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Volatility Forecasting during Extreme Market Events Using the (JSE) Small Cap Share Index

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Abstract: The lost decade of the JSE in small-cap companies from 2010 to 2019 as a result of a decline in investments indicated the high risk of investing in small-cap shares. Therefore, the study aimed to make use of two volatility models, the exponentially weighted moving average (EWMA) and the general autoregressive conditional heteroskedasticity model (GARCH) to forecast volatility of share price returns. Considering the South African market, the outbreak of COVID-19 had an impact on small-cap shares and the optimal lambda used throughout the analysis. Also, the study aimed to determine the optimal lambda amid COVID-19. A comparison was made between the two models where the use of the small-cap index (J202) was applied. The models highlighted the key weakness of the standard deviation, assigning the same weight to all share price returns in the period under analysis. The models captured share price shocks during extreme events whereas a negative relationship between share price returns and volatility in small-cap shares was encountered.

Keywords: EWMA model; GARCH (1,1) model; historical standard deviation; volatility forecasting; extreme event; optimal lambda, COVID-19.

JEL Classification: G01; G11; G17

1. Introduction

Small capitalisation shares are more domestically exposed businesses that derive their large portion of profits within the boundaries of South Africa (Bergman, 2019). This brings to light their vulnerability towards economic tensions to which they are

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not immune. They are more affected by changes in the economic environment (Thune, 2020). Besides, they are at risk from global economic recessions. During the burst of the dot-com bubble in 2002, and the financial crisis in 2008, the performance of the small-cap share index against the JSE Top 40 Index was at negative 67.80 per cent and negative 32.68 per cent respectively (Friedrich, 2019). In both instances, there was a significant sell-off in small-cap shares as a result of investors pushing towards the diversification of their investment portfolios amongst large-cap shares. Mtemeri (2019) makes a definite point on the benefit of dual-listing by large-cap shares from the investors' perspective, such as taking advantage of two varying markets with different investment opportunities based on risk and return is viewed as an opportunity for diversification.

Nonetheless, in both instances of the dot-com bubble in 2002 and the financial crisis in 2008, small-cap shares tend to outperform the large- and mid-cap shares much faster after each economic downturn (Switzer, 2010). Thune (2020) highlights that during this period small-cap companies sold their undervalued shares to raise capital while large-cap companies borrowed capital through the issuing of bonds. Once again, in March 2017, small-cap companies listed on the JSE performed at negative 38.90 per cent, this was an opportunity realised by investors to invest in better-quality small-cap shares (Friedrich, 2019). Likewise, South African Market Insights defines the Economic progress index as an index that measures the progress in the growth of the South African economy (2019). The economic progress index declined by a negative 20.2 per cent from 2010 to 2018 (South African Market Insight, 2019). Lipper (2020) argues that besides the economic progress index, investors with an appropriate time horizon buying small-cap shares only based on their inexpensiveness doesn't guarantee higher returns.

Fundamental analysis with a competent management team is important because focusing solely on valuation puts an investor at a disadvantage of the possibilities of being invested in a company that assigns the same weight to its shares and being inconsiderate of modelling share volatility forecasting based on EWMA and GARCH (1, 1). Additionally, Bollen (2015) explains that factoring in lambda as a decay factor also known as a smoothing parameter is important in determining exponentially decreasing weighting scheme of the observed data in which a high optimal lambda will indicate a slow decay in the analysis, implying, a high variance persisting for a long period. The objective of this article is therefore to examine EWMA and GARCH (1,1) models when forecasting volatility in small-cap shares using the small-cap index (J202). Investigate the models' relationship about the use of daily data and analyse whether COVID-19 can be categorised as an extreme event while calculating the optimal lambda for the period under analysis which will be recognised in the respective models.

2. Literature Review

Before an investor decides to make a purchase of shares from companies listed on the JSE, it is important to know various index categories and how companies listed in these categories vary based on their market capitalisation. According to Panagiotidis (2005) market capitalisation makes it understandable for an investor to determine a company's size in assessing the risk of investing in its shares. Thus, it is the total market value of a company's outstanding shares (Khrawish *et al*, 2010). Likewise, small-cap companies have a market capitalisation that is below one billion rands (JSE, 2020). Wyatt (2009) states that small-cap companies should not be confused with start-up companies. Small-cap companies have moved past the start-up phase since they were able to establish themselves by being publicly traded and owned entity through the initial public offering (IPO). However, small-cap companies tend to be overlooked by the Top 40 Index which consists of companies that are investable in the All Share Index as it is favoured among investors that earn a higher return when selling the most shares.

The small-cap index is representative of 96 per cent to 99 per cent of the full market cap value of all qualified ordinary securities that are listed on the main board of the JSE (JSE, 2020). The index is fairly overlooked since it is particularly unfavourable among investors based on the high level of volatility and lack of extensive history (Eun *et al*, 2008). Small-cap companies have made their initial impact in their respective industries which have placed them on the stock exchange. They are set to grow and have a big impact in the coming years. For that reason, an investor that considers small-cap shares will benefit from the following advantages. Van Vuuren (2019) points out that on average, companies listed in the Top 40 Index consist of eleven research analysts covering each company meanwhile, there are approximately two analysts in small-cap companies. Imbert (2019) states that this creates an investment opportunity for potential investors that are willing to conduct thorough research on undervalued shares.

Although large- and mid-cap shares provide an investor with attractive returns given the level of risk to which they are exposed, small-cap companies should be perceived as a long term investment (Thune, 2020). They are probable to yield a return that is greater than that of the latter through their reputation of being more volatile while steadily building and diversifying their investment portfolio. There is less competition when buying shares as the market is smaller than that of the large and mid-cap shares (Reyes, 2001). Buying shares during the initial public offering implies that their value will increase as the company gains popularity with investors and based on its reputation. Equally so, Tinic (1988) adds that in the long run, an investor will potentially gain a high return on investment. Small-cap companies take advantage of their efficiency and effectiveness since large-cap companies have many employees and various layers of management to which they report to namely a large number of board of directors which can extend to 31 members (Reddy *et al*, 2008). This hinders the large-cap companies' decision-making process. Whereas, small-cap companies are in a position of implementing new strategies quickly with a small board of directors of approximately seven to fifteen members (Natesan & du Plessis, 2018). In the interest of having a balanced understanding of small-cap shares, the companies in this category encounter the following disadvantages.

Even though they outperform large- and mid-cap companies during economic recoveries, their vulnerability to economic downturns due to less financial resources leads to investors opting for small-cap mutual funds and Exchange Traded Funds (Bontis *et al*, 2007). This option helps to lower the risk of loss. Their high level of risk in comparison to other companies based on high volatility might make it difficult for investors to invest hence, diversifying an investment portfolio by having a small percentage of small-cap shares can turn out to be useful (Eun *et al*, 2008). There is no guarantee that these companies will break into the Top 40 Index and pay dividends (Schwartz & Zimmer, 2013). Instead of making dividend payments, small-cap companies make use of the profits for growth. Menkveld and Wang (2013) deduce that these companies tend to have fewer outstanding shares. This creates a liquidity problem.

Investors holding shares could find it challenging to find buyers to whom they will unload their shares since the market is volatile (Cacheche *et al.*, 2015). The optimal liquidity theory supports this phenomenon. It states that as an asset or market liquidity increase, that asset or market’s risk declines to a point in which the increased liquidity attracts an adequate number of speculators to offset the lower risk and, ultimately, increase the asset or market’s risk (Han & Lee, 2012). Therefore, there is an optimal point at which maximum liquidity is reached. This point is optimal liquidity. It is best illustrated in Figure 1

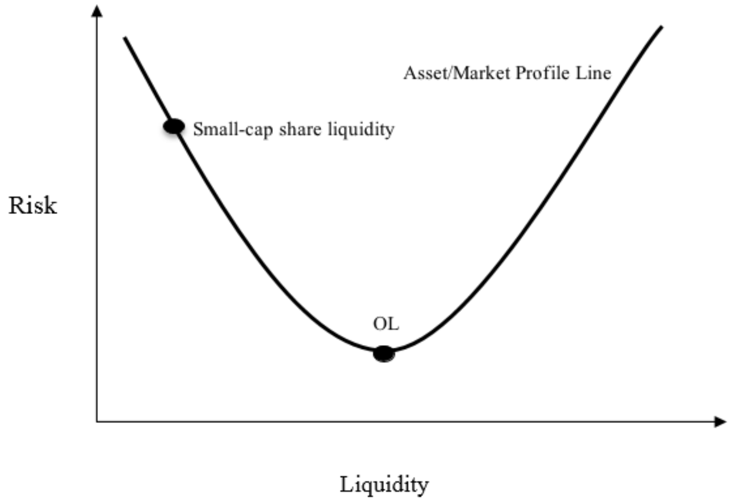

Figure 1. Optimal Liquidity

The above theoretical figure illustrates the optimal liquidity and small-cap share liquidity. In support of the optimal liquidity theory, as an asset or market liquidity increases, a decline in the risk profile occurs up to an optimal point whereby maximum liquidity is reached. However, increasing the liquidity of an asset attracts traders to speculate in the asset or market. On the other hand, small-cap shares are in general less liquid, more volatile, and overall risk of shares. Chen *et al* (2009) make a definite point on the issuance of mispricing as they tend to occur whereby they lead to difficulties in determining the actual share price.

Investors in small-cap shares may attempt to maximise their gains by identifying shares that are mispriced, under this premise, they dismiss the theory of efficient markets (Friedrich, 2020). The theory states that all information regarding shares and investment securities are included in the prices of those securities (Thune, 2020). There are three forms of efficient market hypothesis namely, weak, semi-strong, and strong. The weak form implies all previously held information is priced into securities as a result, the use of technical analysis of past share prices in outperforming the market cannot be used. Meanwhile, the semi-strong form suggests that new and recent information is priced into securities whereby neither fundamental nor technical analysis can be in use in achieving abnormal returns (Thune, 2020). The strong form implies that both public and private information is priced into shares and that investors cannot have an advantage over the entire market (Ball, 2009). As small-cap shares tend to outperform both the mid-and large-cap shares, they pose a challenge to the strong form of the efficient market hypothesis.

This challenge is in support of the small company bias theory which states that shares of small-cap companies, earn abnormal returns and outperform mid-and large-cap companies (Simonson, 2014). This occurrence is often overlooked since small-cap companies are rarely prioritised in comparison to their counterparts. This gives rise to the neglected firm effect which explains the notion of small companies outperforming better-known companies due to their higher risk and reward potential (Akhter *et al.*, 2015). The negligence is well explained particularly in the extreme event of the outbreak of the global pandemic coronavirus or commonly referred to as COVID-19 providing an opportunity for investors with a long-term horizon (Friedrich, 2020). Friedrich (2020) explains that as of 16 June 2020, the small-cap index was down by 18 per cent since the commencement of COVID-19 in comparison to a decrease of nine per cent for the JSE all-share index. This illustrates the high risk of investing in small-cap shares. Despite this, such risks are manageable while creating an advantage to capitalise on such mispricing opportunities (Gilbert, 2019).

Since small-cap companies have smaller earnings, they make way for better growth potential as with the case in the bank, Capitec, which started as a small-cap and is recently the third largest bank in South Africa (Vermeulen, 2018). It reported consistently strong financial results for the period ending February 2020 with earnings increasing by 19 per cent to 6.28 billion rands (Capitec bank, 2020). The growth is largely due to having a strong client growth with over 2.5 million new clients, provided with a score of 84.0 for customer satisfaction, ahead of Nedbank, and FNB at 80.2 and 79.9 respectively (Consulta, 2020). On the other hand, Figure 2 makes provision for the behaviour of small-cap share prices concerning the Top 40 share prices during the financial crisis in 2008.

Figure 2. Behaviour of Small-Cap Share Prices Against the Top 40 Companies during Recessions and Recoveries
Source: IRESS INET BFA (2020)

Figure 2 illustrates the performance of the share prices of the JSE Top 40 and that of small-cap companies. During the period 10 November 2007 and 25 October 2008, small-cap shares were at 370.36 rands and 213.78 rands respectively. Small-cap shares have undergone a negative percentage change of 42.28 per cent. In the same period, the Top 40 shares were at R280.54 and R167.24 respectively. Top 40 shares experienced a negative percentage change of 40.39 per cent. From this, not only were small-cap shares experiencing a significant price decline, so did the Top 40 shares. However, in theory, small-cap shares tend to rise faster than large-cap shares during economic recoveries (Hulbert, 2020). Figure 2 supports the theory as in the year 2009 shares in small-cap companies were trading higher than that of the Top 40. Gurdus (2020) discusses that the rapid rise in recovery for small-cap share companies is based on having undergone a steeper decline in a recession, their shares tend to be undervalued. Small-cap shares get a performance reinforcement as they present attractive valuations when they are acquired and merged with large-cap companies that are looking for ways to grow. Nevertheless, the riskiness and high volatility in small-cap shares should be taken with caution.

The great recession of 2008 began in 2006 due to the significant decline in the housing prices of the United States which was referred to as the subprime mortgage crisis (Vieira, 2011). By the third quarter of 2008, the United States of America GDP declined by 0.3 per cent (Bureau of Economic Analysis, 2008). This occurred as the subprime crisis had affected the United States of America economy. By September 2008, the Dow Jones Industrial Average dropped 777.68 points in intra-day trading, the high rate in the decline was due to congress having failed to accept the bank bailout bill (Perman, 2008). Both the subprime crisis and the crash in the New York Stock Exchange fuelled the recession which became a financial crisis that spilt over to the world economy. Maredza and Ikhide (2013) point out that South Africa was not exempt from the global financial crisis as it had been negatively affected. The economy declined by 1.8 per cent in the fourth quarter of 2008 (Marais, 2009).

3. Research Methodology

Data were obtained from an online tool named IRESS INET (Pty) Ltd. It was used to extract financial data used for analysis. The period under analysis was from 01 January 2010 to 31 July 2020, making use of daily data. The date 01 January 2010 was chosen as a starting period because that was the year shares in small and mid-cap companies listed in the JSE began to experience a significant decline. The date 31 December 2019 was the period in which share prices were at their lowest in the decade at negative four per cent. The period was extended to 31 July 2020 for the inclusion of the extreme events as a result of the outbreak of the COVID-19 global pandemic and its effect on the optimal lambda. Analysis using daily returns was based on the findings that information based on volatility was likely to be lost at lower frequencies particularly in periods of a crisis (Danielsson *et al*, 2018). Hence, Ederington and Guan (2005) pointed out that both models, EWMA and GARCH (1,1) in using daily data, performed better with a more accurate forecast of the volatility after day n-1.

The methodology took into account the current volatility based on historical data using EWMA and GARCH (1, 1). But first, it was important to derive these models to get an understanding of the analysis.

3.1. Forecasting Volatility

The following models namely, EWMA and GARCH (1,1) were derived as the section read and chosen because they aligned with providing solutions to research questions, highlighting volatility and optimal lambda at the end of the study analysis 31 July 2020 amid the outbreak of COVID-19, and in the achievement of the empirical objectives of the article. This section derived both models in the following manner:

The symbol σ_n was well defined as the volatility of a share price on day n, as forecasted at the end of day n-1. The square of the volatility, σ_n^2 on day n was the variance. Using the historical data, this was forecasted in the following manner:

The value of a share at the end of day i was S_i. The variable u_i was the continuous compound return between the end of day $i-1$ and the end of day i which gave the following equation:

$$u_i = \ln(\frac{S_i}{S_i - 1})$$

The square of the volatility σ_n^2 using the most recent m observation on the u_i to have a fair estimate of the variance per day was provided with this equation:

$$\sigma_n^2 = \frac{1}{m-1}\sum_{i=1}^{m}(u_{n-i} - \overline{u})^2 \tag{1.1}$$

Whereby u bar implied the mean of the u_i's:

$$\overline{u} = \frac{1}{m}\sum_{i=1}^{m} u_n - i$$

Since the analysis was based on daily volatility, the above formula was changed based on the following assumptions:

- u_i was the percentage change in the share price between the end of day $i-1$ and day i which allowed

$$u_i = \frac{S_i - S_{i-1}}{S_{i-1}} \tag{1.2}$$

- The assumption was that $\overline{u}$ was zero.
- $m-1$ was replaced by m.

Even though these changes contributed small changes to the estimate calculated, they simplified the formula for the variance to be as follows

$$\sigma_n^2 = \frac{1}{m}\sum_{i=1}^{m} u_{n-i}^2 \tag{1.3}$$

Whereby u_i was provided in (1.2).

3.2. Weighting Schemes

Though equal weight was provided to the m observations as depicted in the above equation, the current level of volatility (σ_n) was of interest in forecasting and more weight was provided to recent data.

In accordance, the formula will then be revised to the following weighting scheme

$$\sigma_n^2 = \frac{1}{m}\sum_{i=1}^{m} \alpha_i u_{n-i}^2 \tag{1.4}$$

The variable α_i was defined as the amount of weight that was provided to the observation i in previous days. The value of α (the weights) was positive (Nilakantan & Mistry, 2013). A long-run variance (V_L) and its weight (γ) is assigned which transforms the model to a new form which is as follows

$$\sigma_n^2 = \gamma V_L + \sum_{i=1}^{m} \alpha_i u_{n-1}^2 \tag{1.5}$$

As suggested by Engle, the above formula was an ARCH (m) model. The forecast of the variance was based on a long-run average variance and m observations. The long-run variance and its weight (ω), in reference to the above equation it was written as

$$\sigma_n^2 = \omega + \sum_{i=1}^{m} \alpha_i u_{n-1}^2 \tag{1.6}$$

According to equation (1.4) and (1.6), various approaches were developed to monitor volatility. These approaches are the EWMA and GARCH (1,1) models.

The EWMA model was part of the model in equation (1.4). Whereby weights α_i declined exponentially when moving back in time. In particular, α_{i+1} was defined as $\lambda\alpha_i$ whereby λ was a constant between zero and one. Such a weighting scheme was simplified with a formula for volatility forecasting as follows:

$$\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2 \tag{1.7}$$

The forecast of the volatility (σ_n) of a variable for day n (made after day n-1) was calculated from σ_{n-1} (the forecast that was made after day n-2 of the volatility for day n-1) and σ_{n-1} (the foremost recent daily rate change within the variable).

The EWMA approach was designed to track changes within the volatility (Adewuyi, 2016). If there was a huge move within the market variable on day n-1, and u_{n-1}^2

was huge, as depicted in Equation (1.7) the forecast of volatility was moved upward. The value of λ administered how responsive the forecast of the daily volatility was subject to the foremost daily percentage change (Finance Train, 2020). On the other hand, a higher weight was provided to the u_{n-1}^2 when volatility (σ_n) was calculated, as such, the value of λ declined. Furthermore, as λ got closer to one, its value increased whereby it led to the forecast of daily volatility that reacted moderately slower to data recently provided by daily percentage change.

The value of λ provided forecasts of variance that were near the approach to the realised variance rate. The value of the optimal lambda was determined. Meanwhile, EWMA was simply a non-stationary version of GARCH (1,1) in which the parameters, namely, α_1 and β_1, added up to the value of one.

Regarding the GARCH (1,1) model, the variance (σ_i^2) was fundamentally calculated from the following: the long-run variance (V_L), σ_{n-1} and u_{n-1}. The formula for volatility forecasting concerning the GARCH (1,1) model took the following form:

$$\sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2 \tag{1.8}$$

The symbol γ implied the weight assigned to V_L. Whereas, α was the weight provided to u_{n-1}^2 and β was the weight provided to σ_{n-1}^2. As the weights added up to unity, the equation $\gamma + \alpha + \beta = 1$

For the weights that added up to one based on the GARCH (1,1) model, the following was different as the EWMA model weights were as follows: $\gamma = 0$, $\alpha = 1 - \lambda$, and $\beta = \lambda$. Since the variance (σ_i^2) was based on the most recent observations of u^2 and the latest forecast of the variance rate, the (1,1) in the GARCH model was determined. The weight provided to V_L was written as ω. Hence, the GARCH (1,1) model was rewritten as follows:

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2 \tag{1.9}$$

Once the parameters ($\omega, \alpha,$ and β) were forecasted, the weight assigned to V_L was calculated such that $\gamma = 1 - \alpha - \beta$. To ensure the GARCH (1,1) model was stable in the analysis, there should be a condition put in place whereby the sum of the parameters (α, β) was less than one i.e. $\alpha + \beta < 1$. When the sum of the parameters (α, β) was greater than one, a negative weight imposed to the long term variance was reflected i.e. $\alpha + \beta > 1$ (Nilakantan & Mistry, 2013). In addition to the negative weight, the GARCH (1,1) model was reduced to the EWMA model such that the sum of the parameters (α, β) was equal to one i.e. $\alpha + \beta = 1$. At this point, the weights experienced an exponential decline at the rate β. The parameter was also referred to as a decay rate (Collins, 2020). It reacted similarly to λ in the EWMA model. It was responsible for its importance towards the observed u's (percentage changes in share prices) in calculating the current variance. The difference between the two models was based on the manner in which the GARCH (1,1) model allocated weights towards the long-run variance volatility and from the analysis, the more preferable model was determined.

4. Results and Findings

The historical standard deviation assigning the same weight to all share price returns in the period regarded as the lost decade of the JSE is considered to be its key weakness (Milton, 2019). This study aimed to realise how investors can benefit from investing in small-cap shares by analysing share price returns using the EWMA and GARCH (1,1) models respectively. EWMA was constructed against the historical standard deviation and the GARCH (1,1) model was also analysed against the historical standard deviation. It was followed by the analysis of the extreme events – financial crisis, the impact of ousting the former Minister of Finance Nhlanhla Nene towards the economy and determining the optimal lambda of small-cap share volatility amid the outbreak of COVID-19.

Small-cap shares are volatile and simply regarded as highly risky investments because they fluctuate more than other share investments in mid-cap or large-cap companies (Imbert, 2019). Since volatility is a statistical value that takes into account the range of returns for a given share price, it measures this dispersion through the standard deviation between returns (Campbell & Lettau, 1999). On the other hand, EWMA model as a weighted average of past volatility takes into account the persistence and clustering of volatility implying that a time frame of high volatility is followed by a period of high volatility (Cont, 2007). A time frame of low volatility is followed by a period of low volatility. Following the theoretical objective which proposes the contextualisation of the significance of volatility forecasting of asset returns in small and mid-cap shares to investors, EWMA and GARCH (1,1) volatilities provide a broader understanding of the construction of share investments than solely focusing on the standard deviation. The following figures illustrated the execution of empirical objectives. The following ten companies were selected as a sample and analysed for the period 1 January 2010 to 31 July 2020 in determining the optimal lambda. The companies were as follows: City Lodge Hotels, Adapt IT Holdings, EOH Holdings, MC Mining, Nampak, Netcare, Purple Group, Sasfin Holdings, Adcorp Holdings and Tongaat Hulett.

Table 1. Optimal lambda for small-cap shares

Small-cap companies	City Lodge Hotels	Adapt IT Holdings	EOH Holdings	MC Mining	Nampak	Purple Group	Sasfin Holdings	Adcorp Holdings	Netcare	Tongaat Hulett
Phi	10.38%	0.05%	6.14%	0.18%	14.56%	0.07%	6.27%	5.35%	29.70%	27.31%
Lambda	0.985	0.988	0.988	1.000	0.962	0.955	0.958%	0.808	0.898	0.936
Optimal lambda	**0.931**									

**Significant at 1%

Table 1 made provision for determining the optimal lambda of small-cap shares with a sample of small-cap companies. The weight applied to the recent share price return on 31 July 2020 determined by adding each phi and lambda of each company in which the decay factor for daily data reflected as the optimal lambda of 0.931 also known as a smoothing constant used in determining the exponentially declining weighting scheme of the observations in the sample period. A low lambda indicated a rapid decay in the series to which a substantial instability in the market led to high volatility as it was evident with the outbreak of COVID-19 as an extreme event in the case of the optimal lambda.

Despite this, it was considered that the negative relationship between volatility and share price return, when forecasting volatility for small-cap shares as they were less liquid, more volatile which were in general viewed as an overall risk for investment, forecasting with historical daily data was made for one day following the end of the period under analysis. This implied the forecast was made by 31 July 2020.

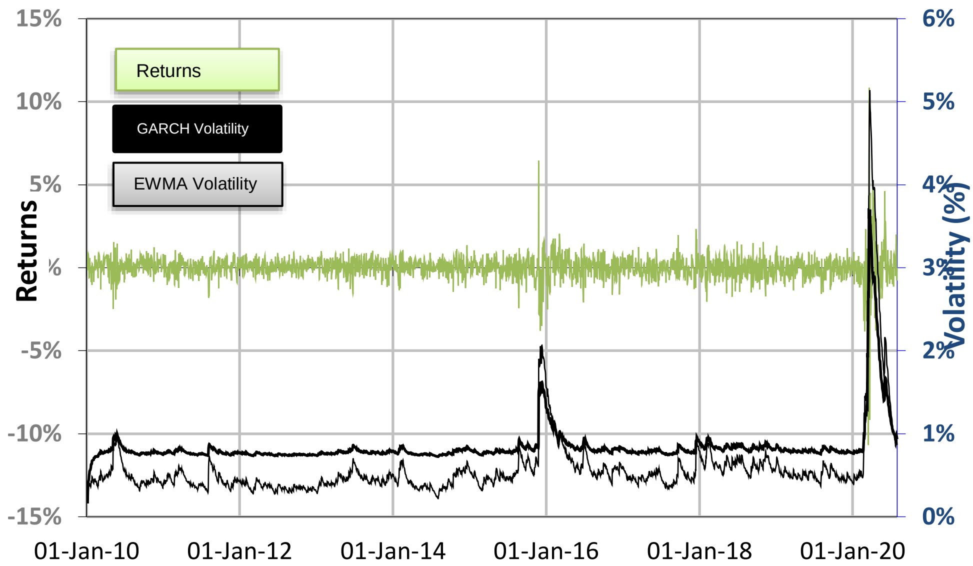

Figure 3. Comparison of the Exponentially Weighted Moving Average and General Autoregressive Conditional Heteroskedasticity (1,1) volatilities for small-cap shares

EWMA and GARCH (1,1) models track a relative similar movement of the volatility of small-cap shares. The distinction between the models lies in the components that make up each equation. The exponential weighted moving average model took into account the optimal lambda expressed as 0.931 for daily data. It was appropriate to make use of 0.931 as data was depicted daily. The forecast of volatility for day n (made at the end of 5 January 2010) was calculated by taking the previous day's volatility (σ_{n-1}) and present-day share price return (negative 0.16 per cent) and optimal lambda of 0.931 for daily data using the equation $\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1 - \lambda)u_{n-1}^2$. A similar principle was applied to share prices that followed up to 31 July 2020 for forecasting. The value 0.931 did the following to the equation and the overall model, it controlled how reactive the forecast of daily volatility was to the leading daily percentage change.

By changing the value 0.931 to a declining value, approaching zero, more weight was assigned to recent share price returns while less weight was assigned to distant share price returns. On the other hand, increasing the value of 0.931 closer to one, share price returns were equally weighted. The change in the value 0.931 is at the centre of EWMA volatility and weights of share price returns. Either the change of the value 0.931 the weight of all share price returns amounted to 100 per cent with an exception of having the value at one. In that regard, all share price returns remained at zero per cent and EWMA volatility remained at constant volatility of an absolute value of 0.16 per cent. Using 0.931 share price return weights declined exponentially which was a feature applied to the model to make aware of the historical standard deviation assigning the same weights to all share price returns in the sampling period. The application of the GARCH (1,1) model was due to noting the variation between its outcomes to that of EWMA. Conversely, the EWMA model suffered from recovering slowly from large share price shocks whereas GARCH (1,1) reacted quickly in recovering from similar large share price shocks (Malz, 2020).

GARCH (1,1) model took into account three distinct weights. Gamma, which is a weight assigned to the long-run variance (V_L). It took the value of 0.0345. Alpha, is the weight assigned to the previous day's squared return as day n-1 (u_{n-1}^2), which makes up the value of 0.0345 as well as beta as the weight assigned previous day's variance as day n-1 (σ_{n-1}^2), taking the value of 0.931. The value 0.931 was yet used, as EWMA is a non-stationary type of GARCH (1,1) model whereby all three weights add up to one. The volatility forecast for day n (5 January 2010) was calculated by considering the previous date (4 January 2010) and the share price return of 5 January 2010 at negative 0.16 per cent. The weights for daily data were factored in the equation $\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$. The equation was recursive as the present-day volatility was forecasted. The same structure of the calculation of forecasting volatility was followed up to 31 July 2020.

In addition to assigning exponentially declining share price weights, the long-run variance was modelled in a way that the series tended to gravitate or pull towards (Carvalho et al, 2018). The product of the long-run variance of one per cent squared and gamma weight led to the value of omega 0.00000345. The weights declined exponentially as greater share price weights were assigned to recent dates and lesser share price weights assigned to more distant dates. Moreover, the weights of the share price returns amounted to 100 per cent. Even with the shared characteristic of exponentially declining share price weights, GARCH (1,1) model could readjust quickly to repercussions of share price shocks (Blasques *et al*, 2017). Following the share price shock on 11 December 2015, GARCH (1,1) had a more constant fluctuation in volatility while the exponentially weighted moving was infrequent in the fluctuation of volatility as it captured more share price shocks. In both instances, the value 0.931 contributed a major role to the equations that made up each model.

The presence of EWMA and GARCH (1,1) models provided a well-explained phenomenon behind investing in small-cap shares. These type of shares were known to emulate the low cost of entry when compared to the issuance of mid-and large-cap shares (Jooste, 2019). Relative low share prices were due to low demand in the market. This followed the performance of small-cap shares during economic uncertainty. Such an occurrence was well captured by the short-term shock in price volatility which took place on 11 December 2015. During a period of economic uncertainty, investors tend to be uncertain about the performance of these companies and created widespread panic which led to the selling of shares while a share market downtrend took place (Stevens, 2020). Both models were able to forecast the highest volatility in the sample period on 11 December 2015 meanwhile the lowest return in shares on the same date was depicted. This was of great importance towards the negative volatility-return relation.

As short-term share volatility is a result of news effect or economic uncertainty giving rise to political risk – the risk investment in share return could undergo due to political instability in a country (Lensink et al, 2000). Share price returns plunged at a low of negative 3.50 per cent. The news effect was due to the ousting of former South African finance minister Nhlanhla Nene (Letsoalo, 2015). The negative return associated with increased volatility is a principle adapted which helps in distinguishing the relations between volatility and returns. When short-term volatility increases, it is followed by declining levels of returns and as long-term volatility increase, it is followed by increasing levels of returns (Dimitriou & Simos, 2011). The share price returns were fairly consistent between 10 October 2010 and 1 October 2015, averaging a return of 0.05 per cent. On the other hand, following an announcement by South African President Cyril Ramaphosa about the 21 days national lockdown from 26 March to 16 April 2020, as a result of a steep increase in coronavirus cases from 61 to as greater than 400 in a week, small-cap companies inevitably have undergone a decline in share price return at negative 9.17 per cent (Burke, 2020). The outbreak of coronavirus proved to have been impactful as the reported share price return on 23 March 2020 was the lowest return in the period under analysis. Figure 4 examined comparisons of EWMA and the historical standard deviation whereby the latter failed to realise share price volatility shocks.

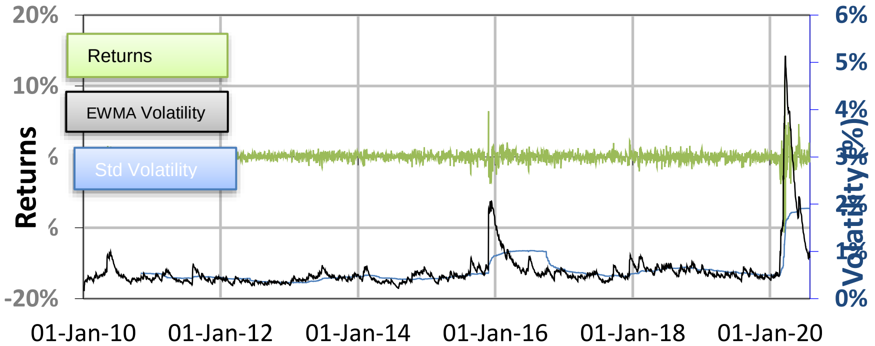

Figure 4. Comparison of the Exponentially Weighted Moving Average (EWMA) and the Historical Standard Deviation for small-cap shares

Following the historical standard deviation volatility application of a fixed and equal weight to all observations during the sampling period, it is thus showcasing a various volatility trend to that of EWMA volatility as shown in Figure 4. The historical standard deviation volatility starting sampling period was on 1st November 2010. This was the date the highest share prices were reported at 12 per cent in the commencement of what was to be called the lost decade of the JSE. Moreover, in the calculation of the historical standard deviation volatility, the period of small-cap share returns before 2nd November 2010, namely, 4th January 2010 to 1st November 2010 was factored in. This gave volatility of 0.54 per cent, implying a variance in a set of small-cap share returns in comparison to the average of the returns at negative 0,063 per cent.

As the simplest measure of volatility available, the historical standard deviation volatility remained as a drawback towards the weight of share price since all the daily share price returns are getting the same weight, following an equal-weighted approach (Larequad, 2015). Errais and Bahri (2016) argue that this is a weakness because whenever share price shocks or spikes occur, the historical standard deviation volatility will fail to capture such shocks. There were three distinct share price shocks that occurred on 7 August 2010, 11 December 2015, and 23 March 2020 respectively and it was evident from Figure 4 that only EWMA was able to capture these share price shocks. Not only was EWMA able to capture share price shocks, but its presence also overcame the weakness of the historical standard deviation volatility by assigning the same weight to recent and distant share price returns in the sampling period. The continuous update of forecast in volatility subject to new information availability provided a more preference of EWMA than the historical standard deviation. It did this with a single parameter denoted as lambda (0.931).

Secondary sector performance to the economy was the lowest amongst the tertiary and primary in which the price shock on 7 August 2010 with EWMA volatility of 0.84 per cent, and a share price return of negative 2.48 per cent, it recorded a growth rate of negative 3.9 per cent while the tertiary and primary sectors had growth rates of 2.0 and 24.5 per cent respectively (StatsSA, 2010). The largest contributors were the mining and quarrying industry in the primary sector at 6.5 per cent meanwhile, the transport, storage and communication industry categorised in the tertiary sector was at 3.4 per cent giving an increase in GDP by 2.6 per cent quarter-on-quarter (StatsSA, 2010). The inverse relationship between volatility and share price return was also captured on 4 August 2014 with an exponential weighted moving average volatility of 0.23 per cent. This was as a result of the information regarding the third quarter of GDP.

The secondary sector declined by negative 2.0 per cent (StatsSA, 2014). The sector was made up of manufacturing with a decline of negative 3.4 per cent and electricity, gas and water which declined by negative 1.1 per cent. Also, the model reacted quickly not due to the shock itself, but due to the response to the growth of GDP in the fourth quarter. On December 2014, the primary and secondary sector grew by 13.3 per cent and 7.2 per cent respectively (StatsSA, 2014). Both sectors included small-cap companies with a core focus on mining, agriculture, manufacturing, construction, and electricity. It was also important to include the tertiary sector as it contributed fairly to the economy and encompass small-cap companies with shares traded on the JSE. Hence, StatsSA (2014) stated that the tertiary sector grew by 1.8 per cent.

It was evident that using EWMA, the model uncovered the tendency of small-cap shares being influenced by market fluctuations, making them volatile. The price shock on 11 December 2015 with EWMA volatility of 1.99 per cent illustrated small-cap shares a high level of volatility. The model thoroughly captured the high level of volatility compared to the historical standard deviation whereby its volatility was at 0.83 per cent. With the departure of the former South African finance minister Nhlanhla Nene as a result of heightened political risk, the business confidence index stood at an all-time low of 79,6 (Dludlu & Mapenzauswa, 2016). Meanwhile, a high level of volatility had not reached that of 11 December 2015 as investors with shares in small-cap companies were vigilant on which companies to construct their investments.

In all three price shocks discussed due to low-level performance in the secondary sector and political risk, the price shock that occurred on 23 March 2020 was the biggest as EWMA volatility was at 4.87 per cent with share price returns declining to a low of negative 9.17 per cent. Depletion of investment portfolios, job losses, and consumers defaulting on their credit loans were amongst the major consequences of coronavirus (Haasbroek, 2020). McKay (2020) argues that the lack of diverse income in small mining companies which were not on par with large companies that produced commodities namely, coal, platinum, and iron ore had an upper hand in remaining active during the lockdown. Even so, small-cap mining companies and other small-cap companies' employees benefited from the Temporary Employee Relief Scheme and those that were infected by the virus at their workplace were paid through the Workmen's Compensation Fund (Botha, 2020). This was to ensure that companies remained active and cautious while conducting their respective daily work activities to continuously contribute to the struggling economy. Similarly, Figure 5 captured share price shocks by GARCH (1,1) volatility and it was evaluated as follows.

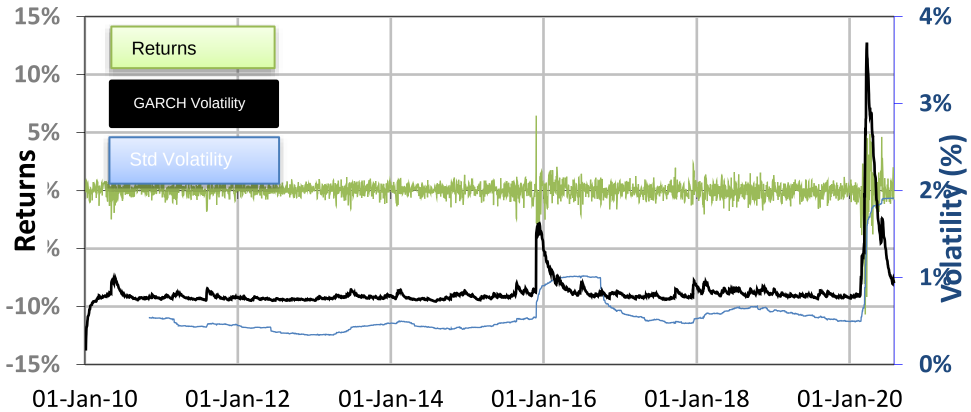

Figure 5. Comparison of the General Autoregressive Conditional Heteroskedasticity (1,1) and the Historical Standard Deviation for small-cap shares

The historical standard deviation volatility displays a starting sampling period of 1 November 2010 similarly to the analysis on EWMA volatility. The date was of the highest share price of 12 per cent recorded at the beginning of the lost decade of the JSE. The beginning calculation of the historical standard deviation volatility small-cap share return was at 0.54 per cent which results in a negative percentage of 0,06 of average returns. Generalising EWMA using the GARCH (1,1) model whereby exponentially declining weights of share prices are analysed, implying, more weight to recent shares are assigned and lesser weights are assigned to distant weights. This phenomenon allowed to abide by GARCH (1,1) that the next day's level of share price volatility was based on the condition of the most recent volatility (Szylar, 2010).

GARCH (1,1) showcased two important features from Figure 5. Firstly, volatility was none constant due to its fluctuations throughout the sampling period. This also applied to the historical standard deviation volatility. Secondly, the next day's volatility is a regressed function of today's volatility (Williams, 2011). In essence, this principle followed volatility clustering as GARCH (1,1) had moderately been constant throughout the sampling period with a few major shocks on 5 January 2010, 11 December 2015, and 23 March 2020. The share price shocks were that of the lowest volatility, moderately high volatility, and highest volatility recorded respectively. The lowest volatility in the sampling period under GARCH (1,1) was a negative 0.16 per cent.

The low volatility is the result of the 2008/09 financial crisis which had negatively impacted all sectors of the economy. This resulted in an overall GDP of 2009 recorded at negative 1.5 per cent (Plecher, 2020). However, the model captured the rise in share price volatility during the period of economic recovery as the share price volatility was at 0.98 per cent in the second quarter of 2010 captured on 26 May 2010. The tertiary sector had a major recovery of 4.5 per cent meanwhile the primary and secondary sectors contributed negative 13.2 per cent and 3.7 per cent respectively (StatsSA, 2010). Most small-cap companies operated within the tertiary sector making provision on personal services, finance, and trade (Bergman, 2019). GARCH (1,1) had its highest volatility on the same date as in the EWMA volatility at 11 December 2015 and 23 March 2020.

The moderately high volatility of the GARCH (1,1) model was at 1.57 per cent which was less by 0.42 per cent volatility in share prices using the EWMA model. Hogg (2016) argued that the repercussions of the firing of the former minister Nhlanhla Nene and bringing in an unknown figure to the public Des van Rooyen resembled a heightened political risk to the economy. Hogg (2016) further elaborated that losses stemmed from the sale of South African bonds, and sensitive rand currency with interest-rate sensitive shares were at R500 billion within two days of the appointment of Minister Des van Rooyen. On the other hand, the highest volatility of the GARCH (1,1) model as captured during the extreme event of COVID-19 was at 3.51 per cent which was less by 1.36 per cent volatility in share prices when using the EWMA model. Extreme events as that of COVID-19, the financial crisis and heightened political risk, had a commonality which was to leave a trace of an unstable economy whenever they do occur. Figure 6 examined such events in detail.

Figure 6. Extreme Events for Small-Cap Shares.

The extreme value is based on the theory that within a probability distribution, and extreme value is the probability of events that are more extreme in comparison to previously observed events (Odening & Hinrichs, 2003). These events have a low probability of occurring however when they do occur, they contribute a significant part towards the distraction of the economy (Trapin, 2016). As such, they leave behind a trail of a destabilised economy that will either take a long period to recover or may not be as functional as before any of the events. The small-cap shares from 2007 to 2020 showcases three instances whereby the companies in this category were hard-hit. Firstly, the 2007/08 financial crisis was concentrated in the financial sector in which the South African economy declined by 1.5 per cent (Maeko & Mathe, 2020). Small-cap shares declined from R372 on 31 October 2007 to R197 on 10 March 2009. Secondly, the ousting of the former minister of Finance Mr Nhlanhla Nene, the rand declined to a record low against the dollar. Oyedele (2015) deduces that the rand declined by three per cent to as low as R15.0563 against the dollar. Small-cap shares declined from R624 on 20 May 2015 to R483 on 21 January 2016. Lastly, the COVID-19 pandemic had caused financial stress in all sectors as the lockdown cost the economy about R13 billion a day (CNBC Africa, 2020).

Value at Risk denoted as VaR as a measure of market risk, measures the worst loss that might be expected from holding security over a while, with a certain probability (Fernandez, 2003). It's used for internal risk control taking into account a 95 per cent confidence level. Meanwhile, the likely behaviour of abnormal large share price losses is quantified using the extreme value theory (Singh *et al*, 2011). Extreme values of small-cap shares amounted to 0.97 per cent and 1.12 per cent which both accounted for the period 02 January 2007 to 31 July 2020. The difference between the two values lied in the manner in which 1.12 per cent was determined using the standard normal distribution, using the standard deviation of a supplied set of share prices returns ranked from lowest to highest. On the other hand, a 95 per cent confidence interval implied a 95 per cent certainty that the accurate extreme value was 0.97 per cent. Figures 7 and 9 were based on the volatility of small-cap shares using EWMA and GARCH (1,1). They illustrated a pattern in these unlikely extreme events, financial risks and the negative relationship between volatility and share price returns.

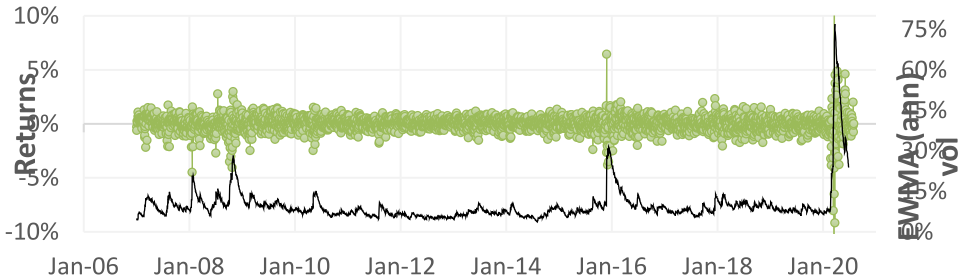

Figure 7. Extreme Values Using the Exponentially Weighted Moving Average Model for Small-Cap Shares

EWMA daily volatility was annualised as there are 250 trading days annually. The period 2 January 2007 to 31 July 2020 was chosen to emphasize the extreme value theory. Meanwhile, the share price return throughout the depicted period averaged negative 0.13 per cent. The small-cap share prices and the share price returns in three extreme events shared a similar trait. During the financial crisis of 2008/09, the share price declined to R196 with a share price return of negative 0.85 per cent on 10 March 2009 from a high of R373 with a share price return of 0.29 per cent on 7 November 2007. On the other hand, the outbreak of COVID-19 occurred in South Africa on 5 March 2020 due to a patient who was diagnosed with the virus in Italy. When the news was made public, they caused investor panic and raised an alarm to citizens amid a spread of the virus (Mkhize, 2020). Uncertainty in the small-cap financial markets led to the share price return on 23 March 2020 at negative 9.17 per cent. This followed after the South African president Mr Cyril Ramaphosa announced the lockdown for 21 days because of a widespread of the virus which on the day of the announcement, there were more than 400 cases (Kiewit *et al*, 2020). The lockdown was in effect from midnight on 26 March 2020 to 16 April 2020 and illustrated in Figure 8 (Burke, 2020). Following all three extreme events, EWMA volatility reacted in the opposing view of the share price and share price return movement. This is due to the negative relation between share price return and volatility which was discussed in Figure 3. The annualised volatilities on 30 October 2008, 11 December 2015, and 23 March 2020 were at 28.3 per cent, 31.4 per cent, and 77 per cent respectively when factoring 250 trading days in a year.

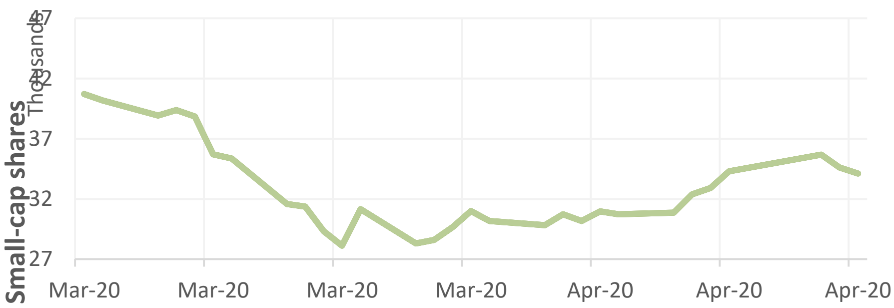

Figure 8. Small-Cap Share Movement during the 21-day Lockdown in South Africa

Before the announcement of the 21-day lockdown following the first case of COVID-19 that was reported on 5 March 2020, share prices declined from 407 to 281 rands respectively on 19 March 2020 which was the lowest share price in the period under analysis. The decline was a result of global unrest of rising cases of the virus and the World Health Organisation elaborating that the virus is a pandemic (World Health Organisation, 2020). After the announcement of the lockdown, share prices averaged 312 rands from 23 March 2020 to the end of the 21-day lockdown on 16 April 2020. Days leading up to the official commencement of the lockdown, agitated consumers rushed to supermarkets and panic bought needed items (Koko, 2020). Equally so, a 30 per cent decline in global equity markets whereby the

FTSE/JSE All Share Index was down by 27 per cent year to date in rands which led to investor panic (Lapping, 2020).

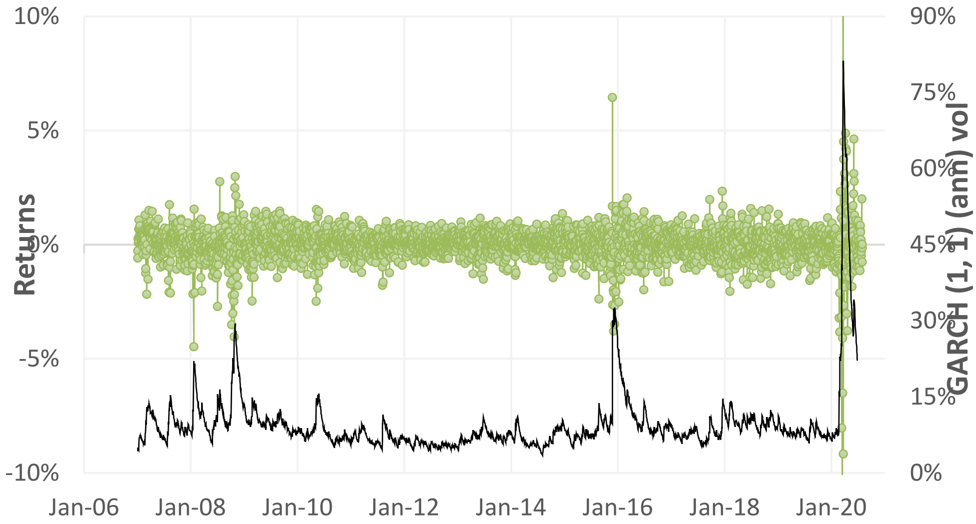

Figure 9. Extreme Values using the General Autoregressive Conditional Heteroskedasticity (1,1) for small-cap shares

GARCH (1,1) daily volatility was annualised since there are 250 trading days each year. Both models made use of the same small-cap share price and the period from 2 January 2007 to 31 July 2020. The average share price throughout the period was negative at 0.13 per cent. Similarly, GARCH (1,1) volatility reacted contrary to the share price and share price movement. The annualised volatility on the following dates, 30 October 2008, 11 December 2015, and 23 March 2020 was at 22.9 per cent, 24.9 per cent, and 55.6 per cent respectively. The stated annualised volatilities were influenced by the weighting components that made up the model. The three weightings were still applicable which had contributed a distinction with the exponential weighted moving average. Beta constituted the value of 0.931. By adjusting 0.931, the weighting was not equal to one as gamma and alpha remained at constants of 0.0345.

At this point, small-cap shares had been reported about the inverse relationship between volatility and share price returns as illustrated in Figures 4, 5, 7, and 9. The ending of the sampling period being 31 July 2020 is of importance in determining the optimal lambda at that date for a sample of small-cap shares provided extreme events that had occurred and the impact they had on the optimal lambda itself which followed by a one-day volatility forecast at 0.51 and 0.79 using EWMA and GARCH (1,1) respectively.

5. Conclusion

The study aimed to depict the preferable model to forecast volatility between EWMA and GARCH (1,1) in small-cap shares using the small-cap share index provided by the JSE named J202 and determine the optimal lambda at the end of the reporting period. In the period for analysis – 01 January 2010 to 31 July 2020, the EWMA model factored in an important element, a decay factor for daily data at 0.931.

Firstly, the value indicated its relevance to the model as share price weights got adjusted when an adjustment was applied to it. When decreasing the value 0.931, share price returns that dated to an earlier period received less weight while those at dates that were approaching the latest i.e. 31 July 2020, received more weight. A similar approach was applied to reducing the value of 0.931. The inclusion of this value, not only was it at the forefront of affecting the adjustment of share price return weights but most importantly, whenever it was adjusted, so was the volatility graph affected. However, it should be noted that in the models' impact by the value, the characteristic of small-cap shares being more volatile was abided to. Moreover, irrespective of the adjustment of 0.931, the EWMA volatility model captured events that were extreme posing large risks for investors in the index.

Secondly, GARCH (1,1) model had an important feature – persistence. By persistence, implying that the model applied three weights namely, gamma, alpha, and beta, in constructing the volatility of small-cap share prices, the model's long-run variance meant a quick reversion to the mean. The model readjusted quicker after the share price shock during extreme events recorded in the period. If the long-run variance had a value closer to one then, the model would have captured share price shocks at a slower rate. A low persistence was more preferred than a high persistence (which would have led to no mean reversion). This quick readjustment was well illustrated after the volatility drastically increased on 7 August 2010 – poor performance in the secondary sector of the economy, 11 December 2015 – heightened political risk and 23 March 2020 – announcement of 21-day lockdown as a result of an outbreak of COVID-19. Both volatilities indicated a relationship between share price return and volatility. On that basis, low volatility implied a high share price return. However, an analysis based solely on that relationship, the general autoregressive conditional heteroskedasticity would have been preferred. Yet, there were instances when EWMA volatility was low and the share price return was high.

Thirdly, the study would have been incomplete without the inclusion of the historical standard deviation as it remained at the core of the analysis. The historical standard deviation as a measure of risk during the share price shock had high volatility which implied that the higher the standard deviation, the riskier the investment in small-cap shares. It wasn't tracking a similar movement of volatility as the two models did. This variation was based on the manner that both models took the optimal lambda into account as it was used as a smoothing parameter. Despite this, the measure of risk was embedded in the analysis of market volatility as the risk in the premise of investing was not viewed negatively as the riskier a security was, the greater potential it had for expected financial returns. Furthermore, the calculated optimal lambda indicated the higher weighting applied to present day's share price returns and lower weighting to share price returns which dated earlier in the reporting period while a notable instability in the market led to high volatility as indicated in the outbreak and impact of COVID-19.

Finally, the decision on which model was preferable was based on an individual investor's preference as both models have shown that they track relatively similar volatility of share price returns and whenever there was an extreme event – financial crisis, heightened political risk, and the outbreak of the coronavirus, they thoroughly captured and recorded such events. Moreover, it was advisable to make use of both models simultaneously allowing a broader view of the share price return volatility forecast when considering investing in small-cap shares.

Recommendations

As the study was focused mainly on forecasting volatility after day n-1, it is recommended that more emphasis be on forecasting volatility and determine the optimal lambda using monthly data and the mispricing of small-cap shares amid COVID-19.

Limitations

The limitation of volatility forecasting was based on the models' calculations with a minor influence of macroeconomic factors which contribute a significant role in fundamental analysis.

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