty. As FMH suggests, the stability of the market indicates the presence of a heterogenous spectrum of investors, from daily traders and speculators to investment funds, generating a stable structure of the

Romanian capital market. Also, the investment horizons of the partipants differ significantly, generating liquidity and financial health for the capital market. As for the closing prices, the values for the Hurst exponent suggest that they should be almost similar with their "fair values", due to the stability generated in the market. As for the smoothness of the Hurst line, increased market maturity generates this flatness. There are no big price changes, nor extreme fluctuations, generated by an unbalanced trading volume, combined with small liquidity.

The particular case for H=0.5, when the first range of values for H (<0.5) reaches the value of H=0.5 is specific for a random process, meaning an independent system. In this case, as compared to the price movement, the Hurst exponent indicates precisely an abrupt decline in price, meaning a disproportion between liquidity and trading volume, where trades are executed at any price. Basically, short and long term investors trade under same horizons, due to uncertainty, which is very present in the market. Also, there is no memory effect, prices reflect only a deep fear emotion of the participants in the transactions.

6. Conclusions

Evolution of the Romanian capital market has been explained using R/S analysis, from an unsecured investment environment to a more stable position, R/S Analysis is a robust indicator, for determining even transition from uncertaity to stability. The evolution of historical daily closing prices of ALRO S.A., starting as anti-persistent process, passing through random walk phase, and reaching to persistent processes is the evolution from a very low stability market to a more stable one. Information about maturity, stability and memory process is encompassed within the Hurst Exponent.

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