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The Asymmetric Effects of Interest Rates on Private Investment in South Africa

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Abstract: Conventional economic theory states that higher interest rates reduce investment levels because interest rates represent the cost of borrowing, thereby implying a linear inverse relationship. However, the relationship between interest rates and investment is beyond a linear one, which has been previously researched. Thus, the novelty of this study is to investigate the differences between the effects of positive and negative shocks in interest rates on private investment, by examining the asymmetric nature of the relationship between these variables in South Africa. We utilize annual time series data from 1971 to 2019 while employing the recent nonlinear autoregressive distributed lag (NARDL) technique. Our study finds that interest rates and private investment exhibit short-run and long-run asymmetric relationships, with private investment responding differently to negative and positive shocks in interest rates. A key conclusion from the study findings is that empirical evidence based on linear analyses of the relationship between interest rates and private investment is insufficient to warrant dependable macroeconomic forecasts. This may lead to misguided policy interventions and management, especially by monetary policy custodians, which ultimately constrains efforts toward sustained economic growth and development.

Keywords: Asymmetry; Investment; NARDL; South Africa

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1. Introduction

Ever since the adoption of financial sector reform policies by numerous economies globally, their effects on economic growth remain contentious among economists. One of the sources of contention is the role of interest rates with investment being one of the key transmission channels through which financial sector reforms usually affect the economy. Its relationship with interest rates is of keen interest to policymakers and has attracted considerable scholarly attention over the years. This is because the interest rate is one of the fundamental determinants of private investment, which is a crucial driver of sustained economic growth. Economic growth is the ultimate goal of monetary policy and for this reason, interest rate is an essential component of monetary policy formulation and management. How changes in interest rates affect investment, especially in the private sector, continues to be of interest to economists. Literature on this topical issue continues to grow with substantial empirical evidence consistently pointing to a significant relationship between interest rates and private investment.

Empirically, many studies have emphasized the centrality of interest rates as a transmission channel of monetary policy into investment mainly by the private sector (Dang et al, 2020, p. 2). Conventional economic theory has it that higher interest rates reduce investment levels because interest rates represent the cost of capital (Mehrara & Rezazadeh, 2011). Once the level of interest rate is high, it requires investment projects to post higher rates of return for them to be profitable. Therefore, *ceteris paribus*, the relationship between investment and the interest rate is inverse, which implies that a fall in interest rate is favorable for investment (Keynes, 1978). Numerous contemporary empirical studies, for instance, Iddrisu and Alagidede (2020), Ngoma, Bonga and Nyoni (2019), George-Anokwuru (2017), and Ndikumana (2008), among others, provide substantial evidence of the negative effects of interest rates on private investment.

Nonetheless, several other empirical studies such as Dang, Pham and Tran (2020), Li and Khurshid (2015), and Obamuyi and Demehin (2012), present divergent evidence indicating that changes in interest rates positively affect private investment. This empirical evidence, which is in support of a positive relationship between investment and interest rate, mirrors the financial liberalisation hypothesis predictions of McKinnon and Shaw (1973). Besides, other studies found no significant relationship between interest rate and investment, for instance, the analysis of the South African economy by Kumo (2006).

Therefore, it is apparent that there is still a lack of empirical consensus regarding the nature of interest rates on private investment, which leaves gaps for further empirical inquiry. The most important evidence and of more cruciality is the scientific novelty of the current study investigating the non-linear relationship between interest rates and private investment in South Africa. Previously available evidence assumed that investment and interest rates are linearly related. This probably accounts for the vast use of linear models in analyses of the investment-interest rate nexus. Thus, the current study adopts non-linear models to examine the asymmetric nature of interest rates on private investment. By so doing, our study answers the key research question of whether the effects of positive and negative shocks in interest rates on private investment differ.

Furthermore, the apparent inconclusiveness is coupled with the fact that many of the available studies have either tended to focus on Western and Asian economies (like Dang et al (2020), Bano (2018), Li et al (2015), and Ang (2009), among others), or they have provided evidence based on analyses of multiple countries (see for example Moyo and Le Roux (2018)). In the latter case, findings usually ignore the effect of structural and characteristic differences among economies under study, which may undermine the reliability of such findings as far as the specific-country application is concerned. This justifies furtherance of specific-country analyses, but more so focusing on sub-Saharan Africa (SSA) – a region that is by far less focused on. We aim to provide specific-country evidence from the South African economy to sidestep the shortcomings of multi-country analyses highlighted earlier. Raising private sector investment is more critical than ever before for the South African economy, but also for the entire sub-Saharan Africa region and generally for developing economies.

Despite the aforementioned gaps, monetary policy formulation and management, especially in SSA, has largely been informed and guided by available, although scanty research evidence (Chetty, 2004). Indeed, in a bid to achieve monetary policy goals in a country like South Africa (and in a considerable number of economies in SSA), instruments like the central bank rate (CBR) have been adopted as reference rates for pricing monetary operations, including interbank rates and repo rates. The CBR has always been raised or otherwise with a view of influencing the level of interest rate to, among other aims, stimulate private investment through its effect on credit to the private sector, as many empirical studies have tended to predict.

However, empirical evidence biased towards linear analyses of the interest rate-private investment nexus may be insufficiently rich to permit dependable macroeconomic policy forecasts. Unfortunately, and as alluded to earlier, many of the existing empirical studies, especially in SSA, have not paid attention to the probable nonlinearity or asymmetrical nature of the relationship between interest rates and private investment. Empirical investigations that have focused on SSA and recognised the asymmetrical nature of the relationship in question are still very scanty. The few that have endeavoured to do so, like Mehrara and Rezazadeh (2011), provide findings based on analyses of multiple developing countries, which negates the effects of structural characteristic differences among economies as earlier noted.

There are important empirical and policy reasons why private investment (and some of its key determinants) ought to be at the center of economic policy debate. Empirical studies have singled out investment as the most robust determinant of economic growth (Ndikumana, 2008). From the policy context, and specifically, regarding South Africa, it will take active participation of the private sector to achieve a growth rate necessary to lift the over 30 million South Africans estimated to be living in poverty at the upper national poverty line of ZAR992 and the over 13 million experiencing food poverty[1]. Yet recent developments in the South African economy indicate that private sector investment has been declining[2] and several manufacturing sub-sectors are operating below production capacity.

Moreover, because the extent to which the private sector contributes to growth and poverty reduction significantly depends on its ability to gain access to critical financial services (Misati & Nyamongo, 2011:140), an empirical inquiry into the nature of the relationship between the price of financial services (interest rate) and private investment in South Africa is timely. Moreover, private investment is more sensitive to changes in interest rates than public investment. Yet persistent improvement in private sector investment also elicits increased inward foreign direct investment, whose role in the growth of the South African economy cannot be overemphasized. Upon this background, this study provides empirical evidence upon which dependable macroeconomic forecasts and management can be made. Ultimately, study findings will guide monetary policymakers in the country immensely as they attempt to resuscitate the domestic economy amid and in the post-COVID-19[3] era.

This study differs from earlier studies, not only in terms of methods employed but also in a time span as we analyse data for a period of 48 years as well as the country-specific, South Africa. To the best of our knowledge, no study has investigated the asymmetric nature of the interest rate-investment nexus in South Africa using the nonlinear autoregressive distributed lag (NARDL). The most recent study observing the linearity of this relationship was by Ngoma, Bonga and Nyoni (2019).

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[1]https://databank.worldbank.org/data/download/poverty/33EF03BB-9722-4AE2-ABC7-AA2972D68AFE/Global_POVEQ_ZAF.pdf.

[2] https://www.idc.co.za/wp-content/uploads/2020/03/IDC-RI-publication-Economic-Overview-External-release-February-2020.pdf.

[3]According to the World Bank poverty and equity brief on SSA, of April 2020, "the number of positive tests for corona virus is rising in South Africa. The minister of health warned that between 60 and 70% of South Africans are expected to become infected, with between 10 and 20% of those developing into severe cases, likely requiring hospitalisation. In addition to the looming health consequences, the epidemic is likely to have devastating economic consequences on the country and on these already impoverished communities. It is estimated that extreme poverty will increase in South Africa by 9% in 2020. Evidence from previous crises, of even far smaller scales, suggest that these negative effects could last across generations and further exacerbate already South Africa's high inequality."

The rest of this paper is organized as follows: section 2 describes the theoretical underpinnings of the study while section 3 presents the empirical literature. Section 4 contains data and methods used while section 5 presents the empirical results and discussion. Section 6 covers the conclusion from the findings, recommendations for policy, and areas for further research.

2. Theoretical Foundation

In this study, the theoretical investigations of the asymmetrical effects of interest rates on private investment are guided by two antagonistic theoretical formulations. These are the classical theory, associated with economists such as Ricardo, JS Mill, Marshall, and Pigou, and the financial liberalization theory of private sector investment, associated with McKinnon and Shaw (1973). The classicals have it that interest rates are inversely related to private investment. The hypothesis in classical theory is that the rate of interest represents the cost of credit for investment. Therefore, private investment is expected to rise in response to falling interest rates.

The second theoretical discourse is that of financial liberalization, which takes the opposite view by asserting that higher interest rates have a net positive effect on investment through saving – an effect the proponents refer to as the "conduit effect". Increases in interest rates are expected to elicit more savings, which ultimately stimulate investment. A reconciliatory attempt to accommodate these two opposing theoretical discourses is what justifies an empirical investigation into the seemingly asymmetrical relationship between interest rate and investment. A policy intervention cognizant of this fact is thought to be sufficiently rich to guide macroeconomic policy management and forecasting.

According to Jorgenson's neoclassical theory of investment, a decline in the interest rate, which is the relative rental price of capital, leads to increased investment through the increase in the demand for capital services (Jorgenson, 1963). Thus, a negative relationship is assumed between interest rates and investment. Investment can be theoretically specified as a flow of net receipts by a firm at time $\boldsymbol{t}$, (Lau and Liew, 2019) which can be specified as follows:

$$\boldsymbol{R(t) = p_t Q_t - w_t L_t - q_t I_t} \quad (2.1)$$

Where $R(t),\ Q_t,\ L_t$, and I_t denote net receipts, firm's output, labor, and net capital purchases respectively, and $\boldsymbol{p_t},\ \boldsymbol{w_t}, \boldsymbol{q_t}$ are the prices for output, labor and capital respectively. The aim of the firm is to maximize its present value, which is integral to discounted net receipts (Jorgenson, 1967). The maximized integral of discounted net receipts is formally specified as:

$$\text{Max } \boldsymbol{W_t = \int_0^{\infty} e^{-\rho_t}(p_t Q_t - w_t L_t - q_t I_t) d_t} \quad (2.2)$$

Where, W_t is the discounted rate of net receipts.

The first step of Jorgenson's neoclassical theory of investment is the optimization problem of the firm and maximizing profits in each period will yield an optimal capital stock for the firm. If we assume certainty, we can reduce and express equation (2.2) by a one-period static profit maximization function (Ang, 2009b as quoted by Lau and Liew, 2019).

$$\boldsymbol{\pi_t = p_t Q_t - w_t L_t - r_t K_t} \quad (2.3)$$

In (2.3) $\boldsymbol{r}$ is the cost of capital. Invoking the putty-putty technology assumption which ensures that substitutability between capital and labor is complete, we can express the production function for the firm by the conventional Cobb-Douglas function in (2.4)

$$\boldsymbol{Q_t = A K_t^{\alpha} L_t^{1-\alpha}} \quad (2.4)$$

Assuming conditions of perfectly competitive markets, profit maximization would require that:

$$\boldsymbol{\frac{\partial \pi}{\partial K_t} = \alpha p_t A K_t^{\alpha-1} L_t^{1-\alpha} - r_t = 0} \quad (2.5)$$

Re-writing (2.5) yields;

$$\boldsymbol{\frac{\partial \pi}{\partial K_t} = \alpha \frac{p_t A K_t^{\alpha} L_t^{1-\alpha}}{K_t} - r_t = 0} \quad (2.6)$$

$$\boldsymbol{\alpha \frac{p_t A K_t^{\alpha} L_t^{1-\alpha}}{K_t} = r_t} \quad (2.7)$$

$\boldsymbol{p_t A K_t^{\alpha} L_t^{1-\alpha}}$ is the nominal output denoted as $\boldsymbol{Y_t}$, hence from (2.7),

$$\boldsymbol{r_t = \alpha \frac{Y_t}{K_t}} \quad (2.8)$$

Where $\boldsymbol{\alpha}$ is the coefficient of nominal output ($\boldsymbol{Y_t}$) to capital ($\boldsymbol{K_t}$) ratio. The optimal level of capital will be;

$$\boldsymbol{K_t^* = \alpha \frac{Y_t}{r_t}} \quad (2.9)$$

Expression (2.9) shows that the desired/optimal capital stock depends on the level of output and the user cost of capital as predicted by Jorgenson (1963). Lags in delivery and decision making create a gap between current and desired capital stock, giving rise to a gross investment equation in the form of;

$$\boldsymbol{I_t = \alpha \sum_{i=0}^{n} \phi_i \Delta \left(\frac{Y_{t-i}}{r_{t-i}}\right) + \psi K_{t-1}} \quad (2.10)$$

And net investment is the sum of the distributed lag on the past changes in desired capital stock,

$$\sum_{i=0}^{n} \phi_i \Delta K_{t-i}^{*} = \alpha \Sigma_{i=0}^{n} \phi_i \Delta \left(\frac{Y_{t-i}}{r_{t-i}} \right) \quad (2.11)$$

And replacement investment is the capital stock lagged by one period that depreciated at a constant rate, $\psi K_{(t-1)}$ with ψ as the depreciation rate of capital stock (Ang, 2009b). It is clear therefore, that the user cost of capital is a key determinant of a firm's optimal capital stock and hence investment. The user cost of capital has two components i.e., interest cost and the depreciation cost.

3. Empirical Review

The effect of interest rate on private investment has drawn empirical attention from many researchers. Literature on this now topical issue continues to grow, with substantial evidence consistently suggesting that interest rate significantly affects private investment, although few studies have recognized the possibility of such effects being asymmetrical.

Ang (2009) employs ordinary least squares (OLS) and auto regressive distributed lag (ARDL) techniques to examine the effect of financial sector policies on private investment in Malaysia and India. Study findings reveal positive effects of interest rates on private investment especially in Malaysia. Similar results are echoed in Hailu and Debele (2015) and Agu (2015) for Ethiopia and Nigeria respectively. Short-run positive effects of real interest rate and exchange rate on investment were revealed in Ethiopia. And in Nigeria interest rate of bank deposit was found to be positively correlated with investment while effects of real output growth and public investment were found to be negative.

Dang et al. (2020) utilises provincial data on a panel of 63 first tier provinces in Vietnam, covering a 9-year period from 2009 to 2017, to examine the relationship between monetary policy and private investment. Their findings demonstrate that interest rate has a significant positive impact on private investment. Findings further reveal positive and significant effects of credit to private sector on investment levels, while exchange rate was found to have no effect at all (Dang et al, 2020, p. 10). Some earlier studies conducted in SSA, for instance, Oshikoya (1992) on Kenya, Dailami and Walton (1992) on Zimbabwe, and Seck and El Nil (1993) on 21 African countries, share similar findings.

In view of the foregone studies, it is worth remarking that interest rate is an integral component of capital cost, which necessitates that the investment guarantees significantly higher rates of return. Investment will only improve if real interest rates rise, but not to the level of the real return on capital (Fowowe, 2013, p. 15). This sort of uncertain guarantee may instead discourage investment. Moreover, higher interest rates may actually encourage more savings and insignificant investments as economic agents prefer to earn more from savings in banks than gamble on investments that must guarantee higher rates of return. Additionally, changes in savings are not always channeled into investments in equal proportions (Fowowe, 2013, p. 10). Under such circumstances, the canonical positive relationship between investment and interest rate may collapse. Indeed, numerous studies have found negative effects of interest rate on private investment.

Ngoma, Bonga and Nyoni (2019) analyse macroeconomic determinants of private investment in SSA. Utilising panel data, covering the period 2000 – 2017 for 35 countries, their study employs pooled regression methods, fixed effects models and panel corrected standard error techniques. Study findings indicated that real interest rates and inflation negatively affected investment by a magnitude of 0.008 and 0.044 units for every unit increase in interest rates and inflation respectively. Additionally, results also reveal negative effects of public investment on private investment just like in Maganga and Edriss (2012).

In a more country-specific study, Ndikumana (2008) examines the key determinants of private investment in South Africa using aggregated panel data on 9 major industries and 27 sub-sectors for the period 1970 to 2001. Employing a two-step generalised method of moment (GMM) estimation procedure, Ndikumana finds evidence of negative and significant effects of both real and nominal interest rate on investment. Specifically, findings indicate that a 20% reduction in real interest rate tends to generate a 3% increase in the level of investment (Ndikumana, 2008:883). Ndikumana further finds positive effects of growth in gross domestic product (GDP) on private investment.

Furthermore, using panel data covering the period 1991 to 2004, from 18 countries in Africa, Misati and Nyamongo (2011) examine the relationship between financial sector development and private investment in SSA. A key finding from their study is that there is a negative relationship between interest rate and private investment. Findings in Ndikumana (2008) and Misati and Nyamongo (2011) are echoed in earlier studies by Ndikumana (2000), Akpokodje (2000), and Greene and Villanueva (1991), among others.

Substantial empirical evidence consistently and linearly explains the relationship between investment and interest rate as either positive or negative. Although still very scanty, especially in SSA, some studies have recognised the nonlinearity of interest rate effects on private investment. Bano (2018) investigates the dynamic relationship between real interest rate and investment for selected Pacific Island countries using a pooled mean group (PMG) - panel autoregressive distributed lag approach for the period 1980 to 2016. The study reveals a significant negative effect of real interest rate on investment in the long run, while positive effects are revealed in the short run across all countries in question, with Samoa being the only exception. The PMG results further show that economic variables such as growth rate, communication, foreign investment, aid and real exchange rate are investment creating in the long run, while savings are investment reducing – a result that contradicts the predictions of the financial liberalisation hypothesis.

In addition, Mehrara and Karsalari (2011) examine the nonlinearity of the relationship between private investment and real interest rates based on dynamic threshold panel models. This study provides evidence from developing countries, although with very few from SSA, for the period, 1970 to 2007. Controlling for bank credit, government expenditures, inflation and exchange rate, findings from nonlinear specifications in this study indicate that the interest rate has varying effects on private investment. Specifically, when interest rate levels are below 5%, a 1% increase tends to generate about a 0.05% increase in private investment, while the effect is negative when interest rates are above 5% (Mehrara & Karsalari, 2011, p. 38). These rather asymmetrical effects of interest rates on investment are fairly echoed in Olaniyi's (2019) study on Nigeria.

In conclusion, much of the empirical evidence re-affirms the twofold earlier suggestion of the neoclassical and financial liberalization theories. The downward-sloping relationship between investment demand and interest rates derived from mainly the neoclassical models of investment, or the upward-sloping relationship based on the financial liberalization hypotheses frequently guide economic policy, whenever such policy is aimed at stimulating private investment in SSA. The review also indicates that studies of the asymmetric nature of the effects of interest rates on private investment are scarce, especially in SSA – one of the gaps that this study attends to.

4. Data and Methodology

In order to incorporate a sound theoretical underpinning of the determinants of investment, we follow Ang (2009b) and Mehrara and Karsalari (2011) approaches while taking into account the unique structural features of South Africa. We derive our empirical specification using the neoclassical model of investment outlined above, augmented with certain important macroeconomic features of South Africa. This helps us to sidestep the likely effects of strict adherence to any narrow model of investment. This approach encompasses the consideration of other macroeconomic variables deemed important in influencing private investment as will be seen in the final estimable model.

4.1. Data Source and Empirical Model Specification

The study used annual data spanning over the period 1971 to 2019, making 48 observations, to estimate the asymmetric effect of interest rates on private investment in South Africa. Annual data was used because a very important variable, bank credit, is only available annually. We also employ linear and latest nonlinear econometric modeling techniques to investigate and confirm the asymmetrical effects of interest rates on private investment. Although linear regression analysis is a vital and primary tool for econometric analysis, there is significant evidence to show that nonlinear modeling is not only equally important, but it is more appropriate and empirical attention is increasingly turning to nonlinear models, especially in the macroeconomic analysis. The choice of variables was informed by theoretical underpinnings as well as empirical literature, focusing on the South African economy. This study closely follows the studies by Ang (2009b) and Mehrara and Karsalari (2011), while adopting the NARDL technique developed by Shin, Yu and Greenwood (2014).

The variables considered as the determinants of private investment are output growth, bank credits, government spending, real interest rates and inflation rate. Private investment, PRI_INV, the dependent variable is the private sector gross fixed capital formation as a percentage of gross domestic product. It was sourced from the World Development Indicators' (WDI) database. Based on the "accelerator effect", the change in gross domestic product or output, GDPG, is seen as one of the major determinants of investment; this is the GDP growth, and it is expected to have a direct relationship with private investment.

Bank credits is an important determinant of private investment in South Africa as a developing country because bank loans do not have a close substitute. This is the growth of domestic credit allocated to the private sector, CREDITG. It is expected to be positively related to private investment because a restriction in bank credits will cause a decline in private investment. Since South Africa is not a high savings country, most of the investment is done using credit extension. Government spending is the ratio of final consumption expenditure by government to GDP, GOVT_GDP. It can either have a positive or negative effect on private investment, depending on whether government investment spending is based on infrastructure or non-infrastructure. All these variables were obtained from the South African Reserve Bank (SARB) database.

The interest rate is the real interest rate (RIR), which is expected to be either inversely related or otherwise to private investment. When the interest rate increases, the opportunity cost of capital increases, thereby making it difficult for investors to obtain capital and private investment spending falls. This rate closely depicts the rate of borrowing, and it was obtained from the WDI database. Inflation rate, INF, has a negative effect on private investment, thus an increase in inflation rate will cause a decline in private investment. Inflation rate was sourced from the International Financial Statistics of the International Monetary Fund's (IMF/IFS) database.

The general model to be estimated is thus specified as follows:

$$PRI\_INV_t = f\,(RIR_t, CREDITG_t, GOVT\_GDP_t, INF_t, GDPG_t) \qquad (4.1)$$

$$\overset{-/+}{} \quad \overset{+}{} \quad \overset{-/+}{} \quad \overset{-}{} \quad \overset{+}{}$$

$$PRI\_INV_t = \alpha_1 RIR_t + \alpha_2 CREDITG_t + \alpha_3 GOVT\_GDP_t + \alpha_4 INF_t + \alpha_5 GDPG_t + \varepsilon_t \quad (4.2)$$

Where the variables are as earlier defined, α's are the coefficients and ε_t is the error term.

4.2. Estimation Techniques

The study first observes the nature of correlation between the variables, after which the stationarity of each variable is determined to avoid any spurious regression. As a result, the study tests all the variables for stationarity and the orders of integration of each variable was determined, using the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests of stationarity. This is followed by the test of cointegration, to determine the long-run relationship, using the popular ARDL of the Bounds test approach, developed by Pesaran, Shin and Smith (2001).

The ARDL model approach, which uses the OLS method of a conditional unrestricted error correction model (UECM), is used to estimate the coefficients and the effects of the long and short-run dynamics of the explanatory variables on private investment. Afterwards, the nonlinear ARDL (NARDL), which measures the asymmetric effects of the exogenous variables on the dependent variable, is estimated. In this case, the NARDL isolates the effects of the positive and negative changes in the independent variable, interest rates (RIR), on the dependent variable, private investments (PRI_INV).

Therefore, to establish a reference point, we first estimate the dynamic linear regression (ARDL) of the explanatory variables on private investment, followed by a dynamic asymmetric regression model (NARDL). This is so that we can compare and thus draw a conclusion on the importance of asymmetric analysis. According to Shin et al. (2014), the NARDL technique exhibits three desirable features that are useful and quite appropriate. These are as follows: the ability to estimate the error correction term, thus improving the performance of the model in small samples, which is the case in the current study; secondly, the technique – although simple – simultaneously provides the estimates for both the short run and long run asymmetries, thus it is flexible; and lastly, the technique uses an asymmetric dynamic multiplier to provide a straightforward means of obtaining joint estimates of the short run and the long run.

While the traditional error correction term measures the speed of adjustment from the short-run to the long-run equilibrium, the NARDL sheds light on the dynamic pattern of adjustment. This is where the nonlinear error correction term shows the gradual movement of the process from the initial equilibrium state to the new equilibrium, via the shock. According to Shin et al. (2014), asymmetry is normally discussed around the differentiation between the short-run and long-run asymmetries, however, NARDL shows three forms of asymmetries. The three forms are: reaction asymmetry, which is associated with testing the equality of the long-run coefficient; impact asymmetry, which observes the possible inequality of the contemporaneous first differences; and the adjustment asymmetry, which is the nonlinear error correction term.

Therefore, the conditional ARDL for the private investment model of equation 4.2 is:

$$\Delta PRI\_INV_t = \sum_{i=1}^{n} \alpha_0 \Delta PRI\_INV_{t-i} + \sum_{i=0}^{n} \alpha_1 \Delta RIR_{t-i} + \sum_{i=0}^{n} \alpha_2 \Delta CREDITG_{t-i} + \sum_{i=0}^{n} \alpha_3 \Delta GOVT\_GDP_{t-i} + \sum_{i=0}^{n} \alpha_4 \Delta INF_{t-i} + \sum_{i=0}^{n} \alpha_5 \Delta GDPG_{t-i} + \lambda_0 PRI\_INV_{t-1} + \lambda_1 RIR_{t-1} + \lambda_2 CREDITG_{t-1} + \lambda_3 GOVT\_GDP_{t-1} + \lambda_4 INF_{t-1} + \lambda_5 GDPG_{t-1} + \varepsilon_t \quad (4.3)$$

In equation 4.3, Δ indicates the first difference, the short run and long run elasticities are $\alpha_0, \ldots, \alpha_5$ and $\lambda_0, \ldots, \lambda_5$ respectively and ε_t is the error term. The long run relationship as well as long run and short run elasticities are first obtained for the dynamic linear regression. Cointegration among the variables is tested using the F-test, through the following null and alternative hypotheses:

H_0: $\lambda_0 = \lambda_1 = \ldots = \lambda_5 = 0$ (there is no cointegration)

H_1: $\lambda_0 \neq \lambda_1 \neq \ldots \neq \lambda_5 \neq 0$ (there is cointegration)

The null hypothesis of "no cointegration" will be rejected, signifying the existence of cointegration if the computed F-statistic exceeds the upper bound. If the F-stat lies below the lower bound, the null hypothesis cannot be rejected, hence there is no cointegration. However, an inconclusive decision on whether there is cointegration is where the computed F-stat lies between the upper and the lower bounds at a chosen level of significance. If cointegration exists, equation 4.3 will be estimated using the standard OLS, while obtaining the long run and short run coefficients, as well as the speed of adjustment of private investment back to the long run equilibrium. The unrestricted error correction model (UECM) of the dynamic linear ARDL regression model, is expressed by re-parameterising equation 4.3 as follows:

$$\Delta PRI\_INV_t = \sum_{i=1}^{n} \alpha_0 \Delta PRI\_INV_{t-i} + \sum_{i=0}^{n} \alpha_1 \Delta RIR_{t-i} + \sum_{i=0}^{n} \alpha_2 \Delta CREDITG_{t-i} + \sum_{i=0}^{n} \alpha_3 \Delta GOVT\_GDP_{t-i} + \sum_{i=0}^{n} \alpha_4 \Delta INF_{t-i} + \sum_{i=0}^{n} \alpha_5 \Delta GDPG_{t-i} + \gamma ECT_{t-1} + \varepsilon_t \quad (4.4)$$

The coefficient of the error correction term (ECT), γ , gives the short-run speed of adjustment back to the long-run equilibrium, showing how private investment gradually returns to its long-run equilibrium path after deviating from it. The one-period lag of the ECT signifies the percentage of the speed of adjustment from a shock in the previous period to the present period equilibrium. The coefficient of the

ECT is expected to be less than one, negative and statistically significant, for the economy to return to equilibrium.

After obtaining the long run and short run coefficients from the ARDL model, the NARDL model, which is developed from ARDL specifications, is estimated. In the NARDL model, we focus on the asymmetric relationship between real interest rates and private investment, while including the control variables, in order to have a well-specified model. This is comparable with the ARDL model. The major difference in these models is the ability to obtain different magnitudes of the coefficient due to a negative shock compared to a positive shock in real interest rates, while employing a nonlinear model. The NARDL technique is recently one of the best approaches to observe asymmetric (nonlinear) relationships between variables. Therefore, the asymmetric long run equation is thus:

$$PRI\_INV_t = \lambda_1^{pos} RIR_t^{pos} + \lambda_1^{neg} RIR_t^{neg} + \sum_{i=1}^{n} \lambda_{2i} Z_{t-i} + \varepsilon_t \qquad (4.5)$$

Where RIR_t^{pos} is the positive shock in the real interest rate, RIR_t^{neg} is the negative shock in the real interest rate, and Z is the group of control variables.

The NARDL-UECM is obtained by combining equations 4.3, 4.4 and 4.5 as follows:

$$\Delta PRI_{INV_t} = \sum_{i=1}^{n} \alpha_0 \Delta PRI_{INV_{t-i}} + \sum_{i=0}^{n} \alpha_{1i}^{pos} \Delta RIR_{t-i}^{pos} + \sum_{i=0}^{n} \alpha_{1i}^{neg} \Delta RIR_{t-i}^{neg} + \sum_{i=0}^{n} \alpha_2 \Delta CREDITG_{t-i} + \sum_{i=0}^{n} \alpha_3 \Delta GOVT_{GDP_{t-i}} + \sum_{i=0}^{n} \alpha_4 \Delta INF_{t-i} + \sum_{i=0}^{n} \alpha_5 \Delta GDPG_{t-i} + \gamma ECT_{t-1} + \lambda_0 PRI_{INV_{t-1}} + \varphi_1^{pos} RIR_{t-1} + \varphi_1^{neg} RIR_{t-1} + \lambda_2 CREDITG_{t-1} + \lambda_3 GOVT\_GDP_{t-1} + \lambda_4 INF_{t-1} + \lambda_5 GDPG_{t-1} + \varepsilon_t \qquad (4.6)$$

Where $\lambda_1^{pos} = -\frac{\phi_1^{pos}}{\alpha_0}$ and $\lambda_1^{neg} = -\frac{\phi_1^{neg}}{\alpha_0}$ are the associated asymmetric long-run parameters, ECT is the nonlinear error correction term.

Following the Chi-square distribution, the Wald test is used to check whether the short run and long run impacts are the same (symmetric) or different (asymmetric) by testing the following null hypothesis against the alternative hypothesis:

H_0: No short run (or long run) asymmetry

H_1: There is short run (or long run) asymmetry

Short run symmetry restrictions can be tested in two ways, using the Wald test:

(i) pairwise, $\alpha_{1i}^{pos} = \alpha_{1i}^{neg}$ for all *i*=0,1, …, n or (ii) additive, $\sum_{i=0}^{q-1} \alpha_{1i}^{pos} = \sum_{i=0}^{q-1} \alpha_{1i}^{neg}$.

Although, the additive symmetry restrictions have been found to be weaker (Shin et al., 2014), the study uses both pairwise and additive symmetry restrictions for comparison.

5. Results

The study observes the relationship among the variables by using the simple correlation probability as shown in table 1. The results show that the variable of interest, the real interest rate, is not only negatively associated with private investment as expected, but the relationship is highly statistically significant, even though at just below 40% correlation. The other control variables also depict some form of relationship with private investment; while some are as expected, but not statistically significant, others are not.

Table 1. Correlation Probability

Probability	**PRV_INV**	**RIR**	**CREDITG**	**GOVT_GDP**	**INF**	**GDPG**
PRV_INV	1.000					
RIR	-0.372*** (-2.751)	1.000				
CREDITG	0.231 (1.627)	-0.205 (-1.434)	1.000			
GOVT_GDP	-0.550*** (-4.510)	0.604*** (5.191)	-0.459*** (-3.546)	1.000		
INF	0.315** (2.273)	-0.377*** (-2.794)	0.319** (2.306)	-0.448*** (-3.439)	1.000	
GDPG	0.027 (0.184)	-0.184 (-1.286)	0.503*** (3.986)	-0.258* (-1.827)	-0.267* (-1.900)	1.000

*t-stat in parentheses; *10%, **5%, ***1%*

Source: Authors' computations based on the data from WDI, IFS, and the reserve bank of South Africa.

While it is not necessary to test for stationarity in the variables, it is, however, required and important to ensure that they are not integrated of order 2, I(2) and also that the dependent variable is of order 1, I(1). Therefore, the stationarity (unit root) tests are carried out using the Augmented Dicky-Fuller (ADF) and Phillips-Perron (PP) tests (table 2).

Table 2. Stationarity Test Results

Variables	Model	ADF Test		Phillips-Perron Test		Decision
		Levels	1st Diff.	Levels	1st Diff.	
PRV_INV	Without Trend	-0.408	-4.734***	0.433	-4.479***	I(1)
GDPG	Without Trend	-4.725***	NA	-4.599***	NA	I(0)
CREDITG	Without Trend	-3.580***	NA	-3.580***	NA	I(0)
GOVT_GDP	With Trend	-1.886	-6.985***	-1.925	-7.453***	I(1)
RIR	With Trend	-3.894**	NA	-3.923**	NA	I(0)
INF	With Trend	-3.214*	-6.676***	-3.081	-9.093***	I(1)

***1%; Test critical values: 1% -2.615; 5% -1.948; 10% -1.612 (without trend);
Test critical values: 1% -4.166; 5% -3.509; 10% -3.184 (with trend)

Source: Authors' computations based on the data from WDI, IFS, and the reserve bank of South Africa.

Given the mix of order of integration, I(0) and I(1), while the dependent variable is I(1), the popular Autoregressive Distributed Lag (ARDL) of the Bounds test approach to cointegration, developed by Pesaran et al. (2001), is best suited to determine whether or not these variables move together in the long run. This technique is appropriate for variables that are purely I(1) or a combination of I(0) and I(1). It is, however, important to determine the optimal lag length, before testing for cointegration. The selected optimal lag length, chosen by Akaike information criterion (AIC) is 4, with the optimal ARDL model selected for the private investment equation as ARDL (1, 3, 1, 0, 0, 1).

The F-test is used to determine the existence of cointegration among the variables. Based on the earlier highlighted interpretation, the result of the ARDL Bounds test approach to cointegration shows that we can reject the null hypothesis of "no cointegration" among the variables at all levels of significance (table 3). The computed F-statistic, 7.937, lies above the upper bound at all levels of significance, thereby depicting that the variables are cointegrated and there exists a long-run relationship between private investment and all the explanatory variables.

Table 3. Cointegration Test Result for Linear Model – ARDL Bounds Test

Test Statistic	Value	k
F-statistic	7.937***	5
	Critical Value Bounds	
Significance	I(0) Bound	I(1) Bound
10%	2.08	3
5%	2.39	3.38
2.5%	2.7	3.73
1%	3.06	4.15

Null Hypothesis: No long run relationships exist; *** 1%
k is the number of explanatory variables
Source: Authors' computations based on the data from WDI, IFS, and the reserve bank of South Africa.

Furthermore, the ARDL result in table 4 shows that while the real interest rate is not statistically significant in the short run at the current period, it becomes highly statistically significant at the first and second lags, although depicting a positive relationship with private investment. Interpreting the short run results should not focus on the signs of the parameters in explaining the private investment-real interest rate nexus because in the short run, variables are in differenced form. When variables are differenced, the long run information is lost. The only sign that should be the focus and interpreted in the short run is the error correction term. Thus, the error correction term of -0.12 satisfies all the conditions that it must be less than one, statistically significant and carry a negative sign. This implies the adjustment back to the long-run equilibrium, where about 12% of disequilibrium is corrected annually. Thus, if private investment deviates from its long-run equilibrium with the other variables in the preceding year, equilibrium will be restored in the following years at the rate of 12%.

Table 4. Dynamic Linear Estimation of the Private Investment-Real Interest Rate Relationship

Long run Coefficients			
Variable	Coefficient	Std. Error	t-Statistic
RIR	-0.951***	0.363	-2.621
CREDITG	0.092	0.266	0.347
GOVT_GDP	0.591***	0.175	3.387
INF	-0.131	0.357	-0.366
GDPG	3.467***	1.250	2.773
Short run Coefficients			
Variable	Coefficient	Std. Error	t-Statistic
ΔRIR	0.004	0.031	0.141
ΔRIR_{t-1}	0.110***	0.033	3.324
ΔRIR_{t-2}	0.069**	0.030	2.304
ΔCREDITG	0.045**	0.020	2.269

ΔGOVT_GDP	0.051	0.169	0.299
ΔINF	-0.001	0.050	-0.026
ΔGDPG	0.125**	0.057	2.201
ECT_{t-1}	-0.115***	0.017	-6.645

*10%, **5%, ***1%

Source: Authors' computations based on the data from WDI, IFS, and the reserve bank of South Africa.

In the long run, the control variables – credit and inflation – although depicting the expected signs, are statistically insignificant. Government spending and GDP growth are, however, both economically and statistically significant. The effect of real interest rate on private investment in the long run depicts the expected sign, thereby economically and highly statistically significant. As the focus of the study, the result shows that if real interest rate increases (or decreases) by 1%, private investment will fall (or increase) by 0.95%. However, imposing long-run symmetry where the underlying relationship is nonlinear will cause a spurious dynamic response because it will confound efforts to establish a stable long-run relationship (Shin et al., 2014). If private investments decline because of a positive shock (increase) in the real interest rates, then private investments may not increase by exactly the same amount after a negative shock (decline) of the same magnitude.

Therefore, this study isolates the effects of positive and negative changes in the real interest rates on private investment. The selected optimal NARDL model for the private investment equation is ARDL (2, 2, 2, 0, 0, 0, 2). The Bounds test for the NARDL model also shows that there is cointegration among the variables (table 5). The dynamic error correction result for the NARDL shows the asymmetric relationship between private investment and real interest rate in table 6.

Table 5. Cointegration Test Result for the NARDL Bounds Test

ARDL Bounds Test		
Test Statistic	Value	k
F-statistic	8.227***	6
Critical Value Bounds		
Significance	I(0) Bound	I(1) Bound
10%	1.75	2.87
5%	2.04	3.24
2.5%	2.32	3.59
1%	2.66	4.05

Null Hypothesis: No long run relationships exist; *** 1%
k is the number of explanatory variables

Source: Authors' computations based on the data from WDI, IFS, and the reserve bank of South Africa.

Table 6. Dynamic Asymmetric Estimation of the Private Investment-Real Interest Rate Relationship

Long run Coefficients			
Variable	Coefficient	Std. Error	t-Statistic
RIR^{pos}	-1.058***	0.379	-2.794
RIR^{neg}	-0.861**	0.385	-2.237
CREDITG	0.472*	0.269	1.754
GOVT_GDP	0.991**	0.493	2.008
INF	-0.530	0.397	-1.334
GDPG	4.405**	1.829	2.409
Short run Coefficients			
Variable	Coefficient	Std. Error	t-Statistic
ΔPRV_INV_{t-1}	-0.170	0.121	-1.403
ΔRIRpos	-0.026	0.036	-0.707
ΔRIR_{t-1}^{pos}	-0.082**	0.035	-2.378
ΔRIRneg	0.008	0.040	0.200
ΔRIR_{t-1}^{neg}	0.268***	0.050	5.387
ΔCREDITG	0.059***	0.018	3.359
ΔGOVT_GDP	0.278*	0.151	1.836
ΔINF	-0.039	0.042	-0.914
Δ GDPG	0.055	0.041	1.330
$\Delta GDPG_{t-1}$	-0.102**	0.047	-2.153
ECT_{t-1}	-0.100***	0.013	-7.936
Wald Test			
SR (pairwise) 7.059 [0.008]			
SR (additive) 3.448 [0.063]			
LR 4.502 [0.033]			

*10%, **5%, ***1%

p-value in parentheses [ ]

Source: Authors' computations based on the data from WDI, IFS, and the reserve bank of South Africa.

The estimated long-run coefficients indicate a different scenario compared to the dynamic linear result. Although both positive and negative shocks of interest rate depict an inverse relationship with private investment and they are statistically significant, their effects differ. When interest rate increases (RIR^{pos}) by 1%, private investment tends to decline by 1.06%, but when interest rate falls (RIR^{neg}) by 1%, private investment tends to increase by only 0.86%. This shows that the fall in private investment as a result of a shock (increase) in the interest rates is larger than when interest rates decline in order to accommodate increased private investment. Also, contrary to the linear effect of 0.95% decline in private investment due to a 1% rise in interest rates, the asymmetric result shows that private investment will fall by a higher magnitude of 1.06%. Likewise, as opposed to the result of the linear model of 0.95% increase in private investment following a 1% fall in real interest rate, the findings from the nonlinear model depict a much lower resultant effect of 0.86%. This therefore confirms the possible misleading result derived from a linear regression.

In the short run, only inflation and GDP growth are statistically insignificant, even though they are economically significant. The positive and negative partial sum processes of real interest rates are statistically significant at one-period lag. The error correction term is once again highly statistically significant; it is less than one and it is negative. Its coefficient of -0.10 is close to the one obtained in the linear model. However, the error correction term of the asymmetric model maps the gradual movement of the process from its initial equilibrium to the new equilibrium, via the shock. This shows that if private investment, along with the other variables, diverge from the long-run equilibrium in the previous year, movement towards equilibrium will be gradually mapped out, at a rate of 10%, through the shock, before it is restored. In all cases, the Wald tests firmly reject the null hypotheses of both short and long-run symmetries, at all levels of significance (table 6).

The pairwise symmetry restrictions reject the null of "no short run asymmetry" at 1%, while additive symmetry restrictions reject it at 10%. This shows the weakness of additive symmetry restrictions, which results in possible non-rejection of the null hypothesis, when it is supposed to be rejected. The model also passed the battery of diagnostic tests. Figure 1 represents the dynamic multiplier to further test the presence of asymmetry. The solid and broken black lines indicate the positive and negative deviations respectively, and the red line is the measure of asymmetry. If the red line is close to zero, there is no justification for asymmetry. Since the red line is further from the zero line, it thus validates the asymmetric relationship.

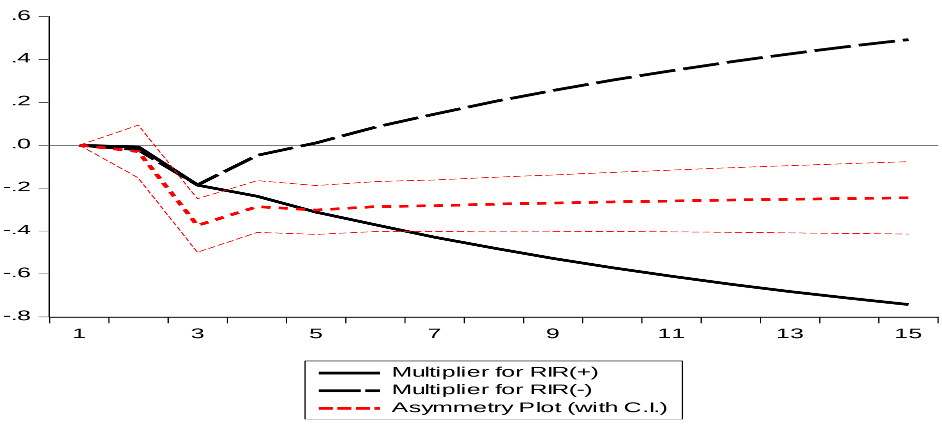

Figure 1. Private Investment-Interest Rate Dynamic Multiplier

6. Conclusion and Recommendation

The findings generally confirm the overarching study hypothesis that effects of interest rate on private investment are not necessarily linear as most studies tend to suggest. Application of only linear ARDL approaches to the analysis of interest rate-private investment nexus tends to mask the real dynamics of the relationship. In this study, what is glaring is that increases in real interest rates are associated with larger reductions in private investment compared to increases in investment associated with reductions in interest rate. Whereas linear analysis tends to show a lighter effect of interest rate increases and decreases, the nonlinear approach shows that the effect is much larger or lower. The ability of the NARDL approach to unpack the effects of interest rate on private investment, based on whether the shock is negative or positive, underlines this technique's efficacy in providing more credible results upon which dependable macroeconomic forecasts can be made.

To this end, although analyses based on linear ARDL and other techniques may indeed provide the requisite guide to any monetary policy direction regarding investment and how the same can be stimulated, nonlinear analyses tend to provide sufficient evidence based on which more credible forecasts can be made. It therefore seems prudent to utilize both approaches simultaneously to guide policy. It is notable that this study proves that the relationship in question is asymmetrical and does not shed light on how far up or low interest rates should go so as to balance up the expected effects on investment. It would therefore be interesting for future research to establish these thresholds to guide policy even further.

6.1. Data Availability Statement

The data that support the findings of this study are available from three sources. These sources are the World Development Indicators (WDI), the International Financial Statistics of the IMF (IFS/IMF), and the South African Reserve Bank (SARB). These data bases are publicly available.

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