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The Analysis of Birth Rate in European Union by Means of Fourier Development

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Abstract: In this article, it will carry out an analysis on the regularity of the indicator of all European Union countries. The research method consists in adjusting the statistical data with the help of Fourier series development and determining the periodicity of the phenomenon. Determining the possible periodicity facilitates the adjustment of the causes that lead to cyclization and therefore the mitigation of possible unwanted effects.

Keywords. Cycle; Fourier; periodicity; interpolation

JEL Clasiffication: C65; E17

1. Introduction

In the literature, the economic cycle designates the fluctuations which accompany the evolution of a nation or, sometimes, it simply is associated with the increasing and decreasing of an economy. Throughout history, many states were faced and have experienced economic fluctuations.

Given the complexity of economic phenomena, in practice there are as many types of economic cycles or economic fluctuations. It can say that almost any segment of the economic life is subject to the fluctuations that, sometimes, may include periods of more than a year.

A more comprehensive approach to the problem of an indicator cycle requires knowledge of all aspects of the market economy or social life.

Over time, many economists have attempted, through analysis of available statistical data, to develop specific models of foresights of changes taking place in the economy

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to come to the aid of the decision-makers to act according to actual economic conditions.

2. Mathematical Considerations on the Fourier Development

Let a function f:**R**→**R**, with f and f' piecewise continous on **R** and periodic with period T, therefore f(x+T)=f(x) ∀x∈**R**.

Considering Fourier series associated with the function f: $F(x)=\frac{a_0}{2}+\sum_{m=1}^{\infty}\left(a_m\cos\frac{2m\pi x}{T}+b_m\sin\frac{2m\pi x}{T}\right)$ it has the following:

Lemma 1 ([2]) $\int_{-\frac{T}{2}}^{\frac{T}{2}}F(x)\cos\frac{2n\pi x}{T}dx=\frac{a_nT}{2}$, n≥0, $\int_{-\frac{T}{2}}^{\frac{T}{2}}F(x)\sin\frac{2n\pi x}{T}dx=\frac{b_nT}{2}$, n≥1.

From Fourier series expression, it is observed that F(x+T)=F(x) ∀x∈**R** so its sum is also a periodic function of period T.

The Dirichlet's theorem (Spiegel, 1974) states that in the conditions above, the Fourier series converges punctually to f in the points of continuity and to $\frac{f(x+0)+f(x-0)}{2}$ in the discontinuity points.

Considering the partial sum of order n corresponding to the series of function F, the n-th Fourier polynomials are:

$F_n(x)=\frac{a_0}{2}+\sum_{k=1}^{n}\left(a_k\cos\frac{2k\pi x}{T}+b_k\sin\frac{2k\pi x}{T}\right)$

It is obvious also that $F_n(x)=F_n(x+T)$ ∀x∈**R**.

The Fourier polynomials have the property of approximating the function through one periodical with the observation that the absolute error tends to fall (due to the convergence points) with the rise of n.

Due to the existence of an important number of cyclical phenomena in many scientific fields, it intend, below, to approximate their development by means of Fourier polynomials of degree conveniently chosen.

In the case of the discretized phenomenons, it put the problem in the generation of functions that will pass through a series of data points. A very useful tool is the Lagrange interpolation polynomial. Therefore, considering a set of data (x_i,y_i), $i=\overline{1,k+1}$, the Lagrange interpolation polynomial has the form:

$L_n(x)=\sum_{i=1}^{k+1}\frac{(x-x_1)\dots(x-x_{i-1})(x-x_{i+1})\dots(x-x_n)}{(x_i-x_1)\dots(x_i-x_{i-1})(x_i-x_{i+1})\dots(x_i-x_n)}y_i$

and is the polynomial of minimum degree (k) passing through the data points.

It will demonstrate, first, the following:

Lemma 2 ([2]) Let $f(x) = ax + b \in \mathbf{R}[X]$. Then:

$\int f(x)\, cos\frac{2n\pi x}{T}\, dx = a\frac{T}{2n\pi}x\, sin\frac{2n\pi x}{T} + b\frac{T}{2n\pi}sin\frac{2n\pi x}{T} + a\left(\frac{T}{2n\pi}\right)^2 cos\frac{2n\pi x}{T}$, n≥1

$\int f(x)\, cos\frac{2n\pi x}{T}\, dx = a\frac{x^2}{2} + bx$, n=0

$\int f(x)\, sin\frac{2n\pi x}{T}\, dx = -a\frac{T}{2n\pi}x\, cos\frac{2n\pi x}{T} - b\frac{T}{2n\pi}\cos\frac{2n\pi x}{T} + a\left(\frac{T}{2n\pi}\right)^2 \sin\frac{2n\pi x}{T}$, n≥1.

3. The Discrete data Analysis using Fourier Development

Consider a discrete data set: Y=(y_1,...,y_n) and the points $(k, y_k)_{k=\overline{1,n}}$. In the following, we consider, for a given T, a sequence of the form: $(s, y_s)_{s=\overline{p,p+T-1}}$ and it will build the corresponding Lagrange interpolation polynomial of degree 1 on each pair of consecutive pairs, where the independent variable would be the sequence number of the corresponding date. The result will be a piecewise function continous on every point.

We will determine the truncation of the Fourier development corresponding to the interpolation polynomial for a number of terms and then the obtained function will be calculated at all points $(k)_{k=\overline{1,n}}$ obtaining the set of values: $(\widetilde{y_k})_{k=\overline{1,n}}$. We will then calculate the average error: $\varepsilon = \frac{\sum_{k=1}^{n}|y_k - \widetilde{y_k}|}{n}$ and we will retain that value of T for which the error is minimal.

For the periodic function to be defined in the range: $\left[-\frac{T}{2}, \frac{T}{2}\right]$, we will consider the points: $\left(\frac{T(2s-2p+1-T)}{2(T-1)}, y_s\right)_{s=\overline{p,p+T-1}}$. For a pair of consecutive points, corresponding to s and s+1, we will consider the interpolation polynomial of degree 1 which passing through: $\left(\frac{T(2s-2p+1-T)}{2(T-1)}, y_s\right)$ and $\left(\frac{T(2s-2p+3-T)}{2(T-1)}, y_{s+1}\right)$, $s = \overline{p, p+T-2}$.

We have ([2]): $g_s(x) = \frac{(T-1)(y_{s+1}-y_s)}{T}x + \frac{(2s-2p+3-T)y_s - (2s-2p+1-T)y_{s+1}}{2}$.

Now, if we decompose the integrals on $\left[-\frac{T}{2}, \frac{T}{2}\right]$ after g_s, we obtain ([2]):

$\frac{a_0}{2} + \sum_{m=1}^{\infty}\left(a_m\, cos\frac{2m\pi x}{T} + b_m\, sin\frac{2m\pi x}{T}\right)$ where:

$a_0 = \frac{2}{T-1}\sum_{s=p}^{p+T-2}(y_s + y_{s+1})$,

$a_n=\frac{1}{n\pi}\sum_{s=p}^{p+T-2}\sin\frac{n(2s-2p+3-T)\pi}{T-1}y_{s+1}-\sin\frac{n(2s-2p+1-T)\pi}{T-1}y_s+\frac{(T-1)(y_{s+1}-y_s)}{nT\pi}\left(\cos\frac{n(2s-2p+3-T)\pi}{T-1}-\cos\frac{n(2s-2p+1-T)\pi}{T-1}\right)$, n≥1,

$b_n=-\frac{1}{n\pi}\sum_{s=p}^{p+T-2}\cos\frac{n(2s-2p+3-T)\pi}{T-1}y_{s+1}-\cos\frac{n(2s-2p+1-T)\pi}{T-1}y_s-\frac{(T-1)(y_{s+1}-y_s)}{nT\pi}\left(\sin\frac{n(2s-2p+3-T)\pi}{T-1}-\sin\frac{n(2s-2p+1-T)\pi}{T-1}\right)$, n≥1.

After this, we will compute the set of values: $(\widetilde{y_k})_{k=\overline{1,n}}$ for abscissas $\frac{T(2s-2p+1-T)}{2(T-1)}$, s=$\overline{1,n}$ and ordinates $(y_k)_{k=\overline{1,n}}$.

4. The Analysis of of Birth Rate

Considering Total population I_k, k=$\overline{1,m}$, we first compute the growth rate (in percents): $r_k=\frac{I_k-I_{k-1}}{I_{k-1}}$, k=$\overline{2,m}$.

After, we will compute the accelerations of the indicator: $\rho_k=r_k-r_{k-1}$, k=$\overline{3,m}$.

We then obtain that: $r_k=r_2+\sum_{j=3}^{k}\rho_j$, k=$\overline{3,m}$.

We will then consider the pairs of elements (k,ρ_k), k=$\overline{3,m}$. For these pairs, we will analyze the Fourier polynomials F_n for n=$\overline{8,20}$ as in the previous section and we will retain that expression for which the mean deviation is minimal. The forecast will be made using the last value of the adjusted growth rate $r_{Fourier,m}$ according to the formula: $I_k=I_{k-1}(1+ r_{Fourier,m}/100)$.

5. The Analysis of Birth Rate Cyclicity

The following analysis will study the indicator for the period 1970-2020. The calculations revealed that the best approximation by Fourier polynomials is relative to the growth rate.

5.1. Austria

Table 1. The Evolution of the Accelerations (Percents) of the Birth Rate during the Period 1972-2018

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	-1,4943	1988	2,6240	2004	4,1672
1973	-1,6941	1989	-1,7544	2005	-4,1672
1974	5,7465	1990	-	2006	1,0093
1975	-2,3498	1991	1,6657	2007	-1,0751
1976	-3,3266	1992	-3,3898	2008	3,2147
1977	3,8654	1993	-	2009	-2,1623
1978	2,5862	1994	-1,6940	2010	3,2492
1979	-	1995	-0,1150	2011	-3,2377
1980	4,3782	1996	3,4483	2012	2,1391
1981	-1,9299	1997	-	2013	-1,0753
1982	-2,5268	1998	3,3929	2014	-
1983	-5,6065	1999	-1,0645	2015	-0,0444
1984	3,9597	2000	3,9216	2016	-0,0425
1985	-0,8546	2001	-	2017	-2,0408
1986	0,8328	2002	7,2731	2018	-
1987	-0,0075	2003	-5,2534	2019	1,9691

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2018], we found that the minimum average absolute error 1,9717 is obtained applying Fourier Analysis for T=7 for the range of years: [2001,2007].

The optimal number of Fourier series terms is 10 being specified in the table 2.

Table 2.

a_0	-0,0502						
a_1	-0,3177	b_1	-0,9098	a_2	0,1076	b_2	0,1558
a_3	-0,8073	b_3	-0,4306	a_4	0,0269	b_4	0,4455
a_5	-0,0127	b_5	-0,2736	a_6	-	b_6	0,2153
a_7	-0,0065	b_7	-0,1768	a_8	0,0067	b_8	0,1308
a_9	-0,0897	b_9	-0,1435	a_{10}	0,0043	b_{10}	0,1488
a_{11}	-	b_{11}	-	a_{12}	-	b_{12}	-
a_{13}	-	b_{13}	-	a_{14}	-	b_{14}	-
a_{15}	-	b_{15}	-	a_{16}	-	b_{16}	-
a_{17}	-	b_{17}	-	a_{18}	-	b_{18}	-
a_{19}	-	b_{19}	-	a_{20}	-	b_{20}	-

The recalculated values of the Birth Rate are:

Table 3. The Evolution of the Accelerations (Percents) of the Birth Rate during the Period 1972-2018

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	-0,3889	1988	-1,2641	2004	-1,1135
1973	1,6011	1989	1,3545	2005	-0,3398
1974	-1,1135	1990	-0,3889	2006	-1,2641
1975	-0,3398	1991	1,6011	2007	1,3545
1976	-1,2641	1992	-1,1135	2008	-0,3889
1977	1,3545	1993	-0,3398	2009	1,6011
1978	-0,3889	1994	-1,2641	2010	-1,1135
1979	1,6011	1995	1,3545	2011	-0,3398
1980	-1,1135	1996	-0,3889	2012	-1,2641
1981	-0,3398	1997	1,6011	2013	1,3545
1982	-1,2641	1998	-1,1135	2014	-0,3889
1983	1,3545	1999	-0,3398	2015	1,6011
1984	-0,3889	2000	-1,2641	2016	-1,1135
1985	1,6011	2001	1,3545	2017	-0,3398
1986	-1,1135	2002	-0,3889	2018	-1,2641
1987	-0,3398	2003	1,6011	2019	-

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2018

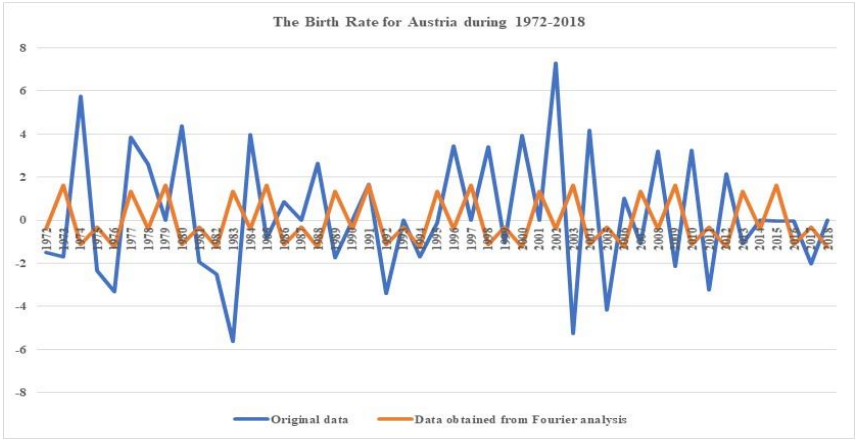

Figure 1.

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**-5,8934**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2019	9,034
2020	8,502
2021	8,001
2022	7,529
2023	7,086

5.2. Belgium

Table 4. The Evolution of the Accelerations (Percents) of the Birth Rate during the Period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	-3,4293	1988	-	2004	1,8099
1973	-0,8904	1989	-0,8543	2005	-1,8423
1974	0,4887	1990	0,8129	2006	0,8694
1975	0,5743	1991	-0,0264	2007	-0,8923
1976	4,7567	1992	-3,2002	2008	0,8473
1977	-0,0067	1993	-1,6385	2009	-2,5497
1978	-0,0065	1994	-0,9409	2010	1,6878
1979	-0,0065	1995	3,2971	2011	-2,5282
1980	-0,8000	1996	1,7468	2012	-0,0287
1981	-	1997	-1,7468	2013	-0,0297
1982	-	1998	-0,8848	2014	0,8541
1983	0,7156	1999	1,7544	2015	-2,6864
1984	0,7783	2000	-	2016	3,5714
1985	0,8260	2001	-3,5401	2017	-
1986	3,4409	2002	-0,9242	2018	1,8254
1987	-2,5862	2003	3,5960	2019	-1,9322

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2019], we found that the minimum average absolute error 1,2801 is obtained applying Fourier Analysis for T=11 for the range of years: [2007, 2017].

The optimal number of Fourier series terms is 18 being specified in the table 5.

Table 5.

a_0	**-0,1492**						
a_1	**-0,0659**	b_1	**0,3871**	a_2	**0,0235**	b_2	**-0,2229**
a_3	**0,1625**	b_3	**0,1546**	a_4	**0,0566**	b_4	**-0,0545**
a_5	**-0,2242**	b_5	**0,0623**	a_6	**0,0251**	b_6	**-0,0623**
a_7	**0,0298**	b_7	**0,0352**	a_8	**0,0015**	b_8	**-0,0347**
a_9	**-0,0008**	b_9	**0,0337**	a_{10}	**-**	b_{10}	**-0,0311**
a_{11}	**-0,0005**	b_{11}	**0,0289**	a_{12}	**0,0007**	b_{12}	**-0,0278**
a_{13}	**0,0087**	b_{13}	**0,0267**	a_{14}	**0,0046**	b_{14}	**-0,0203**
a_{15}	**-0,0249**	b_{15}	**0,0208**	a_{16}	**0,0035**	b_{16}	**-0,0209**
a_{17}	**0,0051**	b_{17}	**0,0167**	a_{18}	**0,0003**	b_{18}	**-0,0165**
a_{19}	**-**	b_{19}	**-**	a_{20}	**-**	b_{20}	**-**

The recalculated values of the Birth Rate are:

Table 6. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	**-0,0692**	**1988**	**-0,8030**	**2004**	**-0,3420**
1973	**0,1547**	**1989**	**-0,0440**	**2005**	**0,7817**
1974	**-0,3420**	**1990**	**-0,6668**	**2006**	**0,2622**
1975	**0,7817**	**1991**	**-0,1711**	**2007**	**0,1514**
1976	**0,2622**	**1992**	**-0,0692**	**2008**	**-0,8030**
1977	**0,1514**	**1993**	**0,1547**	**2009**	**-0,0440**
1978	**-0,8030**	**1994**	**-0,3420**	**2010**	**-0,6668**
1979	**-0,0440**	**1995**	**0,7817**	**2011**	**-0,1711**
1980	**-0,6668**	**1996**	**0,2622**	**2012**	**-0,0692**
1981	**-0,1711**	**1997**	**0,1514**	**2013**	**0,1547**
1982	**-0,0692**	**1998**	**-0,8030**	**2014**	**-0,3420**
1983	**0,1547**	**1999**	**-0,0440**	**2015**	**0,7817**
1984	**-0,3420**	**2000**	**-0,6668**	**2016**	**0,2622**
1985	**0,7817**	**2001**	**-0,1711**	**2017**	**0,1514**
1986	**0,2622**	**2002**	**-0,0692**	**2018**	**-0,8030**
1987	**0,1514**	**2003**	**0,1547**	**2019**	**-0,0440**

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2019

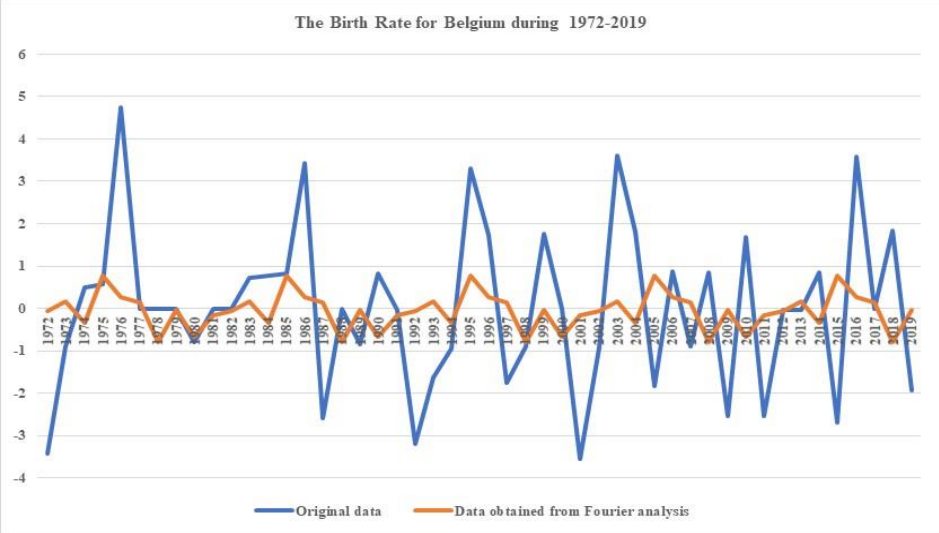

Figure 2

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**-3,5729**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2020	**9,739**
2021	**9,391**
2022	**9,056**
2023	**8,732**
2024	**8,420**

5.3. Bulgaria

Table 7. The Evolution of the Accelerations (Percents) of the Birth Rate during the Period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	**-1,3196**	**1988**	**3,7543**	**2004**	**2,2448**
1973	**9,6560**	**1989**	**-4,5860**	**2005**	**-2,3999**
1974	**0,2904**	**1990**	**-0,1515**	**2006**	**2,1033**
1975	**-9,6612**	**1991**	**-4,2962**	**2007**	**-1,2083**
1976	**2,8860**	**1992**	**1,9582**	**2008**	**0,9072**
1977	**-1,8218**	**1993**	**2,4601**	**2009**	**0,8077**
1978	**-1,3025**	**1994**	**-2,1538**	**2010**	**-11,2297**
1979	**2,4364**	**1995**	**-2,5106**	**2011**	**0,5396**
1980	**-3,9385**	**1996**	**8,5106**	**2012**	**4,8407**

1981	**1,7805**	**1997**	**-**	**2013**	**-2,1162**
1982	**2,7340**	**1998**	**13,0625**	**2014**	**5,3318**
1983	**-0,0051**	**1999**	**8,7950**	**2015**	**-4,3016**
1984	**-0,7299**	**2000**	**-9,1197**	**2016**	**1,0407**
1985	**-0,7566**	**2001**	**-7,8283**	**2017**	**-0,0119**
1986	**2,9578**	**2002**	**5,5556**	**2018**	**-0,0122**
1987	**-3,7370**	**2003**	**-**	**2019**	**-0,0125**

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972, 2019], we found that the minimum average absolute error 3,1302 is obtained applying Fourier Analysis for T=6 for the range of years: [2012, 2017].

The optimal number of Fourier series terms is 8 being specified in the table 8.

Table 8.

a_0	**0,7958**						
a_1	**-0,3755**	**b_1**	**0,2742**	**a_2**	**-0,4869**	**b_2**	**0,3192**
a_3	**0,2164**	**b_3**	**0,5139**	**a_4**	**0,0235**	**b_4**	**-0,1922**
a_5	**-**	**b_5**	**0,1339**	**a_6**	**0,0104**	**b_6**	**-0,1006**
a_7	**0,0397**	**b_7**	**0,0423**	**a_8**	**-0,0304**	**b_8**	**-0,1246**
a_9	**-**	**b_9**	**-**	**a_{10}**	**-**	**b_{10}**	**-**
a_{11}	**-**	**b_{11}**	**-**	**a_{12}**	**-**	**b_{12}**	**-**
a_{13}	**-**	**b_{13}**	**-**	**a_{14}**	**-**	**b_{14}**	**-**
a_{15}	**-**	**b_{15}**	**-**	**a_{16}**	**-**	**b_{16}**	**-**
a_{17}	**-**	**b_{17}**	**-**	**a_{18}**	**-**	**b_{18}**	**-**
a_{19}	**-**	**b_{19}**	**-**	**a_{20}**	**-**	**b_{20}**	**-**

The recalculated values of the Birth Rate are:

Table 9. The Evolution of the Accelerations (Percents) of the Birth Rate During the Period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	0,0338	1988	1,5146	2004	-1,1502
1973	1,5146	1989	-1,1502	2005	0,8056
1974	-1,1502	1990	0,8056	2006	0,7859
1975	0,8056	1991	0,7859	2007	0,0338
1976	0,7859	1992	0,0338	2008	1,5146
1977	0,0338	1993	1,5146	2009	-1,1502
1978	1,5146	1994	-1,1502	2010	0,8056
1979	-1,1502	1995	0,8056	2011	0,7859
1980	0,8056	1996	0,7859	2012	0,0338
1981	0,7859	1997	0,0338	2013	1,5146
1982	0,0338	1998	1,5146	2014	-1,1502
1983	1,5146	1999	-1,1502	2015	0,8056
1984	-1,1502	2000	0,8056	2016	0,7859
1985	0,8056	2001	0,7859	2017	0,0338
1986	0,7859	2002	0,0338	2018	1,5146
1987	0,0338	2003	1,5146	2019	-1,1502

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2019

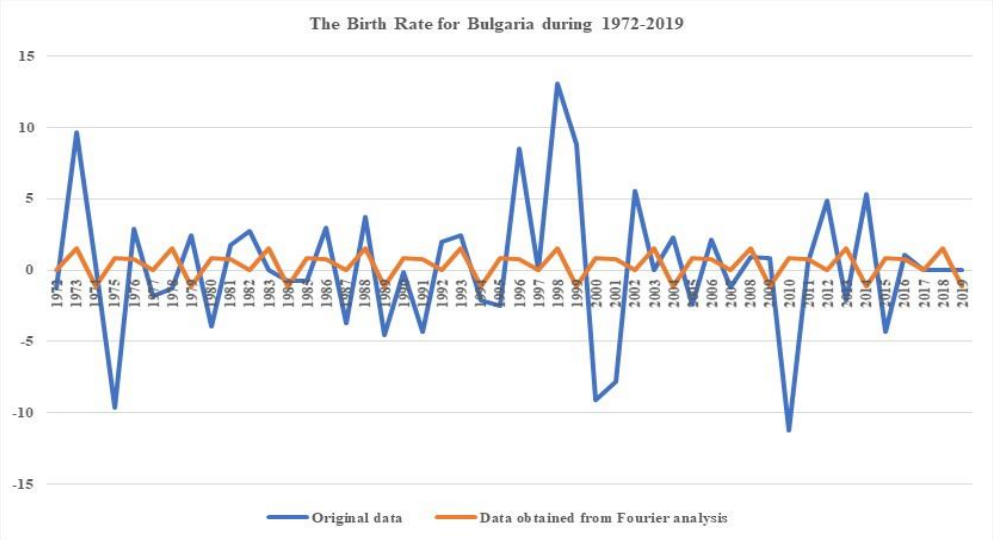

Figure 3.

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**15,8515**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2020	10,195
2021	11,811
2022	13,683
2023	15,852
2024	18,365

5.4. Cyprus

Table 10. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	0,5192	1988	-0,0859	2004	0,5448
1973	0,3202	1989	-0,0687	2005	0,4754
1974	0,2610	1990	-0,1489	2006	0,3930
1975	0,3167	1991	-0,3233	2007	0,2415
1976	0,4180	1992	-0,4633	2008	0,0917
1977	0,4607	1993	-0,5648	2009	-0,0103
1978	0,4011	1994	-0,5486	2010	-0,0968
1979	0,2502	1995	-0,5022	2011	-0,1416
1980	0,0181	1996	-0,3807	2012	-0,1093
1981	-0,2348	1997	-0,2017	2013	-0,0940
1982	-0,4229	1998	0,0251	2014	-0,1319
1983	-0,5359	1999	0,2062	2015	-0,1895
1984	-0,5884	2000	0,4264	2016	-0,2594
1985	-0,5424	2001	0,5717	2017	-0,2515
1986	-0,4021	2002	0,6006	2018	-0,2355
1987	-0,2142	2003	0,5373	2019	-0,1910

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2019], we found that the minimum average absolute error 0,2329 is obtained applying Fourier Analysis for T=27 for the range of years: [1982,2008].

The optimal number of Fourier series terms is 8 being specified in the table 11.

Table 11.

a_0	**-0,1838**						
a_1	**-0,0057**	b_1	**0,1889**	a_2	**-0,0209**	b_2	**-0,0776**
a_3	**0,0021**	b_3	**0,0612**	a_4	**0,0002**	b_4	**-0,0443**
a_5	**0,0002**	b_5	**0,0329**	a_6	**0,0002**	b_6	**-0,0274**
a_7	**-0,0005**	b_7	**0,0241**	a_8	**-0,0003**	b_8	**-0,0209**
a_9	**-**	b_9	**-**	a_{10}	**-**	b_{10}	**-**
a_{11}	**-**	b_{11}	**-**	a_{12}	**-**	b_{12}	**-**
a_{13}	**-**	b_{13}	**-**	a_{14}	**-**	b_{14}	**-**
a_{15}	**-**	b_{15}	**-**	a_{16}	**-**	b_{16}	**-**
a_{17}	**-**	b_{17}	**-**	a_{18}	**-**	b_{18}	**-**
a_{19}	**-**	b_{19}	**-**	a_{20}	**-**	b_{20}	**-**

The recalculated values of the Birth Rate are:

Table 12. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	**-0,0154**	**1988**	**-0,2111**	**2004**	**0,1211**
1973	**0,0071**	**1989**	**-0,2152**	**2005**	**0,1010**
1974	**0,0560**	**1990**	**-0,2145**	**2006**	**0,1632**
1975	**0,0699**	**1991**	**-0,1878**	**2007**	**0,1535**
1976	**0,0695**	**1992**	**-0,1864**	**2008**	**-0,1087**
1977	**0,1126**	**1993**	**-0,1749**	**2009**	**-0,3660**
1978	**0,1211**	**1994**	**-0,1351**	**2010**	**-0,3619**
1979	**0,1010**	**1995**	**-0,1165**	**2011**	**-0,2792**
1980	**0,1632**	**1996**	**-0,0927**	**2012**	**-0,2786**
1981	**0,1535**	**1997**	**-0,0416**	**2013**	**-0,2575**
1982	**-0,1087**	**1998**	**-0,0154**	**2014**	**-0,2111**
1983	**-0,3660**	**1999**	**0,0071**	**2015**	**-0,2152**
1984	**-0,3619**	**2000**	**0,0560**	**2016**	**-0,2145**
1985	**-0,2792**	**2001**	**0,0699**	**2017**	**-0,1878**
1986	**-0,2786**	**2002**	**0,0695**	**2018**	**-0,1864**
1987	**-0,2575**	**2003**	**0,1126**	**2019**	**-0,1749**

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2019

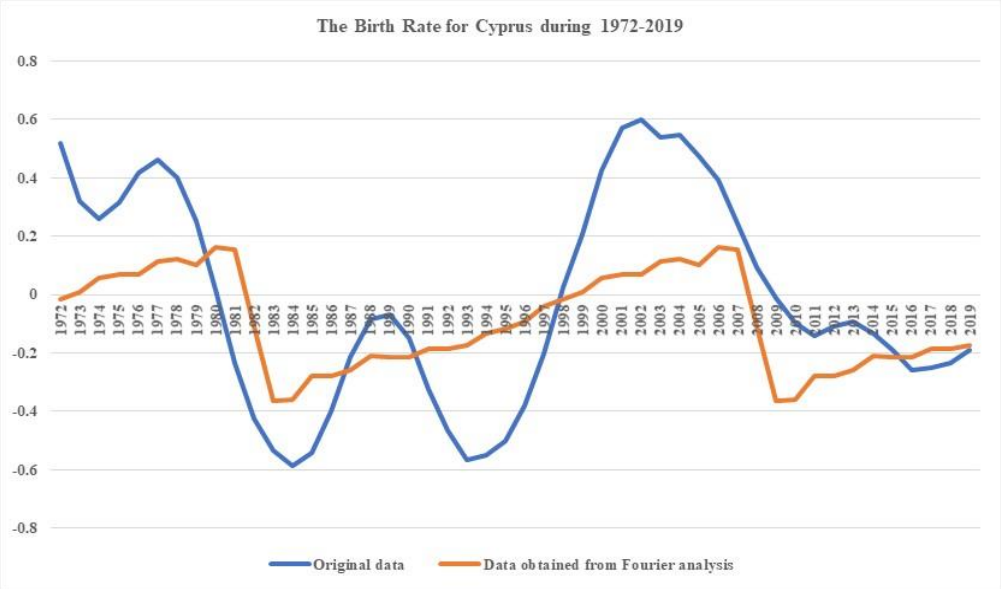

Figure 4.

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**-5,6538**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2020	**9,661**
2021	**9,115**
2022	**8,599**
2023	**8,113**
2024	**7,655**

5.5. Czech Republic

Table 13. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	1,0658	1988	2,3378	2004	3,2489
1973	4,5085	1989	-3,9124	2005	-0,1811
1974	-4,2301	1990	4,7379	2006	-1,1667
1975	-7,5573	1991	-2,4066	2007	4,7670
1976	-1,5950	1992	-4,8063	2008	-4,1634
1977	-0,6424	1993	4,7525	2009	-5,3427
1978	2,0984	1994	-11,1183	2010	0,8541
1979	-2,8860	1995	2,2571	2011	-6,2579
1980	-6,2070	1996	4,3324	2012	6,1814
1981	4,7381	1997	5,3763	2013	-0,0094
1982	3,8974	1998	-	2014	2,9317
1983	-0,7768	1999	-	2015	-0,9993
1984	2,9197	2000	3,4353	2016	0,9433
1985	-	2001	-2,2989	2017	-0,9702
1986	-0,0229	2002	-	2018	-1,8605
1987	-0,0237	2003	-1,1483	2019	-0,9433

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2019], we found that the minimum average absolute error 2,5503 is obtained applying Fourier Analysis for T=15 for the range of years: [1980, 1994].

The optimal number of Fourier series terms is 8 being specified in the table 14.

Table 14.

a_0	-0,2640						
a_1	0,0402	b_1	-0,8759	a_2	0,0145	b_2	0,5316
a_3	-0,0126	b_3	-0,3366	a_4	-0,0316	b_4	0,2846
a_5	-0,0472	b_5	-0,5094	a_6	0,2223	b_6	0,2348
a_7	-0,0246	b_7	-0,1128	a_8	0,1251	b_8	0,0407
a_9	-	b_9	-	a_{10}	-	b_{10}	-
a_{11}	-	b_{11}	-	a_{12}	-	b_{12}	-
a_{13}	-	b_{13}	-	a_{14}	-	b_{14}	-
a_{15}	-	b_{15}	-	a_{16}	-	b_{16}	-
a_{17}	-	b_{17}	-	a_{18}	-	b_{18}	-
a_{19}	-	b_{19}	-	a_{20}	-	b_{20}	-

The recalculated values of the Birth Rate are:

Table 15. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	-0,0125	1988	-0,6706	2004	-0,8576
1973	0,1540	1989	0,0409	2005	-1,2502
1974	-0,6706	1990	-0,8576	2006	-0,4897
1975	0,0409	1991	-1,2502	2007	-2,4001
1976	-0,8576	1992	-0,4897	2008	0,2424
1977	-1,2502	1993	-2,4001	2009	1,3674
1978	-0,4897	1994	0,2424	2010	0,6648
1979	-2,4001	1995	1,3674	2011	0,7682
1980	0,2424	1996	0,6648	2012	0,3712
1981	1,3674	1997	0,7682	2013	0,2234
1982	0,6648	1998	0,3712	2014	-0,0125
1983	0,7682	1999	0,2234	2015	0,1540
1984	0,3712	2000	-0,0125	2016	-0,6706
1985	0,2234	2001	0,1540	2017	0,0409
1986	-0,0125	2002	-0,6706	2018	-0,8576
1987	0,1540	2003	0,0409	2019	-1,2502

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2019

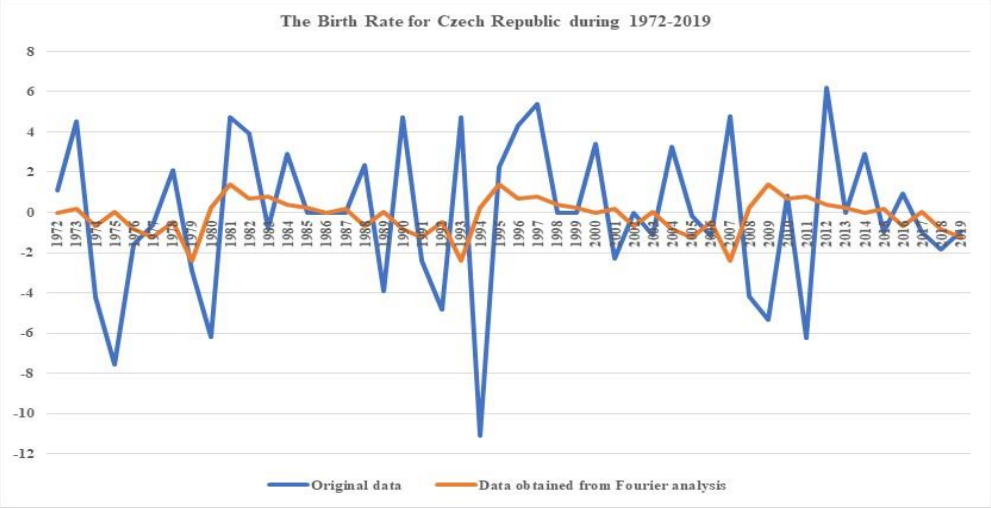

Figure 5.

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**-3,4745**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2020	10,135
2021	9,783
2022	9,443
2023	9,115
2024	8,798

5.6. Germany

Table 16. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	-7,1214	1988	-0,9666	2004	1,1494
1973	0,4179	1989	-3,5401	2005	-
1974	9,4639	1990	3,5401	2006	2,2836
1975	-1,9703	1991	-10,5576	2007	2,4243
1976	5,9715	1992	4,9257	2008	-1,2195
1977	-2,0499	1993	1,8462	2009	-
1978	-0,0095	1994	-1,0612	2010	4,8787
1979	-0,0094	1995	2,0086	2011	-2,4691
1980	4,7528	1996	4,2441	2012	-
1981	-6,6152	1997	-1,1296	2013	-0,0143
1982	0,9009	1998	-5,0922	2014	2,3389
1983	-	1999	0,9470	2015	-1,2567
1984	1,7496	2000	1,0195	2016	4,3940
1985	2,8483	2001	-3,2373	2017	-7,7084
1986	2,8480	2002	2,0539	2018	1,0417
1987	-1,0572	2003	1,0978	2019	-

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2019], we found that the minimum average absolute error 2,2429 is obtained applying Fourier Analysis for T=11 for the range of years: [2008,2018].

The optimal number of Fourier series terms is 8 being specified in the table 17.

Table 17.

a_0	**0,4819**						
a_1	**0,0360**	b_1	**-0,1333**	a_2	**0,2691**	b_2	**0,2587**
a_3	**-0,1937**	b_3	**-0,1161**	a_4	**-0,0669**	b_4	**-0,1591**
a_5	**0,3552**	b_5	**-**	a_6	**-0,0297**	b_6	**0,0707**
a_7	**-0,0356**	b_7	**0,0213**	a_8	**0,0168**	b_8	**-0,0162**
a_9	**-**	b_9	**-**	a_{10}	**-**	b_{10}	**-**
a_{11}	**-**	b_{11}	**-**	a_{12}	**-**	b_{12}	**-**
a_{13}	**-**	b_{13}	**-**	a_{14}	**-**	b_{14}	**-**
a_{15}	**-**	b_{15}	**-**	a_{16}	**-**	b_{16}	**-**
a_{17}	**-**	b_{17}	**-**	a_{18}	**-**	b_{18}	**-**
a_{19}	**-**	b_{19}	**-**	a_{20}	**-**	b_{20}	**-**

The recalculated values of the Birth Rate are:

Table 18. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	**0,2349**	**1988**	**0,2683**	**2004**	**0,0695**
1973	**0,5923**	**1989**	**0,9981**	**2005**	**0,8657**
1974	**0,0695**	**1990**	**-0,1458**	**2006**	**-0,9980**
1975	**0,8657**	**1991**	**0,1975**	**2007**	**0,3272**
1976	**-0,9980**	**1992**	**0,2349**	**2008**	**0,2683**
1977	**0,3272**	**1993**	**0,5923**	**2009**	**0,9981**
1978	**0,2683**	**1994**	**0,0695**	**2010**	**-0,1458**
1979	**0,9981**	**1995**	**0,8657**	**2011**	**0,1975**
1980	**-0,1458**	**1996**	**-0,9980**	**2012**	**0,2349**
1981	**0,1975**	**1997**	**0,3272**	**2013**	**0,5923**
1982	**0,2349**	**1998**	**0,2683**	**2014**	**0,0695**
1983	**0,5923**	**1999**	**0,9981**	**2015**	**0,8657**
1984	**0,0695**	**2000**	**-0,1458**	**2016**	**-0,9980**
1985	**0,8657**	**2001**	**0,1975**	**2017**	**0,3272**
1986	**-0,9980**	**2002**	**0,2349**	**2018**	**0,2683**
1987	**0,3272**	**2003**	**0,5923**	**2019**	**0,9981**

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2019

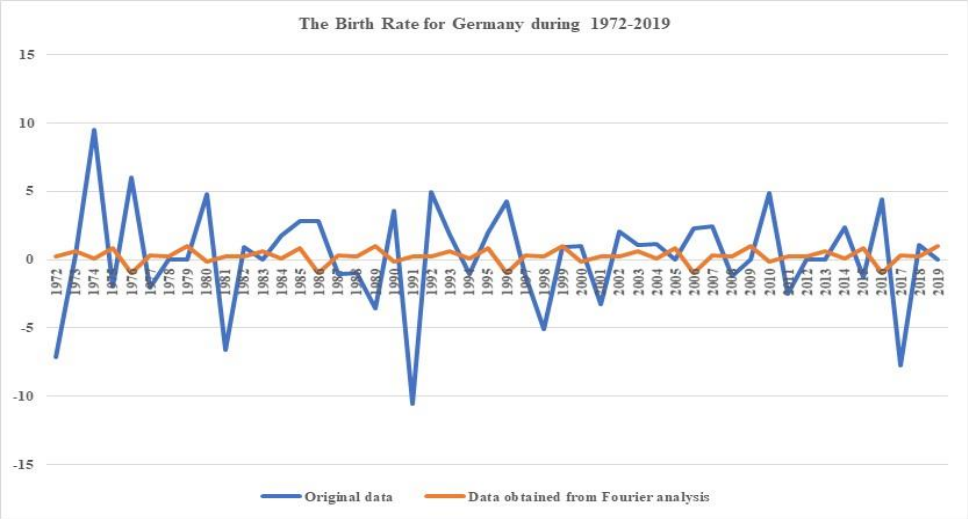

Figure 6.

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**8,2655**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2020	10,285
2021	11,135
2022	12,056
2023	13,052
2024	14,131

5.7. Denmark

Table 19. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	-6,2135	1988	2,6936	2004	-0,8403
1973	-4,6401	1989	-0,1977	2005	-
1974	3,8994	1990	-1,8478	2006	1,6736
1975	2,1078	1991	-0,8740	2007	-3,3403
1976	-9,8641	1992	3,1740	2008	3,3547
1977	3,7285	1993	-5,5634	2009	-4,2445
1978	5,4264	1994	3,8403	2010	3,3898
1979	-	1995	-3,8232	2011	-
1980	1,4697	1996	-2,2612	2012	5,1307
1981	-3,6946	1997	2,2323	2013	-1,9594

1982	**6,1814**	**1998**	**-1,5686**	**2014**	**4,8462**
1983	**-2,9220**	**1999**	**1,5438**	**2015**	**-0,0099**
1984	**5,9037**	**2000**	**2,4129**	**2016**	**4,8923**
1985	**1,9402**	**2001**	**-4,7875**	**2017**	**-7,7343**
1986	**-1,1033**	**2002**	**0,7156**	**2018**	**1,8519**
1987	**-1,0052**	**2003**	**3,2993**	**2019**	**-**

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2019], we found that the minimum average absolute error 2,5573 is obtained applying Fourier Analysis for T=5 for the range of years: [2006, 2010].

The optimal number of Fourier series terms is 8 being specified in the table 20.

Table 20.

a_0	**0,8298**						
a_1	**-0,4173**	b_1	**0,7982**	a_2	**-1,0261**	b_2	**-0,5316**
a_3	**-0,0464**	b_3	**0,3839**	a_4	**-**	b_4	**-0,2658**
a_5	**-0,0167**	b_5	**0,2020**	a_6	**-0,1140**	b_6	**-0,1772**
a_7	**-0,0085**	b_7	**0,1573**	a_8	**-**	b_8	**-0,1329**
a_9	**-**	b_9	**-**	a_{10}	**-**	b_{10}	**-**
a_{11}	**-**	b_{11}	**-**	a_{12}	**-**	b_{12}	**-**
a_{13}	**-**	b_{13}	**-**	a_{14}	**-**	b_{14}	**-**
a_{15}	**-**	b_{15}	**-**	a_{16}	**-**	b_{16}	**-**
a_{17}	**-**	b_{17}	**-**	a_{18}	**-**	b_{18}	**-**
a_{19}	**-**	b_{19}	**-**	a_{20}	**-**	b_{20}	**-**

The recalculated values of the Birth Rate are:

Table 21. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	-1,2141	1988	-1,2141	2004	-1,2141
1973	2,0141	1989	2,0141	2005	2,0141
1974	-0,2363	1990	-0,2363	2006	-0,2363
1975	1,0960	1991	1,0960	2007	1,0960
1976	-1,2141	1992	-1,2141	2008	-1,2141
1977	2,0141	1993	2,0141	2009	2,0141
1978	-0,2363	1994	-0,2363	2010	-0,2363
1979	1,0960	1995	1,0960	2011	1,0960
1980	-1,2141	1996	-1,2141	2012	-1,2141
1981	2,0141	1997	2,0141	2013	2,0141
1982	-0,2363	1998	-0,2363	2014	-0,2363
1983	1,0960	1999	1,0960	2015	1,0960
1984	-1,2141	2000	-1,2141	2016	-1,2141
1985	2,0141	2001	2,0141	2017	2,0141
1986	-0,2363	2002	-0,2363	2018	-0,2363
1987	1,0960	2003	1,0960	2019	1,0960

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2019

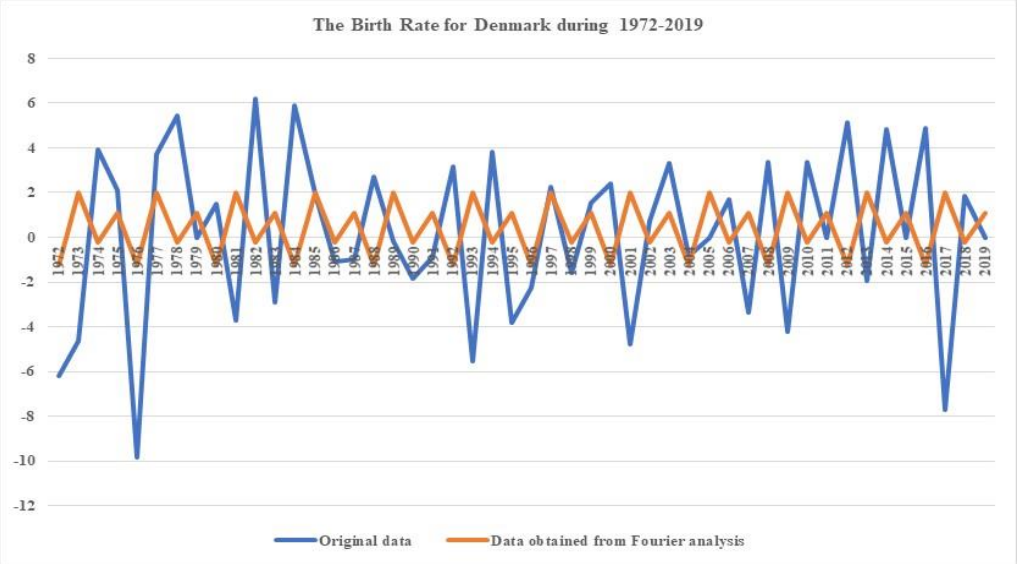

Figure 7.

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**25,4720**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2020	**13,300**
2021	**16,688**
2022	**20,939**
2023	**26,272**
2024	**32,964**

5.8. Euro area

Table 22. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	**-2,9081**	**1988**	**1,0321**	**2004**	**0,6197**
1973	**0,2068**	**1989**	**-2,3422**	**2005**	**-1,8414**
1974	**3,3013**	**1990**	**2,1895**	**2006**	**1,8671**
1975	**-2,5358**	**1991**	**-3,2221**	**2007**	**-0,7155**
1976	**2,2835**	**1992**	**1,4418**	**2008**	**1,6151**
1977	**-1,3437**	**1993**	**-0,8720**	**2009**	**-3,6500**
1978	**0,5501**	**1994**	**-1,0036**	**2010**	**2,1500**
1979	**-0,2611**	**1995**	**2,1622**	**2011**	**-2,0194**
1980	**1,4484**	**1996**	**1,9825**	**2012**	**0,2635**
1981	**-1,4453**	**1997**	**-0,3518**	**2013**	**-1,0308**
1982	**1,2128**	**1998**	**-1,2455**	**2014**	**3,2245**
1983	**-1,7948**	**1999**	**0,8160**	**2015**	**-1,9038**
1984	**1,2643**	**2000**	**1,4948**	**2016**	**1,7770**
1985	**-0,0419**	**2001**	**-3,6850**	**2017**	**-2,8034**
1986	**0,8833**	**2002**	**1,6310**	**2018**	**0,2715**
1987	**-0,0484**	**2003**	**0,6724**	**2019**	**0,1156**

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2019], we found that the minimum average absolute error 1,0935 is obtained applying Fourier Analysis for T=5 for the range of years: [1973,1977].

The optimal number of Fourier series terms is 8 being specified in the table 23.

Table 23.

a_0	**0,3297**						
a_1	**0,0580**	b_1	**-0,4595**	a_2	**0,6556**	b_2	**0,4379**
a_3	**0,0064**	b_3	**-0,3382**	a_4	**-**	b_4	**0,2189**
a_5	**0,0023**	b_5	**-0,1585**	a_6	**0,0728**	b_6	**0,1460**
a_7	**0,0012**	b_7	**-0,1336**	a_8	**-**	b_8	**0,1095**
a_9	**-**	b_9	**-**	a_{10}	**-**	b_{10}	**-**
a_{11}	**-**	b_{11}	**-**	a_{12}	**-**	b_{12}	**-**
a_{13}	**-**	b_{13}	**-**	a_{14}	**-**	b_{14}	**-**
a_{15}	**-**	b_{15}	**-**	a_{16}	**-**	b_{16}	**-**
a_{17}	**-**	b_{17}	**-**	a_{18}	**-**	b_{18}	**-**
a_{19}	**-**	b_{19}	**-**	a_{20}	**-**	b_{20}	**-**

The recalculated values of the Birth Rate are:

Table 24. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	**-0,7099**	**1988**	**-0,7099**	**2004**	**-0,7099**
1973	**0,8254**	**1989**	**0,8254**	**2005**	**0,8254**
1974	**-0,4174**	**1990**	**-0,4174**	**2006**	**-0,4174**
1975	**0,9613**	**1991**	**0,9613**	**2007**	**0,9613**
1976	**-0,7099**	**1992**	**-0,7099**	**2008**	**-0,7099**
1977	**0,8254**	**1993**	**0,8254**	**2009**	**0,8254**
1978	**-0,4174**	**1994**	**-0,4174**	**2010**	**-0,4174**
1979	**0,9613**	**1995**	**0,9613**	**2011**	**0,9613**
1980	**-0,7099**	**1996**	**-0,7099**	**2012**	**-0,7099**
1981	**0,8254**	**1997**	**0,8254**	**2013**	**0,8254**
1982	**-0,4174**	**1998**	**-0,4174**	**2014**	**-0,4174**
1983	**0,9613**	**1999**	**0,9613**	**2015**	**0,9613**
1984	**-0,7099**	**2000**	**-0,7099**	**2016**	**-0,7099**
1985	**0,8254**	**2001**	**0,8254**	**2017**	**0,8254**
1986	**-0,4174**	**2002**	**-0,4174**	**2018**	**-0,4174**
1987	**0,9613**	**2003**	**0,9613**	**2019**	**0,9613**

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2019

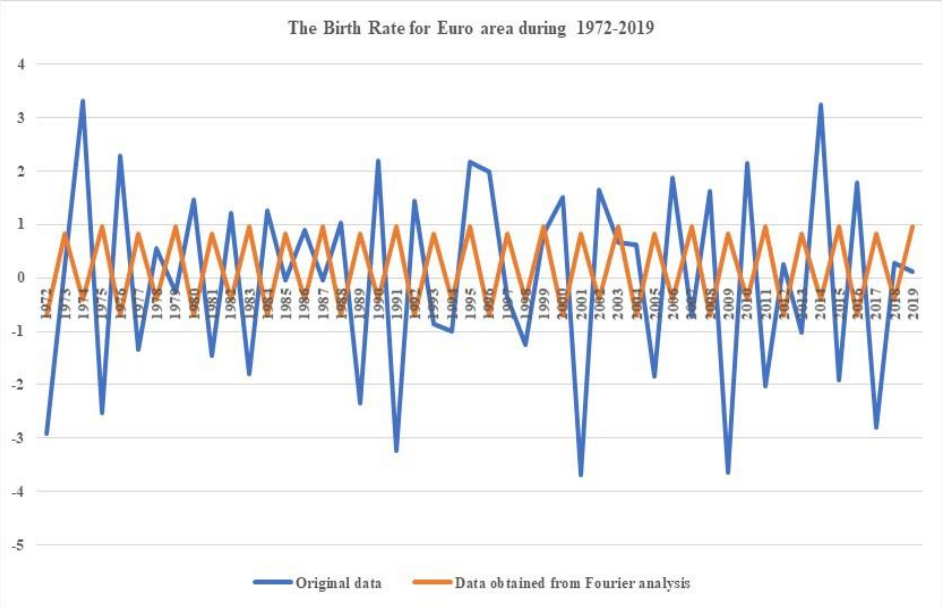

Figure 8.

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**6,5427**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2020	**9,697**
2021	**10,332**
2022	**11,008**
2023	**11,728**
2024	**12,495**

5.9. Spain

Table 25. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	-1,5332	1988	1,6906	2004	-1,0472
1973	-0,0105	1989	-0,9596	2005	-1,9231
1974	2,5934	1990	0,8730	2006	-
1975	-5,6651	1991	0,9339	2007	-0,9609
1976	4,1026	1992	-0,0095	2008	2,7438
1977	-	1993	-1,9899	2009	-9,8644
1978	-0,1456	1994	-1,1113	2010	4,3079
1979	-2,4695	1995	0,8901	2011	-0,9978
1980	0,1856	1996	3,1915	2012	-1,0758
1981	-1,0640	1997	-	2013	-2,2252
1982	3,6907	1998	-2,1859	2014	7,2845
1983	-3,0715	1999	4,3837	2015	-3,2728
1984	3,4680	2000	0,9586	2016	-0,0483
1985	-0,1024	2001	-3,2349	2017	-2,3233
1986	-0,9497	2002	0,9998	2018	-1,4069
1987	0,6929	2003	0,9501	2019	2,1549

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2019], we found that the minimum average absolute error 1,8544 is obtained applying Fourier Analysis for T=24 for the range of years: [1994,2017].

The optimal number of Fourier series terms is 13 being specified in the table 26.

Table 26.

a_0	-0,7997						
a_1	-0,0565	b_1	-0,6974	a_2	-0,0340	b_2	0,3338
a_3	0,0444	b_3	-0,2704	a_4	0,0112	b_4	0,2038
a_5	-0,0105	b_5	-0,1587	a_6	0,1367	b_6	0,1931
a_7	0,0331	b_7	-0,0689	a_8	-0,1257	b_8	0,1252
a_9	-0,0108	b_9	-0,0821	a_{10}	-0,0465	b_{10}	0,0760
a_{11}	-0,0580	b_{11}	-0,1452	a_{12}	0,0487	b_{12}	-0,0052
a_{13}	0,0275	b_{13}	-0,0545	a_{14}	-	b_{14}	-
a_{15}	-	b_{15}	-	a_{16}	-	b_{16}	-
a_{17}	-	b_{17}	-	a_{18}	-	b_{18}	-
a_{19}	-	b_{19}	-	a_{20}	-	b_{20}	-

The recalculated values of the Birth Rate are:

Table 27. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	0,9669	1988	-0,9564	2004	-0,4280
1973	0,4404	1989	-1,0938	2005	-0,3004
1974	0,2866	1990	-0,6114	2006	-0,5351
1975	0,6154	1991	-1,3493	2007	-0,3634
1976	0,2421	1992	-1,2273	2008	-1,2725
1977	-0,0351	1993	-1,7058	2009	-0,4548
1978	0,0430	1994	-0,3787	2010	-0,8758
1979	0,0113	1995	0,9669	2011	-0,9564
1980	-0,2141	1996	0,4404	2012	-1,0938
1981	-0,4280	1997	0,2866	2013	-0,6114
1982	-0,3004	1998	0,6154	2014	-1,3493
1983	-0,5351	1999	0,2421	2015	-1,2273
1984	-0,3634	2000	-0,0351	2016	-1,7058
1985	-1,2725	2001	0,0430	2017	-0,3787
1986	-0,4548	2002	0,0113	2018	0,9669
1987	-0,8758	2003	-0,2141	2019	0,4404

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2019

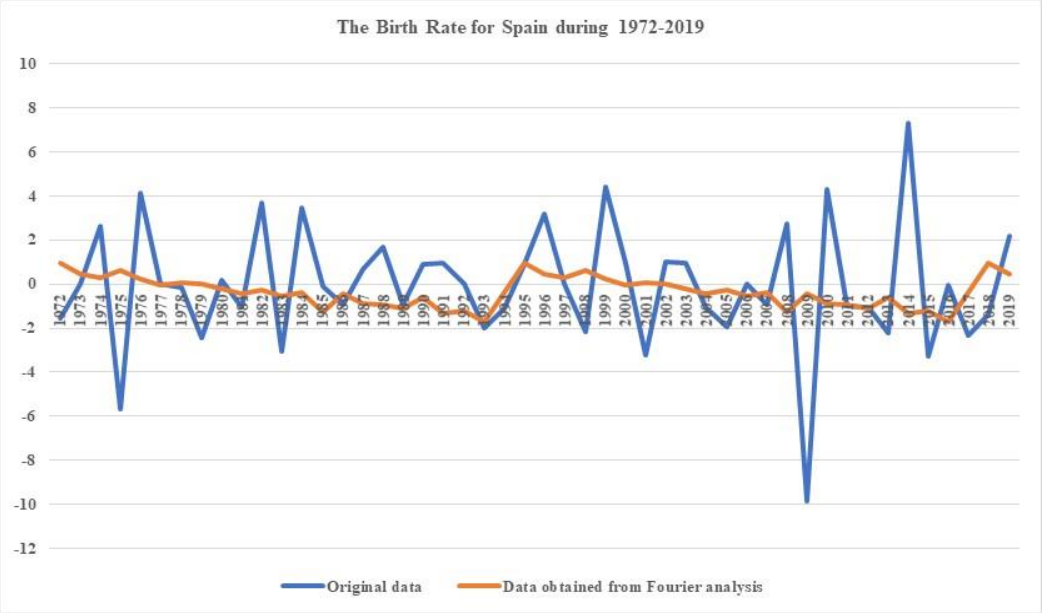

Figure 9.

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**-16,4723**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2020	**6,348**
2021	**5,302**
2022	**4,429**
2023	**3,699**
2024	**3,090**

5.10. Estonia

Table 28. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	**-5,0043**	**1988**	**-4,4193**	**2004**	**7,3573**
1973	**-0,0995**	**1989**	**-1,8904**	**2005**	**-5,5085**
1974	**3,2051**	**1990**	**-5,2621**	**2006**	**0,8610**
1975	**-**	**1991**	**-4,2890**	**2007**	**3,4991**
1976	**2,6668**	**1992**	**7,8374**	**2008**	**-5,5778**
1977	**-0,6800**	**1993**	**-8,7206**	**2009**	**-3,3616**
1978	**-1,9781**	**1994**	**8,6573**	**2010**	**2,5142**
1979	**0,6491**	**1995**	**1,8092**	**2011**	**-7,5702**
1980	**1,3378**	**1996**	**3,0928**	**2012**	**2,2182**
1981	**1,9956**	**1997**	**-**	**2013**	**1,6743**
1982	**-2,6667**	**1998**	**2,0331**	**2014**	**2,8302**
1983	**-**	**1999**	**4,4949**	**2015**	**-**
1984	**-3,8961**	**2000**	**2,1717**	**2016**	**-1,9692**
1985	**-**	**2001**	**-7,6359**	**2017**	**-2,8126**
1986	**4,4153**	**2002**	**6,4882**	**2018**	**5,6787**
1987	**1,8944**	**2003**	**-2,2329**	**2019**	**-6,5618**

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2019], we found that the minimum average absolute error 3,0086 is obtained applying Fourier Analysis for T=15 for the range of years: [2000,2014].

The optimal number of Fourier series terms is 12 being specified in the table 29.

Table 29.

a_0	-0,1790						
a_1	-0,2364	b_1	1,9817	a_2	-0,0173	b_2	-1,1341
a_3	0,1107	b_3	0,6958	a_4	-0,2344	b_4	-0,6294
a_5	-0,2127	b_5	0,2576	a_6	-0,1536	b_6	-0,3627
a_7	0,2625	b_7	0,3472	a_8	-0,0864	b_8	-0,3277
a_9	-0,0657	b_9	0,3406	a_{10}	-0,0375	b_{10}	-0,2396
a_{11}	0,0082	b_{11}	0,2295	a_{12}	-0,0005	b_{12}	-0,2048
a_{13}	-	b_{13}	-	a_{14}	-	b_{14}	-
a_{15}	-	b_{15}	-	a_{16}	-	b_{16}	-
a_{17}	-	b_{17}	-	a_{18}	-	b_{18}	-
a_{19}	-	b_{19}	-	a_{20}	-	b_{20}	-

The recalculated values of the Birth Rate are:

Table 30. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	-0,4859	1988	-2,6022	2004	-2,2286
1973	-1,9198	1989	-1,2135	2005	-1,0435
1974	-2,6022	1990	-2,2286	2006	-0,2271
1975	-1,2135	1991	-1,0435	2007	-0,7524
1976	-2,2286	1992	-0,2271	2008	0,0285
1977	-1,0435	1993	-0,7524	2009	1,2030
1978	-0,2271	1994	0,0285	2010	0,5690
1979	-0,7524	1995	1,2030	2011	2,1145
1980	0,0285	1996	0,5690	2012	2,4820
1981	1,2030	1997	2,1145	2013	2,8230
1982	0,5690	1998	2,4820	2014	-0,4859
1983	2,1145	1999	2,8230	2015	-1,9198
1984	2,4820	2000	-0,4859	2016	-2,6022
1985	2,8230	2001	-1,9198	2017	-1,2135
1986	-0,4859	2002	-2,6022	2018	-2,2286
1987	-1,9198	2003	-1,2135	2019	-1,0435

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2019

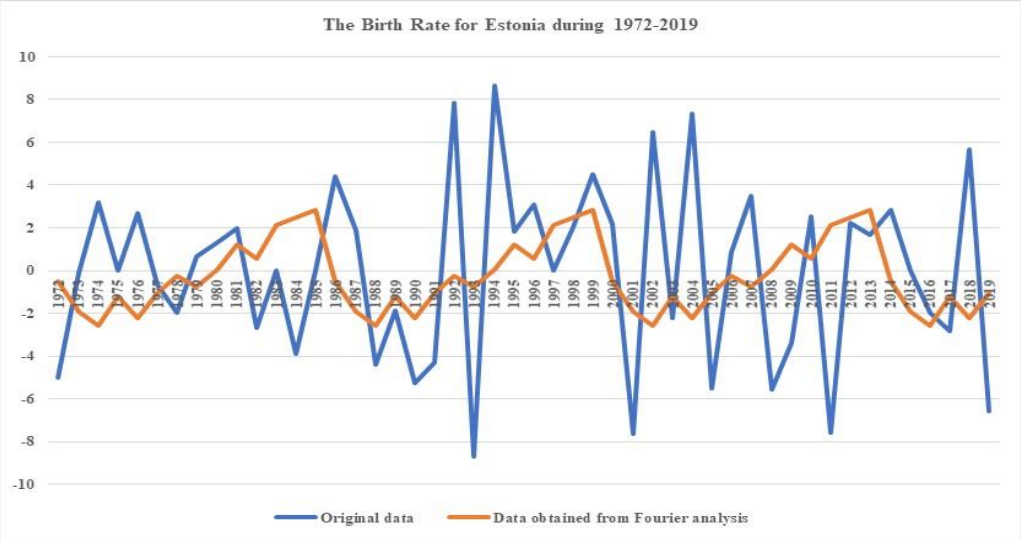

Figure 10

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**-11,3538**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2020	**9,396**
2021	**8,330**
2022	**7,384**
2023	**6,546**
2024	**5,802**

5.11. European Union

Table 31. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	-2,1010	1988	1,0790	2004	0,8748
1973	0,6653	1989	-2,1604	2005	-1,3358
1974	3,7747	1990	1,0613	2006	1,4147
1975	-3,6635	1991	-2,0075	2007	-0,6272
1976	1,5283	1992	0,6135	2008	1,8870
1977	-1,3473	1993	-0,1032	2009	-3,8684
1978	0,3957	1994	-0,7757	2010	1,2076
1979	0,2642	1995	0,5133	2011	-2,5780
1980	0,2330	1996	2,8431	2012	1,7448
1981	-1,3389	1997	-0,2247	2013	-1,6909
1982	1,4663	1998	-1,0894	2014	3,9465
1983	-1,1043	1999	1,0018	2015	-2,3062
1984	1,3356	2000	1,6697	2016	2,1850
1985	-0,2869	2001	-3,6689	2017	-2,2540
1986	0,2976	2002	1,5549	2018	-0,9046
1987	-0,0945	2003	0,9074	2019	-0,1240

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2019], we found that the minimum average absolute error 1,1594 is obtained applying Fourier Analysis for T=7 for the range of years: [2009,2015].

The optimal number of Fourier series terms is 8 being specified in the table 32.

Table 32.

a_0	0,5417						
a_1	0,1651	b_1	0,4236	a_2	-0,0259	b_2	-0,0283
a_3	-0,5389	b_3	0,1037	a_4	-0,0065	b_4	-0,1096
a_5	0,0066	b_5	0,0577	a_6	-	b_6	-0,0519
a_7	0,0034	b_7	0,0467	a_8	-0,0016	b_8	-0,0309
a_9	-	b_9	-	a_{10}	-	b_{10}	-
a_{11}	-	b_{11}	-	a_{12}	-	b_{12}	-
a_{13}	-	b_{13}	-	a_{14}	-	b_{14}	-
a_{15}	-	b_{15}	-	a_{16}	-	b_{16}	-
a_{17}	-	b_{17}	-	a_{18}	-	b_{18}	-
a_{19}	-	b_{19}	-	a_{20}	-	b_{20}	-

The recalculated values of the Birth Rate are:

Table 33. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	-0,0248	1988	-0,1271	2004	-0,6524
1973	0,6007	1989	1,3153	2005	0,5133
1974	-0,6524	1990	-0,0248	2006	-0,1271
1975	0,5133	1991	0,6007	2007	1,3153
1976	-0,1271	1992	-0,6524	2008	-0,0248
1977	1,3153	1993	0,5133	2009	0,6007
1978	-0,0248	1994	-0,1271	2010	-0,6524
1979	0,6007	1995	1,3153	2011	0,5133
1980	-0,6524	1996	-0,0248	2012	-0,1271
1981	0,5133	1997	0,6007	2013	1,3153
1982	-0,1271	1998	-0,6524	2014	-0,0248
1983	1,3153	1999	0,5133	2015	0,6007
1984	-0,0248	2000	-0,1271	2016	-0,6524
1985	0,6007	2001	1,3153	2017	0,5133
1986	-0,6524	2002	-0,0248	2018	-0,1271
1987	0,5133	2003	0,6007	2019	1,3153

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2019

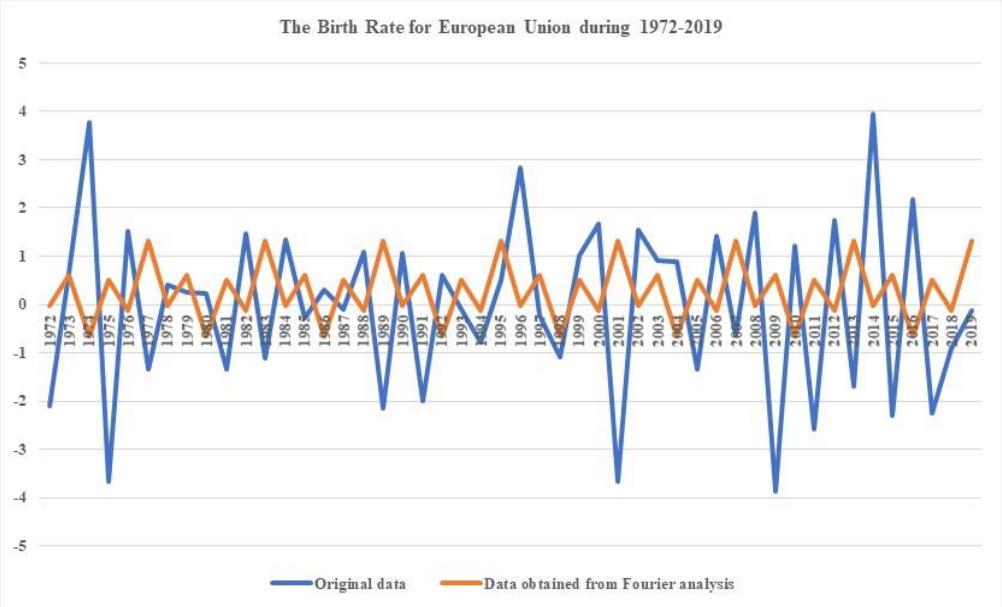

Figure 11.

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**11,9118**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2020	**10,397**
2021	**11,636**
2022	**13,022**
2023	**14,573**
2024	**16,309**

5.12. Finland

Table 34. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	**1,9264**	**1988**	**7,4111**	**2004**	**-0,9518**
1973	**-0,1491**	**1989**	**-5,7851**	**2005**	**-0,9174**
1974	**12,9534**	**1990**	**-**	**2006**	**-**
1975	**-4,5051**	**1991**	**-3,1071**	**2007**	**-2,7111**
1976	**-3,0725**	**1992**	**2,3019**	**2008**	**1,7938**
1977	**-2,8572**	**1993**	**-4,5688**	**2009**	**-0,0080**
1978	**-1,4593**	**1994**	**3,0303**	**2010**	**-0,0079**
1979	**1,3962**	**1995**	**-**	**2011**	**-3,5166**
1980	**0,7296**	**1996**	**-0,1588**	**2012**	**1,7307**
1981	**0,7519**	**1997**	**1,5226**	**2013**	**-1,8264**
1982	**-**	**1998**	**-0,9359**	**2014**	**0,8581**
1983	**-3,0580**	**1999**	**3,4783**	**2015**	**-1,9403**
1984	**-4,3531**	**2000**	**-**	**2016**	**-1,1410**
1985	**-0,1362**	**2001**	**-0,9173**	**2017**	**-0,2578**
1986	**-0,1468**	**2002**	**0,8923**	**2018**	**-0,2862**
1987	**2,2802**	**2003**	**2,7951**	**2019**	**2,0061**

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2019], we found that the minimum average absolute error 1,8644 is obtained applying Fourier Analysis for T=10 for the range of years: [1984,1993].

The optimal number of Fourier series terms is 8 being specified in the table 35.

Table 35.

a_0	**-0,0745**						
a_1	**-0,0817**	b_1	**0,3977**	a_2	**-0,3866**	b_2	**-0,4722**
a_3	**-0,2648**	b_3	**0,3435**	a_4	**0,1378**	b_4	**0,1153**
a_5	**-0,0882**	b_5	**0,4366**	a_6	**0,0662**	b_6	**-0,1661**
a_7	**0,0316**	b_7	**0,1466**	a_8	**0,0013**	b_8	**-0,1355**
a_9	**-**	b_9	**-**	a_{10}	**-**	b_{10}	**-**
a_{11}	**-**	b_{11}	**-**	a_{12}	**-**	b_{12}	**-**
a_{13}	**-**	b_{13}	**-**	a_{14}	**-**	b_{14}	**-**
a_{15}	**-**	b_{15}	**-**	a_{16}	**-**	b_{16}	**-**
a_{17}	**-**	b_{17}	**-**	a_{18}	**-**	b_{18}	**-**
a_{19}	**-**	b_{19}	**-**	a_{20}	**-**	b_{20}	**-**

The recalculated values of the Birth Rate are:

Table 36. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	**-0,1846**	**1988**	**-1,1394**	**2004**	**-0,1930**
1973	**0,9485**	**1989**	**0,0158**	**2005**	**0,8812**
1974	**0,0299**	**1990**	**-0,1846**	**2006**	**-1,1394**
1975	**0,1845**	**1991**	**0,9485**	**2007**	**0,0158**
1976	**-0,8781**	**1992**	**0,0299**	**2008**	**-0,1846**
1977	**-0,1930**	**1993**	**0,1845**	**2009**	**0,9485**
1978	**0,8812**	**1994**	**-0,8781**	**2010**	**0,0299**
1979	**-1,1394**	**1995**	**-0,1930**	**2011**	**0,1845**
1980	**0,0158**	**1996**	**0,8812**	**2012**	**-0,8781**
1981	**-0,1846**	**1997**	**-1,1394**	**2013**	**-0,1930**
1982	**0,9485**	**1998**	**0,0158**	**2014**	**0,8812**
1983	**0,0299**	**1999**	**-0,1846**	**2015**	**-1,1394**
1984	**0,1845**	**2000**	**0,9485**	**2016**	**0,0158**
1985	**-0,8781**	**2001**	**0,0299**	**2017**	**-0,1846**
1986	**-0,1930**	**2002**	**0,1845**	**2018**	**0,9485**
1987	**0,8812**	**2003**	**-0,8781**	**2019**	**0,0299**

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2019

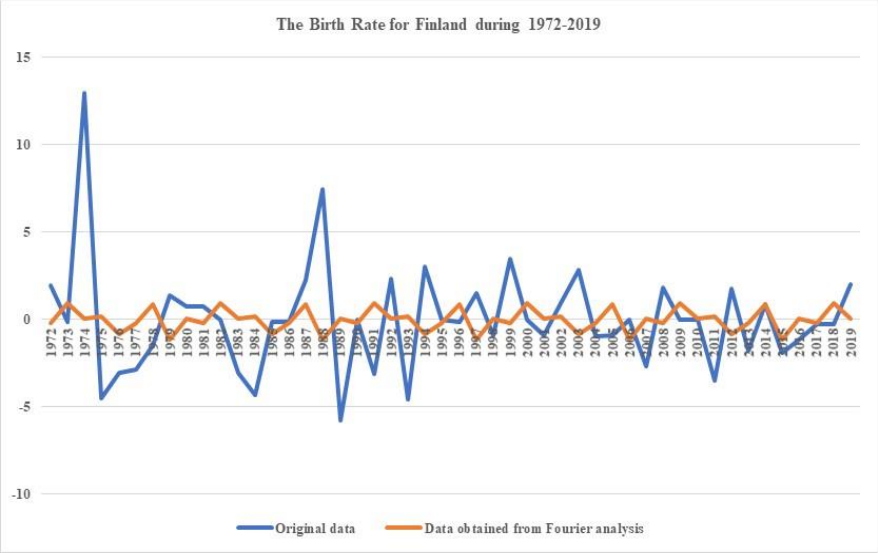

Figure 12

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**-6,5965**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2020	**7,752**
2021	**7,241**
2022	**6,763**
2023	**6,317**
2024	**5,901**

5.13. France

Table 37. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	-0,6305	1988	-0,7194	2004	0,7752
1973	-0,0588	1989	-0,0207	2005	-
1974	-0,0617	1990	-0,0213	2006	-
1975	-0,7101	1991	-0,0219	2007	-4,6338
1976	0,5591	1992	0,7349	2008	3,0713
1977	0,6119	1993	-0,0058	2009	-1,5564
1978	0,6562	1994	-3,0828	2010	1,5564
1979	0,6894	1995	6,2462	2011	-2,3316
1980	0,7092	1996	-2,4000	2012	0,7630
1981	-	1997	-	2013	-0,7999
1982	-0,0051	1998	1,5687	2014	1,5873
1983	-0,7092	1999	-0,0062	2015	-
1984	-	2000	2,3196	2016	1,5591
1985	-1,4035	2001	-4,6046	2017	-0,8757
1986	-0,0049	2002	-0,0229	2018	0,8033
1987	-0,0050	2003	0,7515	2019	0,8541

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2019], we found that the minimum average absolute error 0,9928 is obtained applying Fourier Analysis for T=25 for the range of years: [1990,2014].

The optimal number of Fourier series terms is 15 being specified in the table 38.

Table 38

a_0	-0,0135						
a_1	-0,0240	b_1	-0,0221	a_2	-0,0044	b_2	0,0012
a_3	0,0066	b_3	0,0259	a_4	0,0346	b_4	0,0186
a_5	0,0080	b_5	0,0804	a_6	0,0377	b_6	0,0187
a_7	-0,0092	b_7	-0,0483	a_8	-0,0054	b_8	0,0029
a_9	0,0721	b_9	-0,0279	a_{10}	0,0021	b_{10}	-0,0570
a_{11}	-0,0316	b_{11}	0,0571	a_{12}	-0,0373	b_{12}	-0,0006
a_{13}	-0,0226	b_{13}	-0,0399	a_{14}	0,0011	b_{14}	0,0282
a_{15}	0,0260	b_{15}	0,0108	a_{16}	-	b_{16}	-
a_{17}	-	b_{17}	-	a_{18}	-	b_{18}	-
a_{19}	-	b_{19}	-	a_{20}	-	b_{20}	-

The recalculated values of the Birth Rate are:

Table 39. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	0,0035	1988	-0,0494	2004	-0,0059
1973	0,0986	1989	0,1212	2005	-0,0170
1974	-0,0113	1990	-0,0037	2006	-0,3181
1975	0,1459	1991	0,0290	2007	0,1937
1976	-0,3215	1992	-0,0112	2008	-0,1064
1977	-0,0237	1993	-0,2072	2009	0,0997
1978	0,0470	1994	0,3999	2010	-0,1519
1979	0,0497	1995	-0,1635	2011	0,0408
1980	-0,0059	1996	0,0035	2012	-0,0494
1981	-0,0170	1997	0,0986	2013	0,1212
1982	-0,3181	1998	-0,0113	2014	-0,0037
1983	0,1937	1999	0,1459	2015	0,0290
1984	-0,1064	2000	-0,3215	2016	-0,0112
1985	0,0997	2001	-0,0237	2017	-0,2072
1986	-0,1519	2002	0,0470	2018	0,3999
1987	0,0408	2003	0,0497	2019	-0,1635

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2019

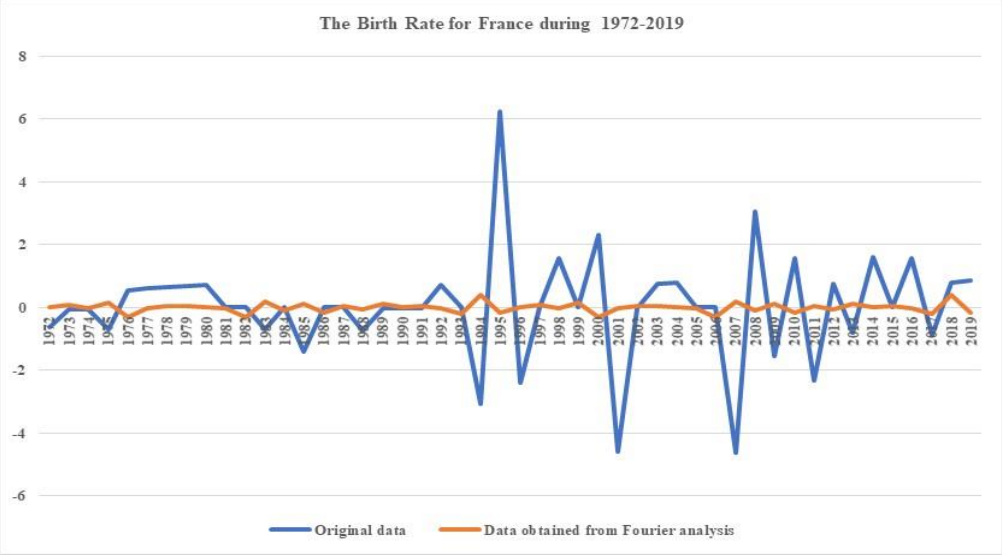

Figure 13

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**-2,0883**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2020	**10,966**
2021	**10,737**
2022	**10,513**
2023	**10,293**
2024	**10,078**

5.14. Greece

Table 40. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	**2,4053**	**1988**	**7,1381**	**2004**	**-1,0526**
1973	**-2,5197**	**1989**	**-6,5509**	**2005**	**-**
1974	**7,6902**	**1990**	**4,6174**	**2006**	**1,9983**
1975	**-7,0300**	**1991**	**-0,0099**	**2007**	**-5,0620**
1976	**4,3953**	**1992**	**2,0101**	**2008**	**6,9210**
1977	**-5,6608**	**1993**	**-4,0101**	**2009**	**-6,8752**
1978	**4,3994**	**1994**	**5,0619**	**2010**	**-1,8956**
1979	**-0,6494**	**1995**	**-5,0922**	**2011**	**-3,9659**
1980	**-**	**1996**	**1,9886**	**2012**	**1,5878**
1981	**-5,1990**	**1997**	**2,0943**	**2013**	**-0,2862**
1982	**2,3959**	**1998**	**-3,1359**	**2014**	**4,3317**
1983	**-0,1231**	**1999**	**2,0833**	**2015**	**1,1628**
1984	**-2,3545**	**2000**	**-**	**2016**	**-**
1985	**-1,9481**	**2001**	**-4,2110**	**2017**	**-5,8277**
1986	**4,4552**	**2002**	**3,1471**	**2018**	**3,4317**
1987	**-2,7759**	**2003**	**-0,0112**	**2019**	**-2,4842**

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2019], we found that the minimum average absolute error 2,4426 is obtained applying Fourier Analysis for T=5 for the range of years: [1974,1978].

The optimal number of Fourier series terms is 8 being specified in the table 41.

Table 41.

a_0	-0,7058						
a_1	-0,2952	b_1	1,5145	a_2	-1,4830	b_2	-1,0155
a_3	-0,0328	b_3	0,7344	a_4	-	b_4	-0,5078
a_5	-0,0118	b_5	0,3855	a_6	-0,1648	b_6	-0,3385
a_7	-0,0060	b_7	0,3007	a_8	-	b_8	-0,2539
a_9	-	b_9	-	a_{10}	-	b_{10}	-
a_{11}	-	b_{11}	-	a_{12}	-	b_{12}	-
a_{13}	-	b_{13}	-	a_{14}	-	b_{14}	-
a_{15}	-	b_{15}	-	a_{16}	-	b_{16}	-
a_{17}	-	b_{17}	-	a_{18}	-	b_{18}	-
a_{19}	-	b_{19}	-	a_{20}	-	b_{20}	-

The recalculated values of the Birth Rate are:

Table 42. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	-2,3465	1988	-2,3465	2004	-2,3465
1973	2,1598	1989	2,1598	2005	2,1598
1974	-1,6548	1990	-1,6548	2006	-1,6548
1975	0,4299	1991	0,4299	2007	0,4299
1976	-2,3465	1992	-2,3465	2008	-2,3465
1977	2,1598	1993	2,1598	2009	2,1598
1978	-1,6548	1994	-1,6548	2010	-1,6548
1979	0,4299	1995	0,4299	2011	0,4299
1980	-2,3465	1996	-2,3465	2012	-2,3465
1981	2,1598	1997	2,1598	2013	2,1598
1982	-1,6548	1998	-1,6548	2014	-1,6548
1983	0,4299	1999	0,4299	2015	0,4299
1984	-2,3465	2000	-2,3465	2016	-2,3465
1985	2,1598	2001	2,1598	2017	2,1598
1986	-1,6548	2002	-1,6548	2018	-1,6548
1987	0,4299	2003	0,4299	2019	0,4299

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2019

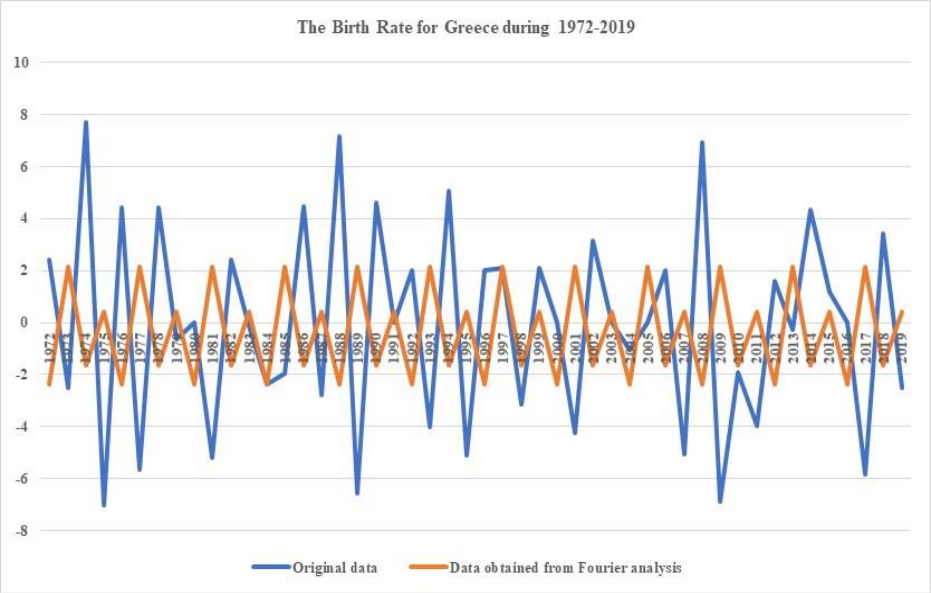

Figure 14.

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**-19,9695**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2020	**6,242**
2021	**4,996**
2022	**3,998**
2023	**3,200**
2024	**2,561**

5.15. Croatia

Table 43. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	-4,4272	1988	0,7438	2004	3,3583
1973	0,6571	1989	-3,2780	2005	2,8407
1974	-2,6893	1990	4,0233	2006	-7,5244
1975	-0,0044	1991	-3,4556	2007	3,1868
1976	-0,0044	1992	-2,8969	2008	4,5688
1977	1,3468	1993	10,1198	2009	-3,2989
1978	-0,0046	1994	-4,7994	2010	-4,9905
1979	-0,0044	1995	6,6945	2011	0,9295
1980	-2,6535	1996	3,4492	2012	4,1241
1981	0,6354	1997	-4,8671	2013	-6,1649
1982	-0,0185	1998	-18,1439	2014	3,0178
1983	-0,7134	1999	10,9079	2015	-3,2373
1984	0,6649	2000	1,8462	2016	5,4247
1985	-2,8981	2001	-4,1224	2017	-2,2347
1986	0,5571	2002	3,9485	2018	2,2347
1987	1,4156	2003	1,0628	2019	-2,2347

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2019], we found that the minimum average absolute error 2,8710 is obtained applying Fourier Analysis for T=10 for the range of years: [1990,1999].

The optimal number of Fourier series terms is 8 being specified in the table 44.

Table 44.

a_0	-0,1514						
a_1	0,0324	b_1	0,5180	a_2	0,7811	b_2	-0,9536
a_3	0,4723	b_3	1,5380	a_4	-0,1743	b_4	-0,9870
a_5	0,1116	b_5	-0,0241	a_6	-0,1181	b_6	-0,0374
a_7	-0,0638	b_7	0,2321	a_8	-0,0005	b_8	-0,2292
a_9	-	b_9	-	a_{10}	-	b_{10}	-
a_{11}	-	b_{11}	-	a_{12}	-	b_{12}	-
a_{13}	-	b_{13}	-	a_{14}	-	b_{14}	-
a_{15}	-	b_{15}	-	a_{16}	-	b_{16}	-
a_{17}	-	b_{17}	-	a_{18}	-	b_{18}	-
a_{19}	-	b_{19}	-	a_{20}	-	b_{20}	-

The recalculated values of the Birth Rate are:

Table 45. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	**-0,1400**	**1988**	**-2,2175**	**2004**	**0,7867**
1973	**-1,9319**	**1989**	**3,1511**	**2005**	**-0,4281**
1974	**0,6193**	**1990**	**-0,1400**	**2006**	**-2,2175**
1975	**-1,4299**	**1991**	**-1,9319**	**2007**	**3,1511**
1976	**0,9092**	**1992**	**0,6193**	**2008**	**-0,1400**
1977	**0,7867**	**1993**	**-1,4299**	**2009**	**-1,9319**
1978	**-0,4281**	**1994**	**0,9092**	**2010**	**0,6193**
1979	**-2,2175**	**1995**	**0,7867**	**2011**	**-1,4299**
1980	**3,1511**	**1996**	**-0,4281**	**2012**	**0,9092**
1981	**-0,1400**	**1997**	**-2,2175**	**2013**	**0,7867**
1982	**-1,9319**	**1998**	**3,1511**	**2014**	**-0,4281**
1983	**0,6193**	**1999**	**-0,1400**	**2015**	**-2,2175**
1984	**-1,4299**	**2000**	**-1,9319**	**2016**	**3,1511**
1985	**0,9092**	**2001**	**0,6193**	**2017**	**-0,1400**
1986	**0,7867**	**2002**	**-1,4299**	**2018**	**-1,9319**
1987	**-0,4281**	**2003**	**0,9092**	**2019**	**0,6193**

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2019

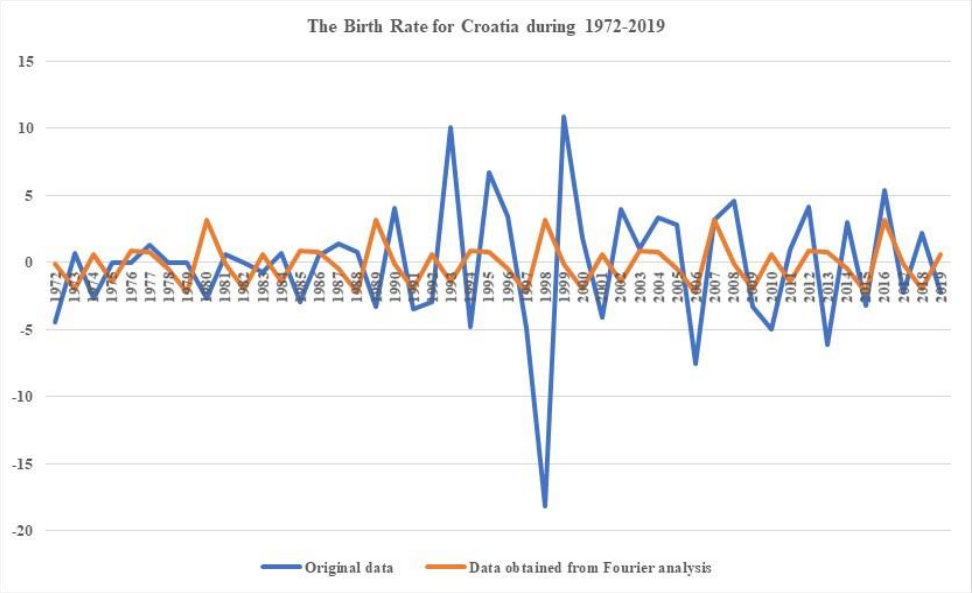

Figure 15.

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**0,9390**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2020	**8,984**
2021	**9,068**
2022	**9,153**
2023	**9,239**
2024	**9,326**

5.16. Hungary

Table 46. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2018

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	2,7398	1988	-0,0278	2004	3,1806
1973	0,6615	1989	2,5354	2005	2,1162
1974	16,6259	1990	1,6877	2006	-1,1296
1975	-15,2959	1991	-0,8895	2007	-4,0821
1976	-8,2621	1992	-6,5309	2008	4,0821
1977	0,3199	1993	1,4592	2009	-5,0922
1978	-1,4166	1994	2,5338	2010	-3,2197
1979	1,5294	1995	-2,6864	2011	4,0278
1980	-2,8747	1996	-1,9842	2012	5,6313
1981	3,0168	1997	1,6340	2013	-4,5080
1982	-1,6985	1998	0,8604	2014	6,6545
1983	1,2150	1999	-0,0967	2015	-6,6082
1984	3,9597	2000	7,5057	2016	4,2441
1985	4,2301	2001	-5,3895	2017	-3,1915
1986	-4,2095	2002	1,0417	2018	-
1987	-0,8332	2003	-	2019	-0,0108

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2018], we found that the minimum average absolute error 3,1782 is obtained applying Fourier Analysis for T=16 for the range of years: [1980,1995].

The optimal number of Fourier series terms is 8 being specified in the table 47.

Table 47

a_0	0,3364						
a_1	-0,0056	b_1	-1,5237	a_2	0,0025	b_2	0,7858
a_3	0,2309	b_3	-0,2552	a_4	0,0055	b_4	0,4178
a_5	-0,1089	b_5	-0,4384	a_6	0,0357	b_6	0,2465
a_7	-0,0373	b_7	-0,1578	a_8	0,0285	b_8	0,2523
a_9	-	b_9	-	a_{10}	-	b_{10}	-
a_{11}	-	b_{11}	-	a_{12}	-	b_{12}	-
a_{13}	-	b_{13}	-	a_{14}	-	b_{14}	-
a_{15}	-	b_{15}	-	a_{16}	-	b_{16}	-
a_{17}	-	b_{17}	-	a_{18}	-	b_{18}	-
a_{19}	-	b_{19}	-	a_{20}	-	b_{20}	-

The recalculated values of the Birth Rate are:

Table 48. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2018

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	0,1931	1988	0,4173	2004	-0,1911
1973	0,4173	1989	-0,1911	2005	-0,6605
1974	-0,1911	1990	-0,6605	2006	-1,5248
1975	-0,6605	1991	-1,5248	2007	-1,1195
1976	-1,5248	1992	-1,1195	2008	-1,0314
1977	-1,1195	1993	-1,0314	2009	-2,4115
1978	-1,0314	1994	-2,4115	2010	0,1613
1979	-2,4115	1995	0,1613	2011	2,3222
1980	0,1613	1996	2,3222	2012	1,7710
1981	2,3222	1997	1,7710	2013	1,9656
1982	1,7710	1998	1,9656	2014	1,6436
1983	1,9656	1999	1,6436	2015	0,4183
1984	1,6436	2000	0,4183	2016	0,5691
1985	0,4183	2001	0,5691	2017	0,1931
1986	0,5691	2002	0,1931	2018	0,4173
1987	0,1931	2003	0,4173	2019	-

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2018

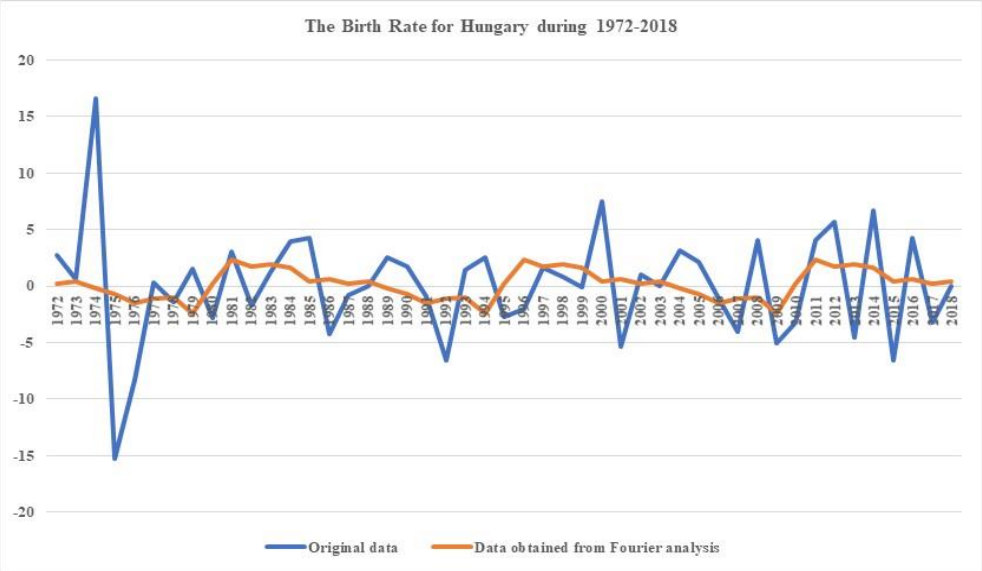

Figure 16.

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**6,8180**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2019	**10,148**
2020	**10,840**
2021	**11,579**
2022	**12,368**
2023	**13,211**

5.17. Ireland

Table 49. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	-3,6697	1988	-0,8882	2004	-
1973	-	1989	1,5445	2005	-1,3329
1974	-0,0179	1990	6,5431	2006	6,0100
1975	-2,7456	1991	-3,3515	2007	2,5040
1976	3,1430	1992	-2,0312	2008	-2,7960
1977	1,4264	1993	-0,8110	2009	-3,0864
1978	-0,0023	1994	1,2681	2010	-
1979	1,4195	1995	3,6449	2011	-0,6206
1980	-0,9655	1996	2,2167	2012	-1,8855
1981	-4,6168	1997	0,6341	2013	-0,7835
1982	0,8158	1998	-2,9027	2014	1,1315
1983	-3,0405	1999	-1,3841	2015	-0,1165
1984	0,6757	2000	0,6897	2016	-0,1249
1985	2,4732	2001	-	2017	-0,1342
1986	1,6260	2002	-1,5000	2018	0,6305
1987	-4,0360	2003	-2,6667	2019	-0,0992

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2019], we found that the minimum average absolute error 1,5070 is obtained applying Fourier Analysis for T=12 for the range of years: [1981,1992].

The optimal number of Fourier series terms is 8 being specified in the table 50.

Table 50

a_0	-0,1755						
a_1	0,1183	b_1	-0,3849	a_2	-0,3691	b_2	0,3574
a_3	-0,3048	b_3	-0,1379	a_4	0,1192	b_4	0,0443
a_5	0,0185	b_5	0,0252	a_6	-0,0128	b_6	0,1757
a_7	-0,0389	b_7	-0,1018	a_8	0,0429	b_8	0,0696
a_9	-	b_9	-	a_{10}	-	b_{10}	-
a_{11}	-	b_{11}	-	a_{12}	-	b_{12}	-
a_{13}	-	b_{13}	-	a_{14}	-	b_{14}	-
a_{15}	-	b_{15}	-	a_{16}	-	b_{16}	-
a_{17}	-	b_{17}	-	a_{18}	-	b_{18}	-
a_{19}	-	b_{19}	-	a_{20}	-	b_{20}	-

The recalculated values of the Birth Rate are:

Table 51. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	0,4890	1988	0,0294	2004	0,0372
1973	0,6164	1989	0,5459	2005	0,4890
1974	0,3107	1990	-1,0442	2006	0,6164
1975	-0,6220	1991	-0,9642	2007	0,3107
1976	-0,2627	1992	-0,1007	2008	-0,6220
1977	0,0294	1993	0,0372	2009	-0,2627
1978	0,5459	1994	0,4890	2010	0,0294
1979	-1,0442	1995	0,6164	2011	0,5459
1980	-0,9642	1996	0,3107	2012	-1,0442
1981	-0,1007	1997	-0,6220	2013	-0,9642
1982	0,0372	1998	-0,2627	2014	-0,1007
1983	0,4890	1999	0,0294	2015	0,0372
1984	0,6164	2000	0,5459	2016	0,4890
1985	0,3107	2001	-1,0442	2017	0,6164
1986	-0,6220	2002	-0,9642	2018	0,3107
1987	-0,2627	2003	-0,1007	2019	-0,6220

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2019

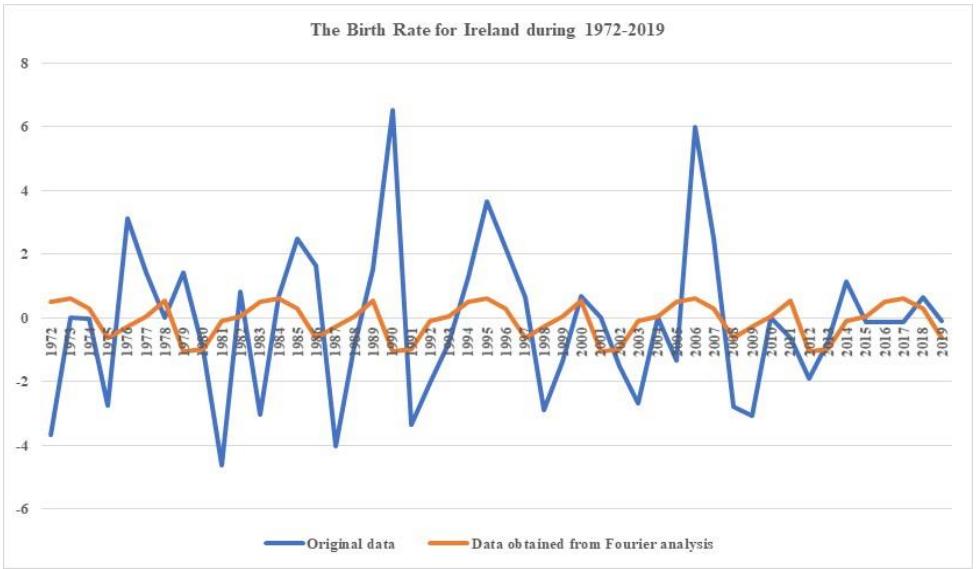

Figure 17.

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**0,6030**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2020	**12,173**
2021	**12,246**
2022	**12,320**
2023	**12,395**
2024	**12,469**

5.18. Italy

Table 52. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	**-3,5750**	**1988**	**5,1441**	**2004**	**2,0941**
1973	**1,1357**	**1989**	**-6,1039**	**2005**	**-5,1987**
1974	**0,5905**	**1990**	**2,9903**	**2006**	**2,0408**
1975	**-4,4462**	**1991**	**-2,0101**	**2007**	**-**
1976	**-0,3441**	**1992**	**2,0101**	**2008**	**-0,0108**
1977	**0,3260**	**1993**	**-4,0101**	**2009**	**-3,0717**
1978	**1,1688**	**1994**	**-0,0928**	**2010**	**0,9991**
1979	**-1,0101**	**1995**	**0,9651**	**2011**	**-2,1162**
1980	**0,5136**	**1996**	**3,2147**	**2012**	**0,9840**
1981	**2,3871**	**1997**	**-0,0117**	**2013**	**-3,3817**
1982	**1,7458**	**1998**	**-1,0753**	**2014**	**3,2027**
1983	**-1,8432**	**1999**	**-**	**2015**	**-1,2616**
1984	**0,8655**	**2000**	**-**	**2016**	**1,1145**
1985	**-0,0363**	**2001**	**-2,1164**	**2017**	**-0,0641**
1986	**-1,9985**	**2002**	**1,0526**	**2018**	**-1,3833**
1987	**2,9012**	**2003**	**-**	**2019**	**-0,1622**

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2019], we found that the minimum average absolute error 1,5425 is obtained applying Fourier Analysis for T=23 for the range of years: [1984,2006].

The optimal number of Fourier series terms is 14 being specified in the table 53.

Table 53.

a_0	-0,0405						
a_1	-0,0008	b_1	-0,0022	a_2	0,0377	b_2	0,0342
a_3	0,0650	b_3	-0,0370	a_4	0,0233	b_4	0,0389
a_5	0,0560	b_5	-0,0255	a_6	0,0266	b_6	-0,1027
a_7	-0,0166	b_7	0,0830	a_8	0,0483	b_8	-0,0564
a_9	-0,0521	b_9	0,0330	a_{10}	-0,0170	b_{10}	0,0341
a_{11}	0,0993	b_{11}	0,0011	a_{12}	-0,0118	b_{12}	-0,0254
a_{13}	-0,0250	b_{13}	-0,0143	a_{14}	0,0158	b_{14}	0,0171
a_{15}	-	b_{15}	-	a_{16}	-	b_{16}	-
a_{17}	-	b_{17}	-	a_{18}	-	b_{18}	-
a_{19}	-	b_{19}	-	a_{20}	-	b_{20}	-

The recalculated values of the Birth Rate are:

Table 54. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	0,0645	1988	-0,4474	2004	-0,3739
1973	0,2286	1989	0,1822	2005	0,1147
1974	-0,0141	1990	-0,1905	2006	-0,0232
1975	-0,0850	1991	0,1115	2007	-0,1530
1976	-0,0222	1992	-0,3151	2008	0,1874
1977	-0,0371	1993	-0,0437	2009	0,3370
1978	-0,1557	1994	0,0645	2010	-0,4474
1979	0,0701	1995	0,2286	2011	0,1822
1980	-0,0243	1996	-0,0141	2012	-0,1905
1981	0,1440	1997	-0,0850	2013	0,1115
1982	-0,3739	1998	-0,0222	2014	-0,3151
1983	0,1147	1999	-0,0371	2015	-0,0437
1984	-0,0232	2000	-0,1557	2016	0,0645
1985	-0,1530	2001	0,0701	2017	0,2286
1986	0,1874	2002	-0,0243	2018	-0,0141
1987	0,3370	2003	0,1440	2019	-0,0850

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2019

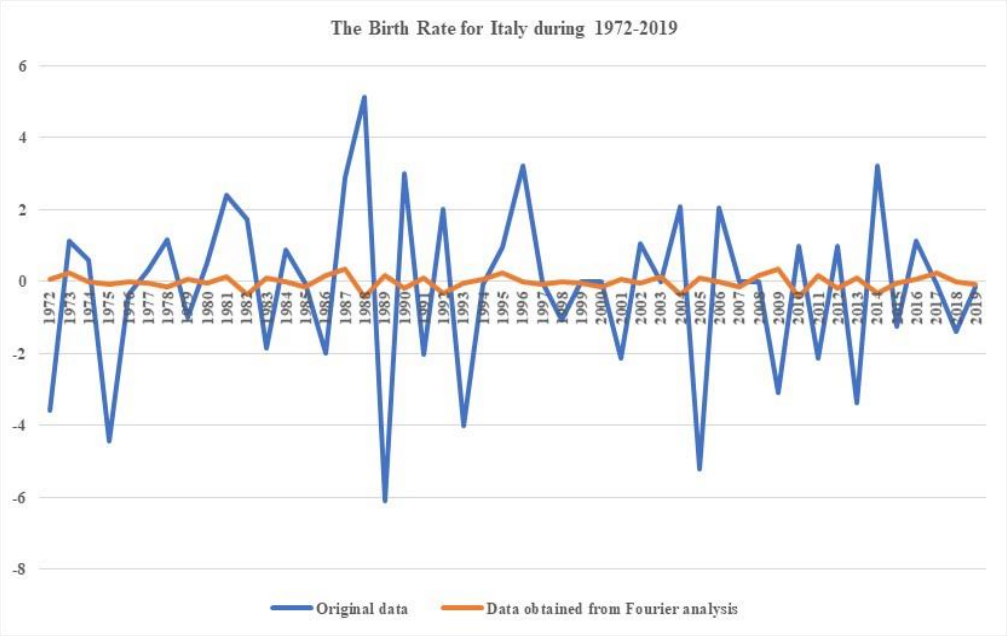

Figure 18

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**-0,0976**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2020	**6,993**
2021	**6,986**
2022	**6,980**
2023	**6,973**
2024	**6,966**

5.19. Lithuania

Table 55. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	-2,8441	1988	-3,6914	2004	-2,3256
1973	-2,4733	1989	2,9072	2005	-
1974	5,2574	1990	4,5674	2006	1,1108
1975	-0,6329	1991	-3,9349	2007	-0,0494
1976	1,2579	1992	-0,7009	2008	4,2538
1977	-	1993	-9,5959	2009	-3,4213
1978	-0,0164	1994	2,1674	2010	-5,9715
1979	1,2903	1995	8,3534	2011	3,9513
1980	-	1996	-3,5391	2012	0,9899
1981	0,6536	1997	3,4113	2013	-2,9804
1982	-	1998	-0,0349	2014	3,9507
1983	5,8271	1999	0,9253	2015	0,8759
1984	-7,7490	2000	-3,8929	2016	-4,7721
1985	1,2159	2001	-3,3089	2017	-4,6816
1986	0,6023	2002	3,7189	2018	4,6174
1987	-3,0085	2003	6,7700	2019	-1,0099

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2019], we found that the minimum average absolute error 2,6669 is obtained applying Fourier Analysis for T=9 for the range of years: [1990,1998].

The optimal number of Fourier series terms is 8 being specified in the table 56.

Table 56

a_0	-0,7217						
a_1	0,4964	b_1	1,7357	a_2	0,5869	b_2	-0,9511
a_3	0,3522	b_3	-0,0849	a_4	0,0624	b_4	-0,3868
a_5	0,1268	b_5	0,5256	a_6	0,0652	b_6	-0,2381
a_7	0,0101	b_7	0,2172	a_8	-	b_8	-0,1934
a_9	-	b_9	-	a_{10}	-	b_{10}	-
a_{11}	-	b_{11}	-	a_{12}	-	b_{12}	-
a_{13}	-	b_{13}	-	a_{14}	-	b_{14}	-
a_{15}	-	b_{15}	-	a_{16}	-	b_{16}	-
a_{17}	-	b_{17}	-	a_{18}	-	b_{18}	-
a_{19}	-	b_{19}	-	a_{20}	-	b_{20}	-

The recalculated values of the Birth Rate are:

Table 57. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	1,1785	1988	1,1785	2004	1,1785
1973	0,9123	1989	0,9123	2005	0,9123
1974	-0,6318	1990	-0,6318	2006	-0,6318
1975	-1,7978	1991	-1,7978	2007	-1,7978
1976	-3,0797	1992	-3,0797	2008	-3,0797
1977	-0,3330	1993	-0,3330	2009	-0,3330
1978	1,3391	1994	1,3391	2010	1,3391
1979	-0,4746	1995	-0,4746	2011	-0,4746
1980	1,1785	1996	1,1785	2012	1,1785
1981	0,9123	1997	0,9123	2013	0,9123
1982	-0,6318	1998	-0,6318	2014	-0,6318
1983	-1,7978	1999	-1,7978	2015	-1,7978
1984	-3,0797	2000	-3,0797	2016	-3,0797
1985	-0,3330	2001	-0,3330	2017	-0,3330
1986	1,3391	2002	1,3391	2018	1,3391
1987	-0,4746	2003	-0,4746	2019	-0,4746

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2019

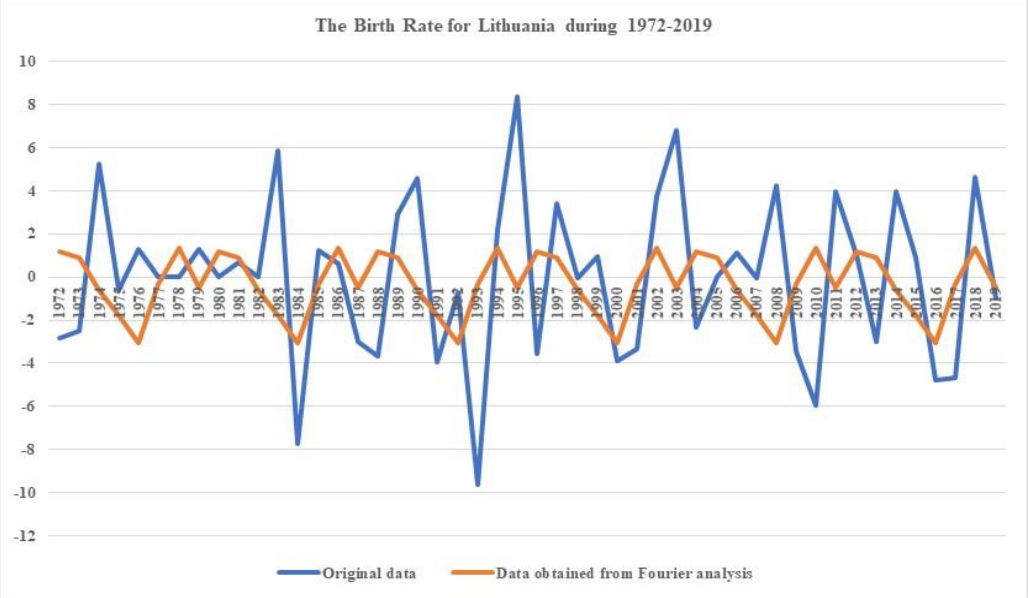

Figure 19.

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**-17,8870**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2020	**8,047**
2021	**6,608**
2022	**5,426**
2023	**4,455**
2024	**3,658**

5.20. Luxembourg

Table 58. The evolution of the accelerations (percents) of the Birth Rate during the period 1973-2018

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	**-**	**1988**	**10,4588**	**2004**	**4,2094**
1973	**0,7562**	**1989**	**-7,0817**	**2005**	**-5,0707**
1974	**11,2524**	**1990**	**3,2193**	**2006**	**5,1004**
1975	**-2,7778**	**1991**	**-4,0323**	**2007**	**-4,3032**
1976	**-**	**1992**	**-**	**2008**	**3,4413**
1977	**4,5541**	**1993**	**1,5030**	**2009**	**-2,6163**
1978	**-2,7523**	**1994**	**-3,0534**	**2010**	**4,3940**
1979	**-**	**1995**	**-**	**2011**	**-8,6894**
1980	**-**	**1996**	**4,4890**	**2012**	**9,7042**
1981	**4,3547**	**1997**	**-7,3871**	**2013**	**-3,6697**
1982	**-8,6197**	**1998**	**1,3262**	**2014**	**-**
1983	**-0,9105**	**1999**	**5,4156**	**2015**	**1,7049**
1984	**4,2670**	**2000**	**-1,5930**	**2016**	**-0,9688**
1985	**-3,4859**	**2001**	**-6,1127**	**2017**	**2,8037**
1986	**7,0730**	**2002**	**2,1177**	**2018**	**-**
1987	**-7,0284**	**2003**	**0,7258**	**2019**	**-1,9511**

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1973,2018], we found that the minimum average absolute error 3,4652 is obtained applying Fourier Analysis for T=10 for the range of years: [1979,1988].

The optimal number of Fourier series terms is 8 being specified in the table 59.

Table 59.

a_0	**1,1414**						
a_1	**-0,0544**	b_1	**-1,4229**	a_2	**0,4240**	b_2	**0,7938**
a_3	**-0,0805**	b_3	**-0,7745**	a_4	**-0,7162**	b_4	**0,2045**
a_5	**0,4584**	b_5	**-0,6806**	a_6	**0,0201**	b_6	**0,3699**
a_7	**-0,0346**	b_7	**-0,3492**	a_8	**0,0008**	b_8	**0,2948**
a_9	**-**	b_9	**-**	a_{10}	**-**	b_{10}	**-**
a_{11}	**-**	b_{11}	**-**	a_{12}	**-**	b_{12}	**-**
a_{13}	**-**	b_{13}	**-**	a_{14}	**-**	b_{14}	**-**
a_{15}	**-**	b_{15}	**-**	a_{16}	**-**	b_{16}	**-**
a_{17}	**-**	b_{17}	**-**	a_{18}	**-**	b_{18}	**-**
a_{19}	**-**	b_{19}	**-**	a_{20}	**-**	b_{20}	**-**

The recalculated values of the Birth Rate are:

Table 60. The evolution of the accelerations (percents) of the Birth Rate during the period 1973-2018

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	**-**	**1988**	**0,0105**	**2004**	**-2,0195**
1973	**1,1814**	**1989**	**3,0967**	**2005**	**0,3955**
1974	**1,5352**	**1990**	**0,4179**	**2006**	**0,0105**
1975	**-0,3035**	**1991**	**1,1814**	**2007**	**3,0967**
1976	**0,8220**	**1992**	**1,5352**	**2008**	**0,4179**
1977	**-2,0195**	**1993**	**-0,3035**	**2009**	**1,1814**
1978	**0,3955**	**1994**	**0,8220**	**2010**	**1,5352**
1979	**0,0105**	**1995**	**-2,0195**	**2011**	**-0,3035**
1980	**3,0967**	**1996**	**0,3955**	**2012**	**0,8220**
1981	**0,4179**	**1997**	**0,0105**	**2013**	**-2,0195**
1982	**1,1814**	**1998**	**3,0967**	**2014**	**0,3955**
1983	**1,5352**	**1999**	**0,4179**	**2015**	**0,0105**
1984	**-0,3035**	**2000**	**1,1814**	**2016**	**3,0967**
1985	**0,8220**	**2001**	**1,5352**	**2017**	**0,4179**
1986	**-2,0195**	**2002**	**-0,3035**	**2018**	**1,1814**
1987	**0,3955**	**2003**	**0,8220**	**2019**	**-**

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1973-2018

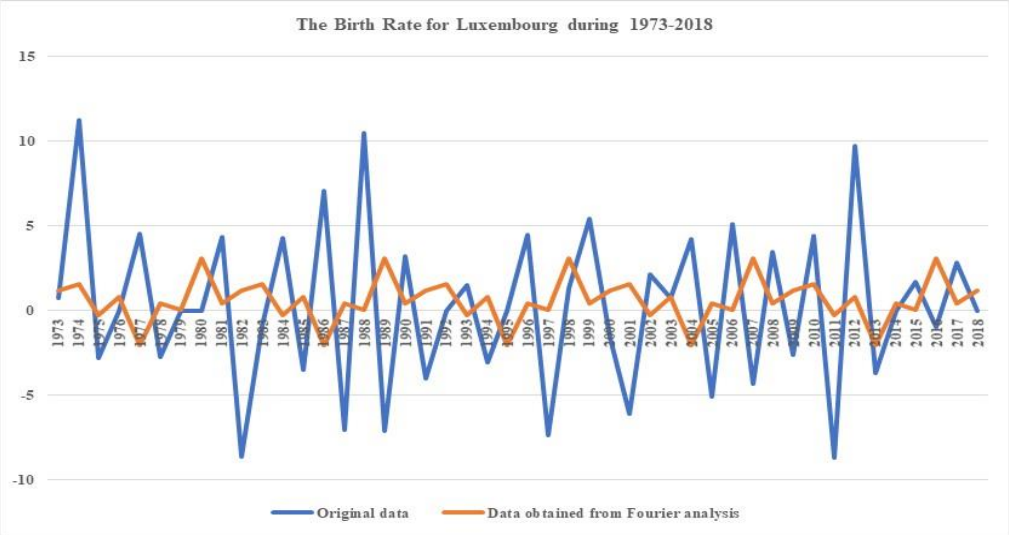

Figure 20.

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**26,8624**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2019	**12,686**
2020	**16,094**
2021	**20,417**
2022	**25,902**
2023	**32,860**

5.21. Latvia

Table 61. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	-2,7213	1988	-1,8789	2004	-6,8341
1973	-2,0733	1989	-3,9103	2005	8,7793
1974	4,8431	1990	3,6706	2006	-2,5903
1975	-2,1177	1991	-5,0068	2007	0,7232
1976	-0,7092	1992	0,1129	2008	-3,0729
1977	-0,0201	1993	-6,4160	2009	-10,7880
1978	0,7040	1994	6,3573	2010	-0,7022
1979	1,4545	1995	-1,6827	2011	5,5464
1980	1,4440	1996	2,4784	2012	10,8838
1981	-1,4647	1997	1,9583	2013	-3,6107
1982	3,5162	1998	3,6396	2014	2,7811
1983	3,2070	1999	7,8776	2015	-5,0278
1984	-7,4324	2000	-0,4061	2016	-0,9340
1985	-	2001	-8,4984	2017	-5,3652
1986	7,6902	2002	5,8970	2018	-2,0778
1987	-5,1666	2003	2,1757	2019	4,5421

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2019], we found that the minimum average absolute error 3,3253 is obtained applying Fourier Analysis for T=15 for the range of years: [1999,2013].

The optimal number of Fourier series terms is 8 being specified in the table 62.

Table 62.

a_0	-0,2582						
a_1	-0,1829	b_1	0,8796	a_2	-0,2248	b_2	-0,9279
a_3	0,4479	b_3	0,1509	a_4	-0,1059	b_4	-0,1882
a_5	-0,1726	b_5	0,4610	a_6	0,2706	b_6	0,0299
a_7	-0,0806	b_7	0,1449	a_8	0,1522	b_8	-0,2387
a_9	-	b_9	-	a_{10}	-	b_{10}	-
a_{11}	-	b_{11}	-	a_{12}	-	b_{12}	-
a_{13}	-	b_{13}	-	a_{14}	-	b_{14}	-
a_{15}	-	b_{15}	-	a_{16}	-	b_{16}	-
a_{17}	-	b_{17}	-	a_{18}	-	b_{18}	-
a_{19}	-	b_{19}	-	a_{20}	-	b_{20}	-

The recalculated values of the Birth Rate are:

Table 63. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	-2,4312	1988	-0,6250	2004	0,4495
1973	-0,4970	1989	-1,4274	2005	-0,6004
1974	-0,6250	1990	0,4495	2006	-0,0252
1975	-1,4274	1991	-0,6004	2007	-0,4067
1976	0,4495	1992	-0,0252	2008	-1,0092
1977	-0,6004	1993	-0,4067	2009	0,4644
1978	-0,0252	1994	-1,0092	2010	1,2412
1979	-0,4067	1995	0,4644	2011	2,1764
1980	-1,0092	1996	1,2412	2012	0,9320
1981	0,4644	1997	2,1764	2013	-0,0488
1982	1,2412	1998	0,9320	2014	-2,4312
1983	2,1764	1999	-0,0488	2015	-0,4970
1984	0,9320	2000	-2,4312	2016	-0,6250
1985	-0,0488	2001	-0,4970	2017	-1,4274
1986	-2,4312	2002	-0,6250	2018	0,4495
1987	-0,4970	2003	-1,4274	2019	-0,6004

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2019

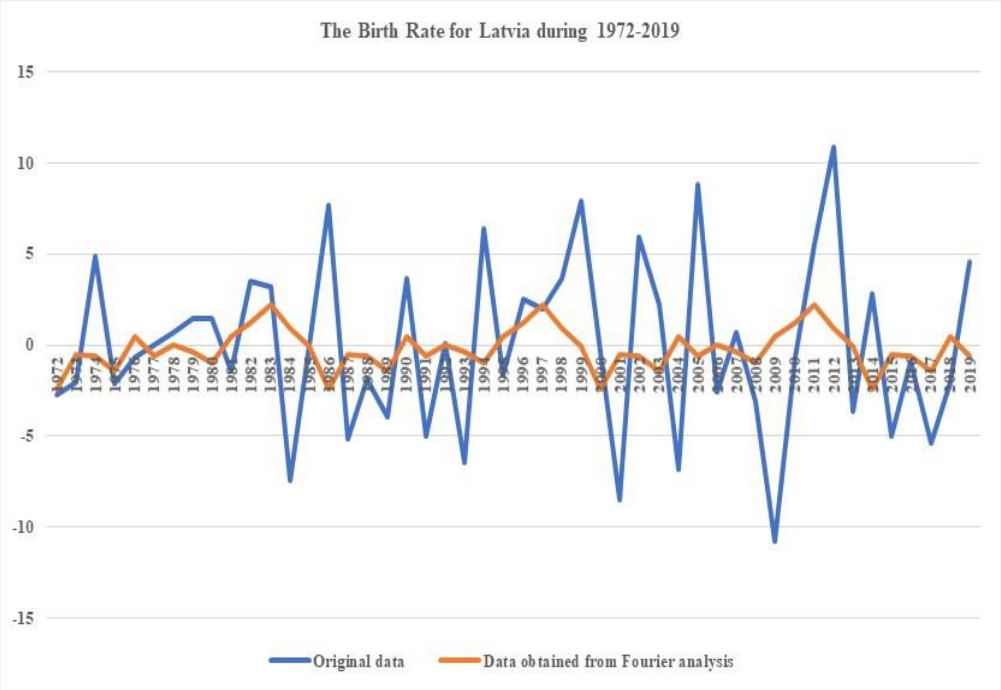

Figure 21.

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**-9,1838**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2020	**8,900**
2021	**8,083**
2022	**7,340**
2023	**6,666**
2024	**6,054**

5.22. Malta

Table 64. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	-7,8064	1988	2,5932	2004	-7,9323
1973	1,0137	1989	-3,2468	2005	3,8711
1974	5,6955	1990	-	2006	-2,0941
1975	0,9961	1991	0,4551	2007	3,1250
1976	-5,5082	1992	6,0022	2008	-
1977	2,6767	1993	-8,7662	2009	-5,3763
1978	-5,3166	1994	-0,4828	2010	-
1979	5,9015	1995	1,7678	2011	10,4646
1980	-7,0365	1996	14,4428	2012	-8,3830
1981	-1,3760	1997	-13,5277	2013	-1,0612
1982	15,2508	1998	-0,2131	2014	4,1138
1983	-14,5610	1999	-1,0607	2015	-0,0109
1984	4,3553	2000	4,9079	2016	-0,0108
1985	-3,5641	2001	-9,7423	2017	-7,1533
1986	-0,2352	2002	8,6393	2018	6,1224
1987	5,6225	2003	5,0105	2019	-

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2019], we found that the minimum average absolute error 4,3112 is obtained applying Fourier Analysis for T=29 for the range of years: [1974,2002].

The optimal number of Fourier series terms is 14 being specified in the table 65.

Table 65.

a_0	0,1402						
a_1	0,0220	b_1	1,1941	a_2	0,0219	b_2	-0,5989
a_3	-0,0148	b_3	0,3831	a_4	0,0858	b_4	-0,3896
a_5	-0,0278	b_5	0,2619	a_6	-0,1380	b_6	-0,1547
a_7	-0,1544	b_7	0,1661	a_8	0,1381	b_8	-0,3324
a_9	-0,0387	b_9	0,1542	a_{10}	-0,1262	b_{10}	-0,0850
a_{11}	0,0714	b_{11}	0,2527	a_{12}	0,1214	b_{12}	-0,2086
a_{13}	-0,0315	b_{13}	0,1592	a_{14}	-0,1474	b_{14}	-0,0913
a_{15}	-	b_{15}	-	a_{16}	-	b_{16}	-
a_{17}	-	b_{17}	-	a_{18}	-	b_{18}	-
a_{19}	-	b_{19}	-	a_{20}	-	b_{20}	-

The recalculated values of the Birth Rate are:

Table 66. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	1,0065	1988	-0,1481	2004	-1,3643
1973	2,6439	1989	0,2303	2005	-1,6626
1974	0,1995	1990	0,3563	2006	-1,0240
1975	-2,2258	1991	0,7907	2007	-1,4966
1976	-1,3643	1992	0,1042	2008	-1,0002
1977	-1,6626	1993	0,7038	2009	-0,1624
1978	-1,0240	1994	1,0213	2010	-1,3703
1979	-1,4966	1995	1,7725	2011	-0,5053
1980	-1,0002	1996	0,4228	2012	-0,6428
1981	-0,1624	1997	1,2324	2013	-0,3006
1982	-1,3703	1998	1,2716	2014	0,0855
1983	-0,5053	1999	1,8781	2015	0,1459
1984	-0,6428	2000	1,0065	2016	-0,1481
1985	-0,3006	2001	2,6439	2017	0,2303
1986	0,0855	2002	0,1995	2018	0,3563
1987	0,1459	2003	-2,2258	2019	0,7907

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2019

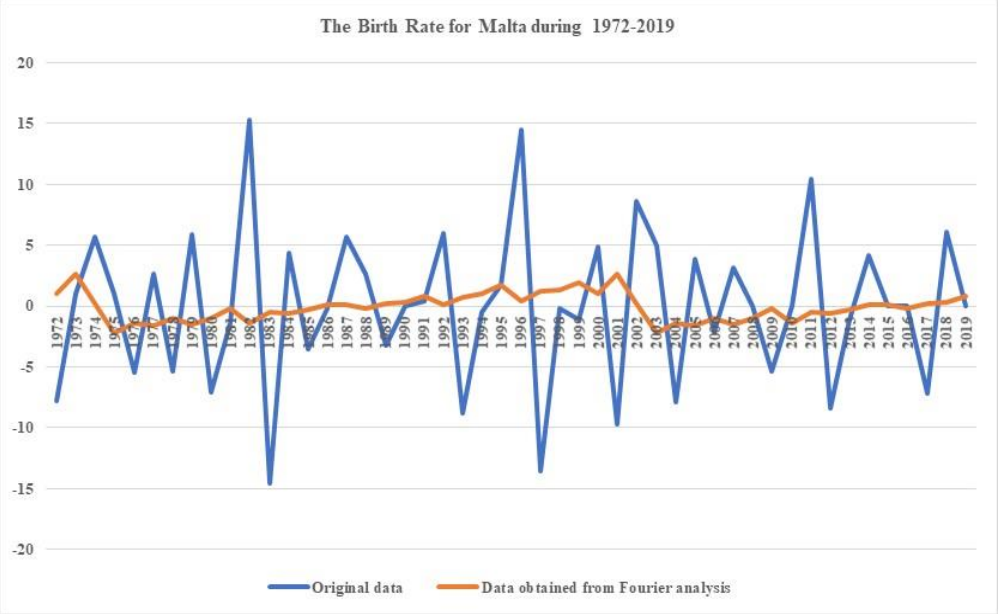

Figure 22

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**0,0634**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2020	**9,206**
2021	**9,212**
2022	**9,218**
2023	**9,223**
2024	**9,229**

5.23. Netherlands

Table 67. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	-0,3844	1988	-	2004	-1,6520
1973	-3,5426	1989	1,5811	2005	-0,1093
1974	4,4207	1990	3,1433	2006	1,6222
1975	0,4077	1991	-3,9370	2007	-0,0308
1976	4,3403	1992	-	2008	2,6708
1977	-2,3316	1993	-0,0233	2009	-0,9009
1978	3,9008	1994	0,7572	2010	-
1979	-1,5937	1995	-2,3683	2011	-1,8098
1980	3,1937	1996	2,3366	2012	-0,0751
1981	-4,7438	1997	1,6327	2013	-0,0793
1982	-1,6562	1998	2,4323	2014	4,8179
1983	2,3333	1999	-3,2520	2015	-4,8454
1984	4,2091	2000	-	2016	2,8846
1985	-0,8895	2001	-5,4391	2017	-
1986	1,5991	2002	2,2832	2018	0,9701
1987	-3,2520	2003	-0,8063	2019	-0,0103

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2019], we found that the minimum average absolute error 1,6837 is obtained applying Fourier Analysis for T=7 for the range of years: [2010,2016].

The optimal number of Fourier series terms is 9 being specified in the table 68.

Table 68.

a_0	1,1985						
a_1	0,1612	b_1	0,3041	a_2	0,3238	b_2	-0,6898
a_3	0,5194	b_3	0,1920	a_4	0,0809	b_4	-0,0436
a_5	0,0064	b_5	0,1261	a_6	-	b_6	-0,0960
a_7	0,0033	b_7	0,0767	a_8	0,0202	b_8	-0,0971
a_9	0,0577	b_9	0,0640	a_{10}	-	b_{10}	-
a_{11}	-	b_{11}	-	a_{12}	-	b_{12}	-
a_{13}	-	b_{13}	-	a_{14}	-	b_{14}	-
a_{15}	-	b_{15}	-	a_{16}	-	b_{16}	-
a_{17}	-	b_{17}	-	a_{18}	-	b_{18}	-
a_{19}	-	b_{19}	-	a_{20}	-	b_{20}	-

The recalculated values of the Birth Rate are:

Table 69. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	**-0,5280**	**1988**	**0,3183**	**2004**	**0,2762**
1973	**1,7429**	**1989**	**1,7722**	**2005**	**0,0141**
1974	**0,2762**	**1990**	**-0,5280**	**2006**	**0,3183**
1975	**0,0141**	**1991**	**1,7429**	**2007**	**1,7722**
1976	**0,3183**	**1992**	**0,2762**	**2008**	**-0,5280**
1977	**1,7722**	**1993**	**0,0141**	**2009**	**1,7429**
1978	**-0,5280**	**1994**	**0,3183**	**2010**	**0,2762**
1979	**1,7429**	**1995**	**1,7722**	**2011**	**0,0141**
1980	**0,2762**	**1996**	**-0,5280**	**2012**	**0,3183**
1981	**0,0141**	**1997**	**1,7429**	**2013**	**1,7722**
1982	**0,3183**	**1998**	**0,2762**	**2014**	**-0,5280**
1983	**1,7722**	**1999**	**0,0141**	**2015**	**1,7429**
1984	**-0,5280**	**2000**	**0,3183**	**2016**	**0,2762**
1985	**1,7429**	**2001**	**1,7722**	**2017**	**0,0141**
1986	**0,2762**	**2002**	**-0,5280**	**2018**	**0,3183**
1987	**0,0141**	**2003**	**1,7429**	**2019**	**1,7722**

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2019

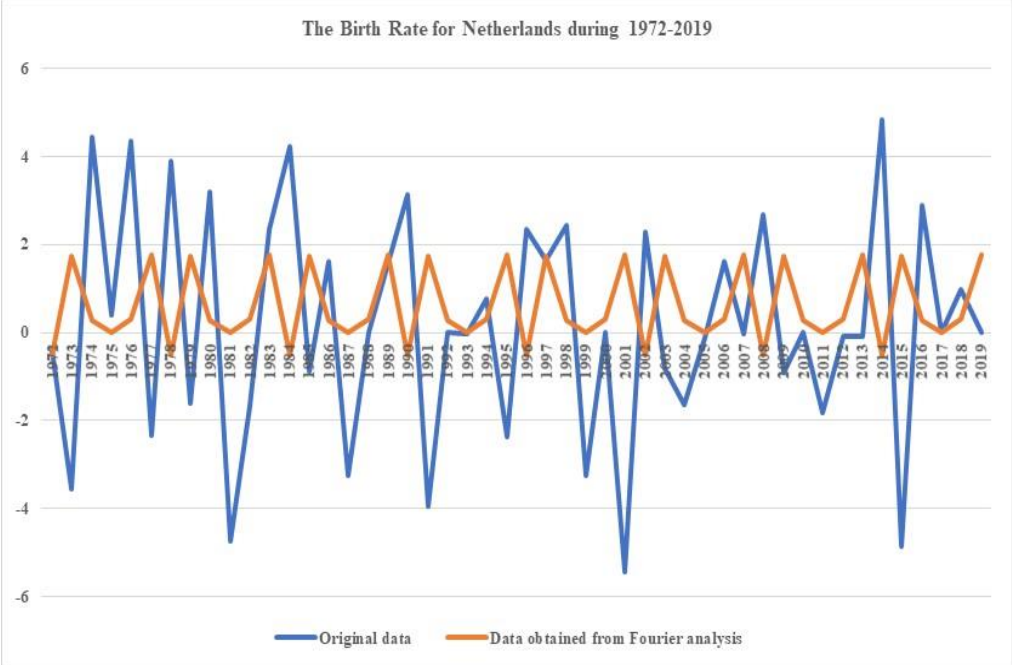

Figure 23.

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**22,7547**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2020	**11,907**
2021	**14,617**
2022	**17,943**
2023	**22,025**
2024	**27,037**

5.24. Poland

Table 70. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	-0,6368	1988	2,1885	2004	2,1623
1973	1,1129	1989	-1,3816	2005	1,0635
1974	-0,0793	1990	1,1315	2006	1,0074
1975	-0,0751	1991	2,6613	2007	0,9237
1976	0,4552	1992	-5,5993	2008	2,7811
1977	-5,1987	1993	2,5624	2009	-6,8627
1978	1,5200	1994	0,6305	2010	-
1979	3,1386	1995	-7,2992	2011	-
1980	-2,6178	1996	9,5071	2012	5,4416
1981	-	1997	-2,7107	2013	-2,9800
1982	5,6928	1998	-1,0693	2014	6,0223
1983	-1,0931	1999	1,7317	2015	-4,0821
1984	-5,5789	2000	2,9412	2016	6,1439
1985	0,3562	2001	-	2017	0,8268
1986	-3,4196	2002	-0,0947	2018	-8,7241
1987	1,8097	2003	2,0497	2019	0,8324

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2019], we found that the minimum average absolute error 2,4093 is obtained applying Fourier Analysis for T=10 for the range of years: [1990,1999].

The optimal number of Fourier series terms is 8 being specified in the table 71.

Table 71.

a_0	0,2464						
a_1	0,0400	b_1	0,2429	a_2	0,1303	b_2	0,0345
a_3	-0,0244	b_3	0,7582	a_4	0,1991	b_4	0,3011
a_5	-0,1274	b_5	0,2248	a_6	0,0061	b_6	0,1673
a_7	-0,0106	b_7	0,0192	a_8	-0,0006	b_8	-0,0087
a_9	-	b_9	-	a_{10}	-	b_{10}	-
a_{11}	-	b_{11}	-	a_{12}	-	b_{12}	-
a_{13}	-	b_{13}	-	a_{14}	-	b_{14}	-
a_{15}	-	b_{15}	-	a_{16}	-	b_{16}	-
a_{17}	-	b_{17}	-	a_{18}	-	b_{18}	-
a_{19}	-	b_{19}	-	a_{20}	-	b_{20}	-

The recalculated values of the Birth Rate are:

Table 72. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	0,5805	1988	-0,0326	2004	1,7448
1973	-0,9248	1989	0,5138	2005	-0,2657
1974	0,4697	1990	0,5805	2006	-0,0326
1975	0,1465	1991	-0,9248	2007	0,5138
1976	-1,1234	1992	0,4697	2008	0,5805
1977	1,7448	1993	0,1465	2009	-0,9248
1978	-0,2657	1994	-1,1234	2010	0,4697
1979	-0,0326	1995	1,7448	2011	0,1465
1980	0,5138	1996	-0,2657	2012	-1,1234
1981	0,5805	1997	-0,0326	2013	1,7448
1982	-0,9248	1998	0,5138	2014	-0,2657
1983	0,4697	1999	0,5805	2015	-0,0326
1984	0,1465	2000	-0,9248	2016	0,5138
1985	-1,1234	2001	0,4697	2017	0,5805
1986	1,7448	2002	0,1465	2018	-0,9248
1987	-0,2657	2003	-1,1234	2019	0,4697

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2019

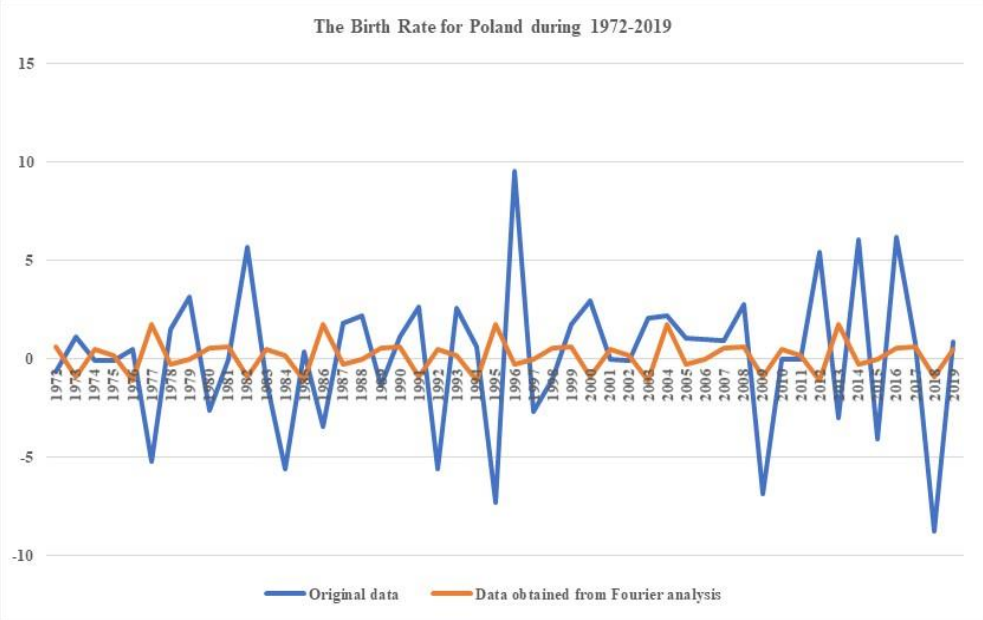

Figure 24.

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**8,0504**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2020	**10,697**
2021	**11,558**
2022	**12,489**
2023	**13,494**
2024	**14,580**

5.25. Portugal

Table 73. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2018

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	-4,7710	1988	1,5680	2004	-1,8855
1973	2,8194	1989	-2,4657	2005	3,7037
1974	-1,0099	1990	2,4312	2006	-
1975	3,0204	1991	0,8475	2007	0,8462
1976	-0,0103	1992	-	2008	5,0619
1977	-5,5101	1993	0,8398	2009	-7,1124
1978	-3,8770	1994	-3,5164	2010	7,1782
1979	3,2341	1995	2,5511	2011	-6,2944
1980	2,7333	1996	4,6386	2012	-3,4420
1981	-2,5287	1997	-0,9855	2013	0,5499
1982	3,6396	1998	-1,8182	2014	7,0588
1983	-3,3066	1999	-	2015	-
1984	3,2260	2000	0,8459	2016	-3,8585
1985	-7,7116	2001	-9,4692	2017	-1,2048
1986	6,0140	2002	7,7550	2018	-
1987	0,6959	2003	-2,7356	2019	-2,3670

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2018], we found that the minimum average absolute error 2,8097 is obtained applying Fourier Analysis for T=21 for the range of years: [1983,2003].

The optimal number of Fourier series terms is 11 being specified in the table 74.

Table 74.

a_0	0,0310						
a_1	0,0577	b_1	-1,5327	a_2	-0,0009	b_2	0,8027
a_3	0,0001	b_3	-0,4253	a_4	-0,1187	b_4	0,3529
a_5	0,0243	b_5	-0,1424	a_6	-0,1359	b_6	0,1665
a_7	0,0011	b_7	-0,2853	a_8	0,0048	b_8	0,2279
a_9	-0,1412	b_9	-0,1984	a_{10}	0,1429	b_{10}	0,1627
a_{11}	-0,0945	b_{11}	-0,1361	a_{12}	-	b_{12}	-
a_{13}	-	b_{13}	-	a_{14}	-	b_{14}	-
a_{15}	-	b_{15}	-	a_{16}	-	b_{16}	-
a_{17}	-	b_{17}	-	a_{18}	-	b_{18}	-
a_{19}	-	b_{19}	-	a_{20}	-	b_{20}	-

The recalculated values of the Birth Rate are:

Table 75. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2018

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	0,2622	1988	0,9224	2004	1,9048
1973	-0,2448	1989	1,2005	2005	2,1879
1974	0,0413	1990	0,7265	2006	1,7295
1975	-0,1443	1991	0,5382	2007	1,5850
1976	-0,6590	1992	0,2622	2008	0,9224
1977	-1,1108	1993	-0,2448	2009	1,2005
1978	-1,1313	1994	0,0413	2010	0,7265
1979	-1,3685	1995	-0,1443	2011	0,5382
1980	-2,3553	1996	-0,6590	2012	0,2622
1981	-1,1195	1997	-1,1108	2013	-0,2448
1982	-2,7150	1998	-1,1313	2014	0,0413
1983	0,0603	1999	-1,3685	2015	-0,1443
1984	1,9048	2000	-2,3553	2016	-0,6590
1985	2,1879	2001	-1,1195	2017	-1,1108
1986	1,7295	2002	-2,7150	2018	-1,1313
1987	1,5850	2003	0,0603	2019	-

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2018

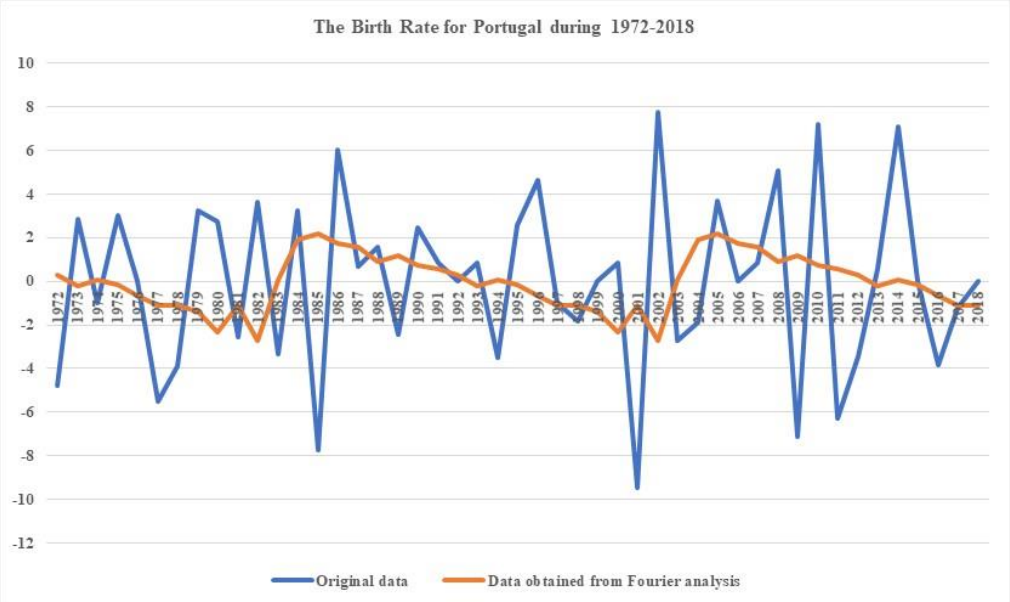

Figure 25

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**-1,4050**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2019	**8,282**
2020	**8,166**
2021	**8,051**
2022	**7,938**
2023	**7,826**

5.26. Romania

Table 76. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	**3,0274**	**1988**	**-2,4097**	**2004**	**2,0303**
1973	**0,8901**	**1989**	**-1,8327**	**2005**	**-0,0909**
1974	**14,7300**	**1990**	**-11,9697**	**2006**	**-2,9703**
1975	**-14,9868**	**1991**	**3,2353**	**2007**	**-**
1976	**2,4279**	**1992**	**6,7647**	**2008**	**5,8159**
1977	**1,5359**	**1993**	**1,4912**	**2009**	**-3,9285**
1978	**-3,0796**	**1994**	**2,5997**	**2010**	**-4,5956**
1979	**0,4588**	**1995**	**-3,6781**	**2011**	**-3,9493**
1980	**-1,1205**	**1996**	**2,6641**	**2012**	**10,7118**
1981	**-2,3298**	**1997**	**4,8643**	**2013**	**-9,0928**
1982	**-4,4444**	**1998**	**-2,9412**	**2014**	**12,3830**
1983	**3,4641**	**1999**	**-**	**2015**	**-4,3830**
1984	**14,9275**	**2000**	**0,9524**	**2016**	**-0,0392**
1985	**-6,4561**	**2001**	**-**	**2017**	**1,8854**
1986	**2,4949**	**2002**	**0,8462**	**2018**	**-7,5499**
1987	**-3,2183**	**2003**	**4,0309**	**2019**	**-3,9886**

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2019], we found that the minimum average absolute error 3,7950 is obtained applying Fourier Analysis for T=11 for the range of years: [1973,1983].

The optimal number of Fourier series terms is 8 being specified in the table 77.

Table 77.

a_0	-1,2983						
a_1	-0,2839	b_1	0,1149	a_2	0,4042	b_2	-0,4700
a_3	-0,4722	b_3	0,7083	a_4	0,5653	b_4	-0,2765
a_5	-0,3017	b_5	0,0126	a_6	0,2512	b_6	0,1054
a_7	-0,0867	b_7	-0,1173	a_8	0,0253	b_8	0,0196
a_9	-	b_9	-	a_{10}	-	b_{10}	-
a_{11}	-	b_{11}	-	a_{12}	-	b_{12}	-
a_{13}	-	b_{13}	-	a_{14}	-	b_{14}	-
a_{15}	-	b_{15}	-	a_{16}	-	b_{16}	-
a_{17}	-	b_{17}	-	a_{18}	-	b_{18}	-
a_{19}	-	b_{19}	-	a_{20}	-	b_{20}	-

The recalculated values of the Birth Rate are:

Table 78. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	0,1211	1988	-0,5475	2004	-2,9645
1973	1,7413	1989	-0,7697	2005	-0,3557
1974	-2,9645	1990	-0,9819	2006	-0,3811
1975	-0,3557	1991	-1,2584	2007	-1,0948
1976	-0,3811	1992	0,1211	2008	-0,5475
1977	-1,0948	1993	1,7413	2009	-0,7697
1978	-0,5475	1994	-2,9645	2010	-0,9819
1979	-0,7697	1995	-0,3557	2011	-1,2584
1980	-0,9819	1996	-0,3811	2012	0,1211
1981	-1,2584	1997	-1,0948	2013	1,7413
1982	0,1211	1998	-0,5475	2014	-2,9645
1983	1,7413	1999	-0,7697	2015	-0,3557
1984	-2,9645	2000	-0,9819	2016	-0,3811
1985	-0,3557	2001	-1,2584	2017	-1,0948
1986	-0,3811	2002	0,1211	2018	-0,5475
1987	-1,0948	2003	1,7413	2019	-0,7697

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2019

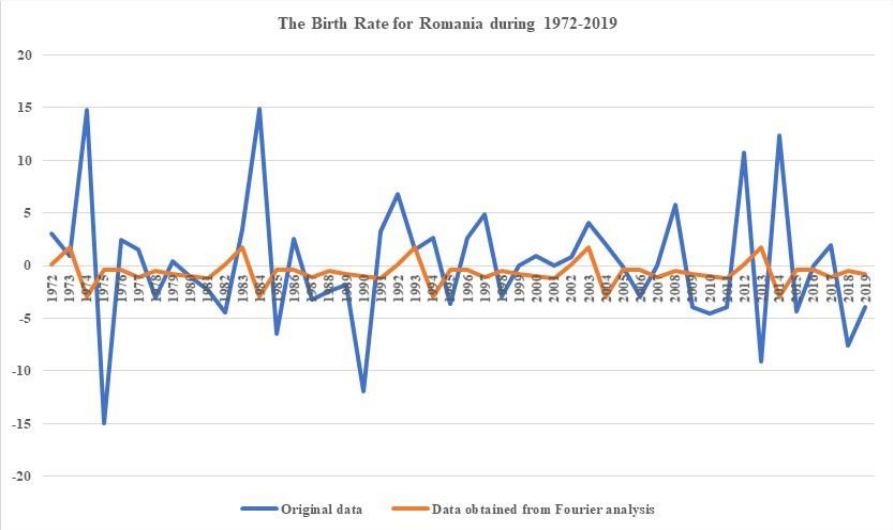

Figure 26.

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**-37,3247**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2020	**6,017**
2021	**3,771**
2022	**2,364**
2023	**1,481**
2024	**0,928**

5.27. Slovak Republic

Table 79. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	2,6979	1988	2,9245	2004	3,1141
1973	-0,2331	1989	-3,1603	2005	-3,1667
1974	-0,7120	1990	3,7446	2006	-1,9901
1975	-4,9615	1991	-1,3289	2007	1,9901
1976	1,9324	1992	-2,7429	2008	4,9406
1977	-1,9324	1993	2,6020	2009	0,6015
1978	0,4761	1994	-8,0172	2010	-8,2965
1979	-0,4902	1995	2,8868	2011	2,6473
1980	-4,9357	1996	4,6494	2012	-9,7425
1981	3,2935	1997	0,8230	2013	6,9079
1982	1,0049	1998	-0,9416	2014	2,9318
1983	0,5200	1999	-0,0764	2015	-0,0097
1984	-1,1170	2000	0,8806	2016	1,9322
1985	1,0800	2001	-4,9396	2017	-1,9692
1986	-2,8701	2002	6,8627	2018	-1,8780
1987	-0,1667	2003	-	2019	-0,0088

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2019], we found that the minimum average absolute error 2,2772 is obtained applying Fourier Analysis for T=8 for the range of years: [1998,2005].

The optimal number of Fourier series terms is 8 being specified in the table 80.

Table 80.

a_0	0,9816						
a_1	0,5498	b_1	-0,3624	a_2	0,2926	b_2	0,3757
a_3	0,0614	b_3	-0,7470	a_4	-0,0345	b_4	-0,1537
a_5	-0,0468	b_5	-0,1105	a_6	-0,0153	b_6	0,1084
a_7	-	b_7	-0,0870	a_8	-0,0086	b_8	0,0723
a_9	-	b_9	-	a_{10}	-	b_{10}	-
a_{11}	-	b_{11}	-	a_{12}	-	b_{12}	-
a_{13}	-	b_{13}	-	a_{14}	-	b_{14}	-
a_{15}	-	b_{15}	-	a_{16}	-	b_{16}	-
a_{17}	-	b_{17}	-	a_{18}	-	b_{18}	-
a_{19}	-	b_{19}	-	a_{20}	-	b_{20}	-

The recalculated values of the Birth Rate are:

Table 81. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	-0,2483	1988	0,4282	2004	-0,7604
1973	2,0530	1989	0,7541	2005	0,1606
1974	0,4282	1990	-0,7604	2006	1,0485
1975	0,7541	1991	0,1606	2007	-0,2483
1976	-0,7604	1992	1,0485	2008	2,0530
1977	0,1606	1993	-0,2483	2009	0,4282
1978	1,0485	1994	2,0530	2010	0,7541
1979	-0,2483	1995	0,4282	2011	-0,7604
1980	2,0530	1996	0,7541	2012	0,1606
1981	0,4282	1997	-0,7604	2013	1,0485
1982	0,7541	1998	0,1606	2014	-0,2483
1983	-0,7604	1999	1,0485	2015	2,0530
1984	0,1606	2000	-0,2483	2016	0,4282
1985	1,0485	2001	2,0530	2017	0,7541
1986	-0,2483	2002	0,4282	2018	-0,7604
1987	2,0530	2003	0,7541	2019	0,1606

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2019

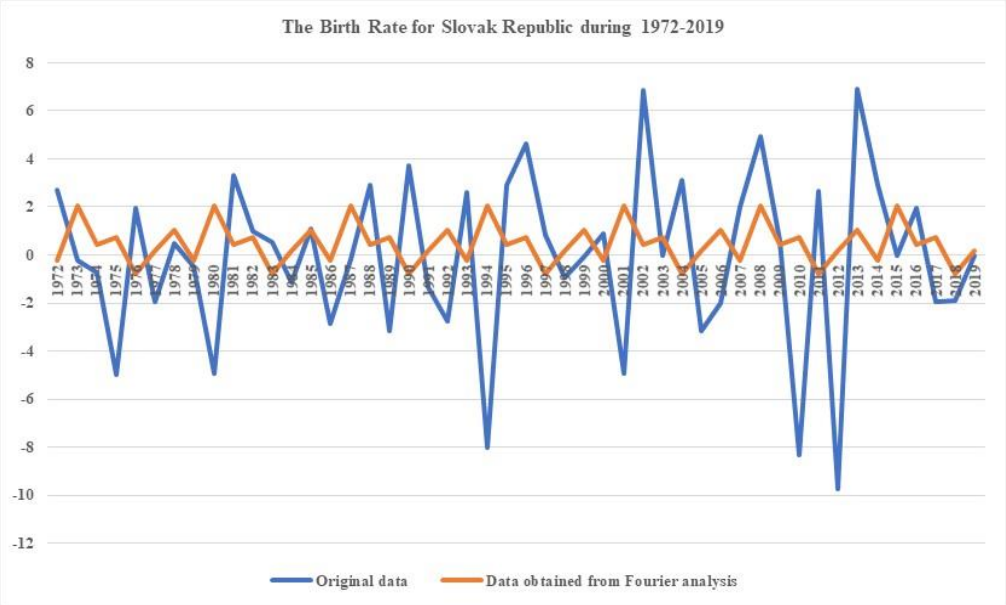

Figure 27.

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**25,2486**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2020	**13,151**
2021	**16,472**
2022	**20,630**
2023	**25,839**
2024	**32,363**

5.28. Slovenia

Table 82. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	**-1,9022**	**1988**	**-1,5564**	**2004**	**4,5847**
1973	**1,2158**	**1989**	**-4,8173**	**2005**	**-2,3372**
1974	**-5,4221**	**1990**	**2,8694**	**2006**	**2,1856**
1975	**6,6984**	**1991**	**0,7021**	**2007**	**0,9586**
1976	**-2,5032**	**1992**	**-3,8360**	**2008**	**5,9488**
1977	**-3,5964**	**1993**	**6,4074**	**2009**	**-11,1300**
1978	**3,6113**	**1994**	**-0,0101**	**2010**	**2,7951**
1979	**-0,6173**	**1995**	**-2,0511**	**2011**	**-3,7041**
1980	**-**	**1996**	**2,0086**	**2012**	**1,8349**
1981	**1,1332**	**1997**	**-2,1389**	**2013**	**-**
1982	**1,2406**	**1998**	**2,0926**	**2014**	**5,6533**
1983	**-5,3153**	**1999**	**-1,1233**	**2015**	**-3,8930**
1984	**3,0764**	**2000**	**5,6313**	**2016**	**1,9126**
1985	**2,0755**	**2001**	**-6,7058**	**2017**	**-0,0101**
1986	**-1,5145**	**2002**	**3,2967**	**2018**	**-3,0715**
1987	**2,2159**	**2003**	**-**	**2019**	**3,0178**

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2019], we found that the minimum average absolute error 2,6212 is obtained applying Fourier Analysis for T=7 for the range of years: [1994,2000].

The optimal number of Fourier series terms is 8 being specified in the table 83.

Table 83

a_0	1,3946						
a_1	0,0886	b_1	-0,7827	a_2	-0,2915	b_2	0,5062
a_3	0,6706	b_3	-0,4939	a_4	-0,0729	b_4	0,4291
a_5	0,0035	b_5	-0,3243	a_6	-	b_6	0,2469
a_7	0,0018	b_7	-0,1974	a_8	-0,0182	b_8	0,1705
a_9	-	b_9	-	a_{10}	-	b_{10}	-
a_{11}	-	b_{11}	-	a_{12}	-	b_{12}	-
a_{13}	-	b_{13}	-	a_{14}	-	b_{14}	-
a_{15}	-	b_{15}	-	a_{16}	-	b_{16}	-
a_{17}	-	b_{17}	-	a_{18}	-	b_{18}	-
a_{19}	-	b_{19}	-	a_{20}	-	b_{20}	-

The recalculated values of the Birth Rate are:

Table 84. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	0,6184	1988	-0,4498	2004	-0,0885
1973	1,0793	1989	2,2946	2005	0,7297
1974	-0,0885	1990	0,6184	2006	-0,4498
1975	0,7297	1991	1,0793	2007	2,2946
1976	-0,4498	1992	-0,0885	2008	0,6184
1977	2,2946	1993	0,7297	2009	1,0793
1978	0,6184	1994	-0,4498	2010	-0,0885
1979	1,0793	1995	2,2946	2011	0,7297
1980	-0,0885	1996	0,6184	2012	-0,4498
1981	0,7297	1997	1,0793	2013	2,2946
1982	-0,4498	1998	-0,0885	2014	0,6184
1983	2,2946	1999	0,7297	2015	1,0793
1984	0,6184	2000	-0,4498	2016	-0,0885
1985	1,0793	2001	2,2946	2017	0,7297
1986	-0,0885	2002	0,6184	2018	-0,4498
1987	0,7297	2003	1,0793	2019	2,2946

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2019

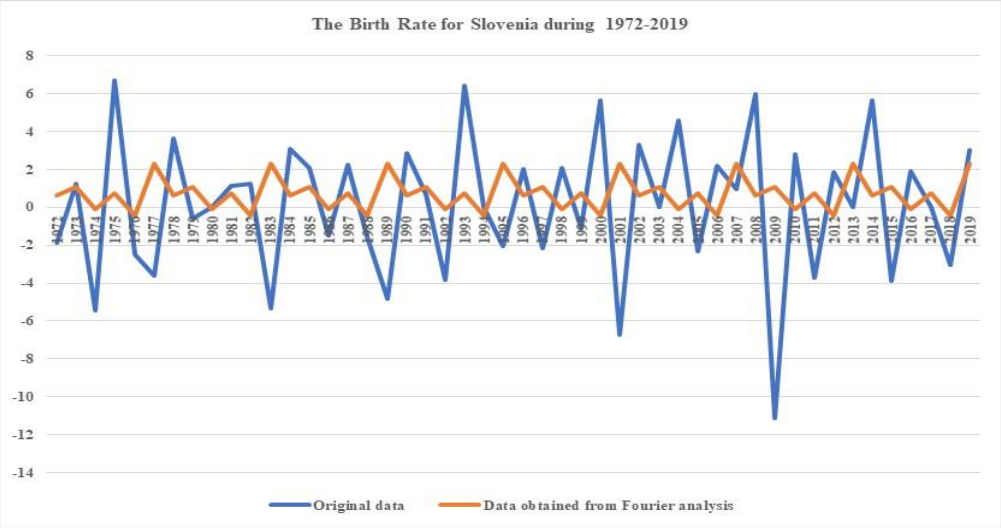

Figure 28

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:35,9853, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2020	**12,647**
2021	**17,198**
2022	**23,386**
2023	**31,802**
2024	**43,246**

5.29. Sweden

Table 85. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	-5,0474	1988	3,9410	2004	-2,8374
1973	-0,0462	1989	-3,3925	2005	-0,9009
1974	2,1739	1990	2,8319	2006	-
1975	-	1991	-6,5291	2007	-4,4643
1976	1,9048	1992	-0,6992	2008	-
1977	1,4286	1993	-3,5407	2009	-0,8691
1978	0,7471	1994	-0,2556	2010	1,6597
1979	5,2411	1995	-3,4086	2011	-6,5650
1980	-1,7928	1996	0,9015	2012	4,9125
1981	-4,2809	1997	2,1367	2013	-1,6878
1982	1,6489	1998	4,5752	2014	1,6878
1983	0,8690	1999	-0,0097	2015	-2,5282
1984	3,6282	2000	2,9901	2016	2,5354
1985	1,6975	2001	-1,0196	2017	-3,3971
1986	-1,0350	2002	2,9031	2018	1,6728
1987	-0,9308	2003	-0,1452	2019	-1,7620

Source: https://data.worldbank.org/ and own calculations

By analysing the data set (k,ρ_k), corresponding to the period [1972,2019], we found that the minimum average absolute error 1,9127 is obtained applying Fourier Analysis for T=5 for the range of years: [2010,2014].

The optimal number of Fourier series terms is 8 being specified in the table 86.

Table 86

a_0	**0,3659**						
a_1	**0,4635**	b_1	**0,4350**	a_2	**-1,0403**	b_2	**-0,6425**
a_3	**0,0515**	b_3	**0,5228**	a_4	**-**	b_4	**-0,3212**
a_5	**0,0185**	b_5	**0,2230**	a_6	**-0,1156**	b_6	**-0,2142**
a_7	**0,0095**	b_7	**0,2009**	a_8	**-**	b_8	**-0,1606**
a_9	**-**	b_9	**-**	a_{10}	**-**	b_{10}	**-**
a_{11}	**-**	b_{11}	**-**	a_{12}	**-**	b_{12}	**-**
a_{13}	**-**	b_{13}	**-**	a_{14}	**-**	b_{14}	**-**
a_{15}	**-**	b_{15}	**-**	a_{16}	**-**	b_{16}	**-**
a_{17}	**-**	b_{17}	**-**	a_{18}	**-**	b_{18}	**-**
a_{19}	**-**	b_{19}	**-**	a_{20}	**-**	b_{20}	**-**

The recalculated values of the Birth Rate are:

Table 87. The evolution of the accelerations (percents) of the Birth Rate during the period 1972-2019

Year	Acceleration	Year	Acceleration	Year	Acceleration
1972	**-0,4300**	**1988**	**-0,4300**	**2004**	**-0,4300**
1973	**1,2732**	**1989**	**1,2732**	**2005**	**1,2732**
1974	**-1,5159**	**1990**	**-1,5159**	**2006**	**-1,5159**
1975	**1,4046**	**1991**	**1,4046**	**2007**	**1,4046**
1976	**-0,4300**	**1992**	**-0,4300**	**2008**	**-0,4300**
1977	**1,2732**	**1993**	**1,2732**	**2009**	**1,2732**
1978	**-1,5159**	**1994**	**-1,5159**	**2010**	**-1,5159**
1979	**1,4046**	**1995**	**1,4046**	**2011**	**1,4046**
1980	**-0,4300**	**1996**	**-0,4300**	**2012**	**-0,4300**
1981	**1,2732**	**1997**	**1,2732**	**2013**	**1,2732**
1982	**-1,5159**	**1998**	**-1,5159**	**2014**	**-1,5159**
1983	**1,4046**	**1999**	**1,4046**	**2015**	**1,4046**
1984	**-0,4300**	**2000**	**-0,4300**	**2016**	**-0,4300**
1985	**1,2732**	**2001**	**1,2732**	**2017**	**1,2732**
1986	**-1,5159**	**2002**	**-1,5159**	**2018**	**-1,5159**
1987	**1,4046**	**2003**	**1,4046**	**2019**	**1,4046**

The comparative graphs of the evolution of the Birth Rate accelerations and their recomputing after the Fourier regression is:

The Fourier analysis for the period 1972-2019

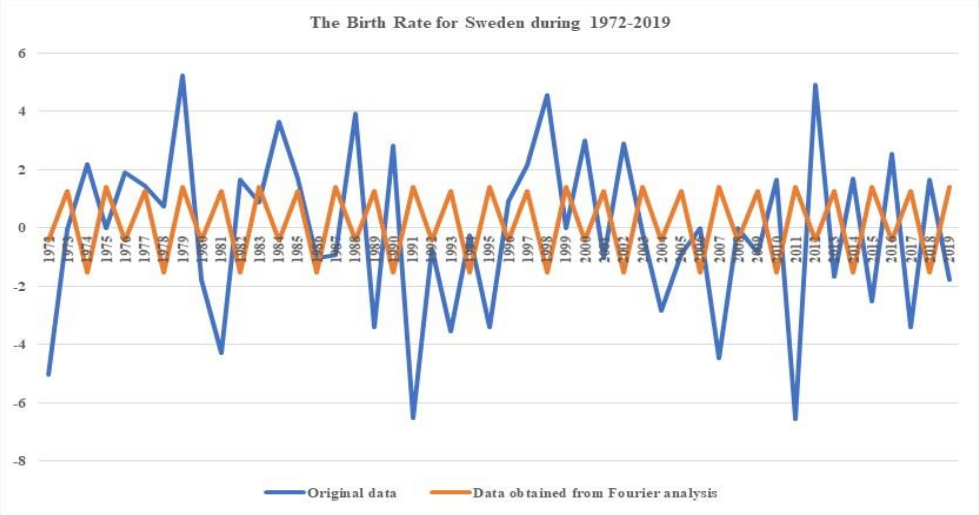

Figure 29.

Based on this analysis, and using the growth rate (in percents) corresponding to 2019:**11,7025**, the forecast of the Birth Rate for the next 5 years is:

Year	Birth Rate
2020	**12,399**
2021	**13,850**
2022	**15,471**
2023	**17,281**
2024	**19,304**

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Ioan C.A., Ioan G. (2011), The Analysis of the Evolution of the Gross Domestic Product by Means of Fourier Development, *Acta Universitatis Danubius, Oeconomica*, no.4, volume 7, pp. 146-161.

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