DIDACTICA DANUBIENSIS

Educational Management

The Rainbow between the GeoGebra Platform and the New Innovative Digital Education Era

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Abstract: The development of information technology has radically changed our way of living, communicating with others, and transferring information. In the future, the role of computers in our lives is expected to become essential. Every student will need to master this field to easily achieve their goals. In a new paradigm of learning, one strategy could involve revisiting select learning tasks proposed and resolved some time ago and reevaluating them using new platforms or dedicated software. The steps of approaching an old geometry problem could be as follows: “Draw, construct, analyze properties, experiment with different shapes and sizes, and note that the properties remain constant. Can you formulate a theorem based on this investigation? Finally, rigorously prove it!” The experience should not only be lived but also shared. Through such actions, any teaching-learning paradigm can evolve to a new and higher stage. To be useful in this technology-driven teaching-learning environment, any software must be dynamic and continually evolve based on daily use. GeoGebra is an excellent example of this kind of software-platform, connecting people and their educational actions with the development of new learning technologies.

Keywords: GeoGebra; educational; platform; innovation; digital didactics

1. Introduction

After more than a decade of its real development, the GeoGebra platform has embraced the paradigm of 2010, described at the Linz GeoGebra Gathering as “The GeoGebra software, the language of the third Millennium”. Since that time until today, the educational platform and the associated community have transformed the software into an accepted working method for exchanging old,

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present, and future ideas in science education. This paper aims to consider and review the most important transformations supported by the Romanian curriculum domain. It will present these transformations in chronological order, as reported by the first local GeoGebra Institute established in Romania in 2009 until the present day. The paper will not just provide a chronological list of actions but will review specific projects that serve as examples of the timely transformation of digital education in various teaching-learning domains and curriculum levels.

2. First stage-GeoGebra Software as an Instrument for Teaching-Learning Geometry

2.1. The Start without Stops of the International GI Platform

Upon its invention by Markus Hohenwarter, the creator envisioned the software's future widespread use across the globe. Consequently, a discussion forum for the community was initiated to facilitate collaboration and relationships among users (Hohenwarter & Lavicza, 2010). This forum's perspective was initially unclear, but it evolved and transformed over the past fifteen years, positively responding to the advancements such as YouTube channels, augmented reality, socialization platforms like Facebook, Twitter, TikTok, and even new artificial intelligence. Presently, we can observe the real extension of the GI Factor in the community. With the optimal dynamics of informational flow and the use of appropriate "catalyzers" for self-multiplying structures, the local GeoGebra Institutes have progressed from latency to development, fueled by an unexpected informational flow (Valerian Antohe, 2010).

2.2. The Connection between Geometry and Algebraic Representation Proper to Analytical Geometry

When the GeoGebra software was designed, it was prepared to handle both geometric figures in the context of Euclidean geometry and their algebraic representation in analytical geometry. These two stages work together and accept inputs either in the visual form through mouse actions or through the software's command line. For example, a geometric point A in the plane can be created on the screen using mouse commands, or alternatively, by inputting the command <A=(2,3)> in the command line. Similarly, lines can be created by selecting two points, A(2,3) and B(5,2), using the mouse, or by utilizing the algebraic representation by entering the command <Line(A,B)> in the command line. The program will then return the line equation "x+3y=11" (Figure 1).

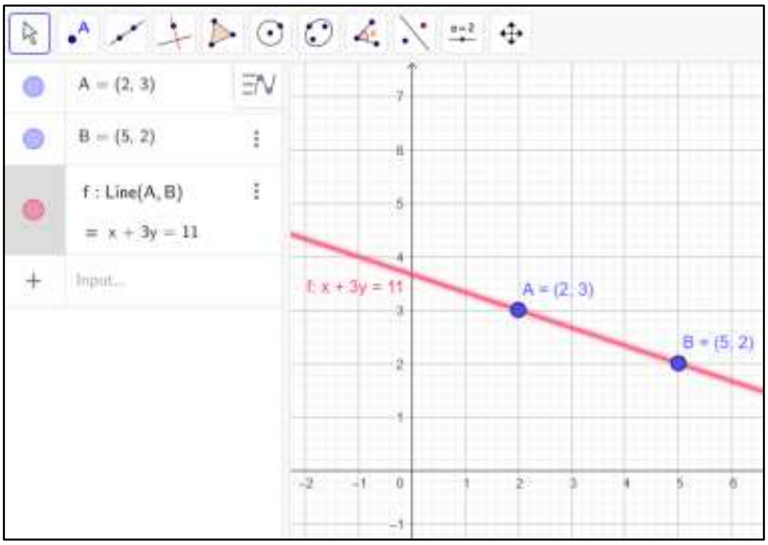

Figure 1. Geometric Figures, Points, and Line in one GeoGebra Project

The same action can be considered for other simple figures as circles, ellipses, or hyperbolas but at the same time for complex figures for that only the algebraic analytical representation it is known. Plane curves are exemplified as it follows in the next figure, (Figure2).

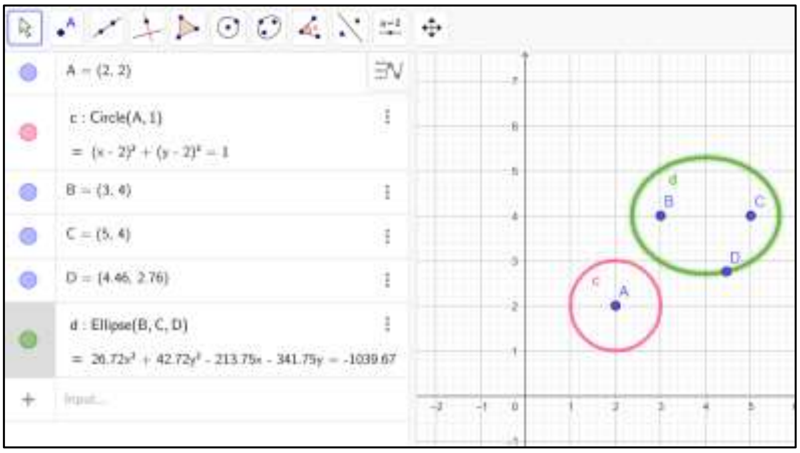

Figure 2. Geometric Figures, Points, and Line in one GeoGebra Project

2.3. The Transformation from the Elementary Geometric Locus Problems to the Augmented Reality

The mathematical software GeoGebra dynamically explores and investigates various geometric problems, combining algebra, geometry, and dynamic representations of geometric tasks. These tools can be used to delve into more complex mathematical results. The initial focus was on points, lines, triangles, and polygons represented in the Euclidean Plane. By understanding the relationship between Euclidean construction and proof, demonstrations can be created using animations and dedicated action buttons within the GeoGebra workspace. For instance, let's consider a complex task that stems from one International Olympiad of Mathematics. While it may seem intricate, a visual demonstration can be devised and agreed upon by many students, even those not involved in the competition. The statement of the problem is as follows: "On the fixed line (d), fixed points A and B in the plane, and a mobile point M are given. Regular polygons with [AM] and [AN] sides are constructed in the plane, where m and n are the numbers of sides, and m, n $\in$ N, m, n $\geq$ 3. The circumscribed circles of these polygons intersect at points M and P. The geometric locus of P is required to be discovered" (A. Dafina, 2003). The initial investigation for m=5 and n=4 is presented in the first figure (Figure 3). After exploring this problem using GeoGebra software, various intriguing results may emerge.

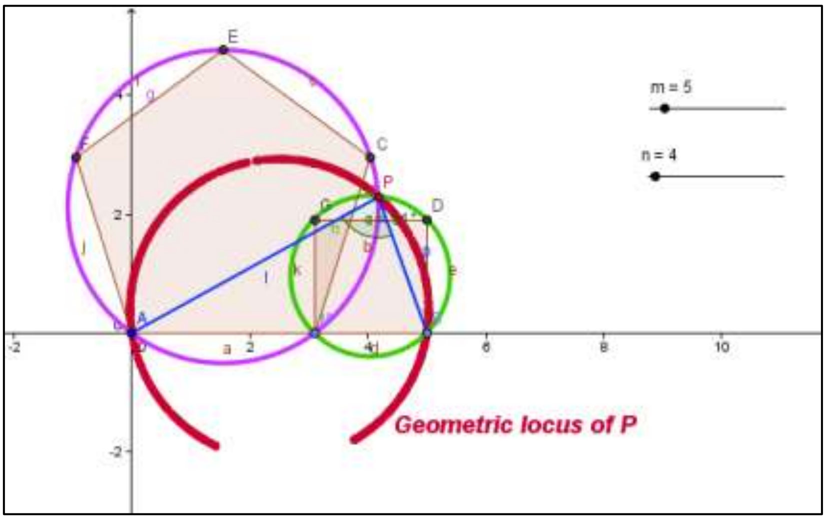

Figure 3. Geometric locus of P for m=5 and n=4

The GeoGebra application displays all changes in real-time and allows the manipulation of variables "m" and "n" using sliders (Figure 4). This example demonstrates how the software can be utilized to solve complex mathematical tasks visually. While the intuitive interpretation of the problem initiates the reasoning, a rigorous demonstration can be constructed after the GeoGebra model is projected (Figure 4). The geometric locus has a visual solution depicted in a Cartesian Plane context, and it relates to the algebraic calculus, automatically solved in the background by the software.

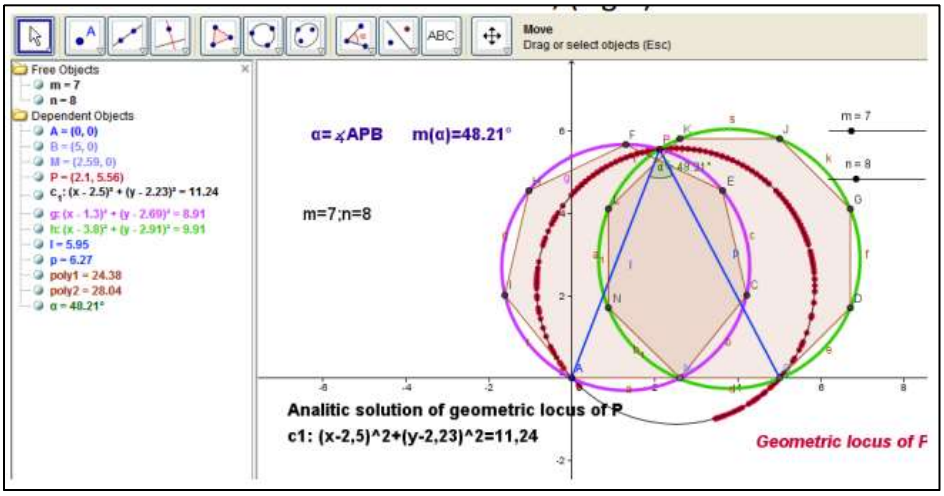

Figure 4. Geometric Locus of P in GeoGebra Project

2.4. The 3-D Corps Represented with GeoGebra Software

Any 3-D structure can be introduced through a command line using algebraic expressions of two variables, and the corresponding visual representation will be generated. For instance, the expression $A(x,y) = (x y^2 - 3x + 2xy - 2x + y^3) / (x + 2y)$ represents a geometric surface (Figure 5). While this example involves a structure relevant to calculus, the software's library of examples, contributed by more than one million users, offers access to typical 3-D structures such as spheres, cylinders, cones, and other space bodies. These examples can be discovered through keywords and explored on the platform, as they have been used by others in the teaching-learning processes. "GeoGebra is a community of millions of users located in just about every country. It has become the leading provider of open sources dynamic mathematics software, supporting science, technology, engineering, and mathematics (STEM) education and innovations in teaching-learning worldwide. GeoGebra's math engine powers hundreds of educational websites worldwide, from simple demonstrations to full online assessment systems," as declared on the GI home site[1].

[1] About GeoGebra: www.geogebra.org/about.

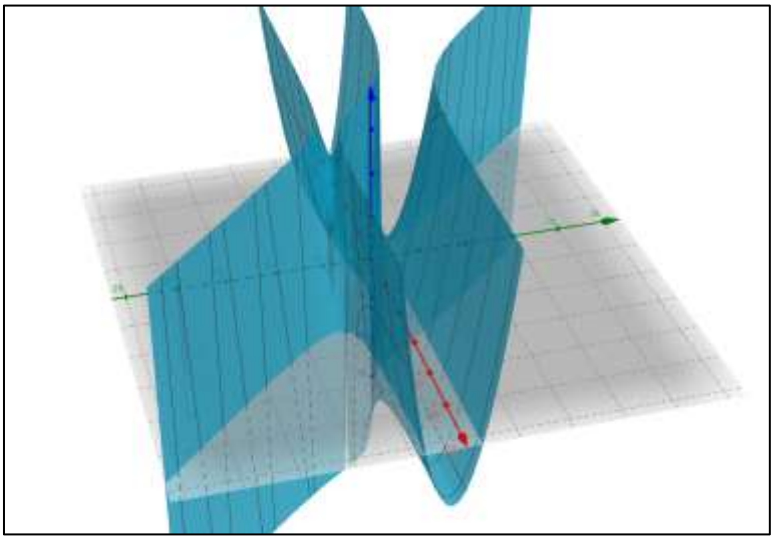

Figure 5. Geometric 3-D Structure in GeoGebra Project

GeoGebra is now a crucial component of the learning platforms for hundreds of millions of students. Its apps, classroom resources, GeoGebra Classroom, and other features remain accessible to the public for free. For instance, let's consider the 3-D representation of a solid in a GeoGebra project presented by Anthony OR[1] from Hong Kong Institute of Educational Research, a GeoGebra Ambassador for 2022-2023.

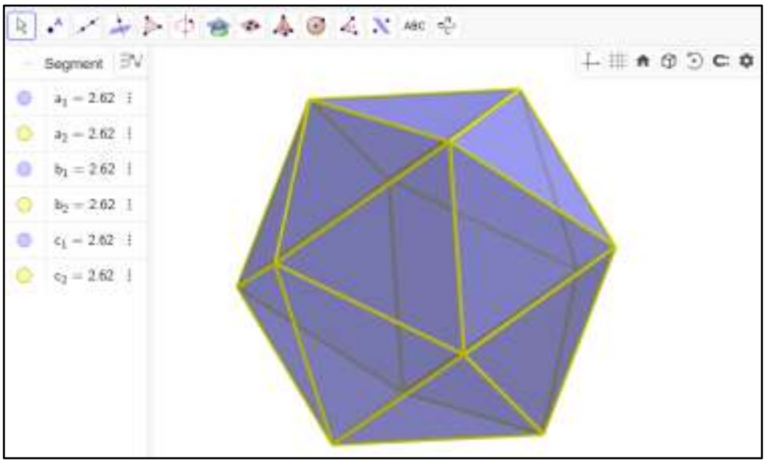

Figure 6. Geometric 3-D Structure in GeoGebra Project

[1] Site GeoGebra of Anthony OR: www.geogebra.org/u/orchiming.

The possibility of constructing GeoGebra projects for later use in the teaching and learning processes covers a wide range of themes and domains, all connected with STEAM education (Science, Technology, Engineering, Arts, and Mathematics) and dynamic structures that involve models and modeling. STEAM education places a focus on these five key areas while encouraging more engaging and integrated teaching in these fields. The inclusion of the 'A' for the arts sets STEAM education apart from the earlier concept of STEM education[1]. Today, the Art of STEM education could not exist without GeoGebra. Hence, the programs can cover various domains, such as mathematics, engineering, arts, statistics, time series, calculus, and the visualization of probabilities for predicting phenomena, economic structures, or tasks involving linear programming.

3. Second Stage-GeoGebra Platform used Around the World Became an Instrument of Teaching Science.

Observing the GI Platform and the resources developed globally, it becomes evident that the presence of geometry in the Romanian curriculum differs from other parts of the world. In the past decade, the study of geometrical forms and 2-D orientation in the plane, as well as 3-D orientation in space for secondary school students and those in engineering fields of study, has become more challenging theoretically and in practical applications. The distinction between studying fundamental theoretical disciplines and applying them in practical tasks has grown, with theoretical structures taking precedence over visual interpretations of studied concepts. Romanian educators' efforts are evident in satellite projects like the common GeoGebra space for all files of the e-twinning project "Math, Art, and Real Life with GeoGebra" initiated by Maria[2]. These initiatives are not yet officially recognized by the Ministry of Education or mandated as a discipline in the curriculum.

[1] Site www.viewsonic.com about the importance of STEM.
[2] E-Twinning project: www.geogebra.org/m/kTDzYqG5.

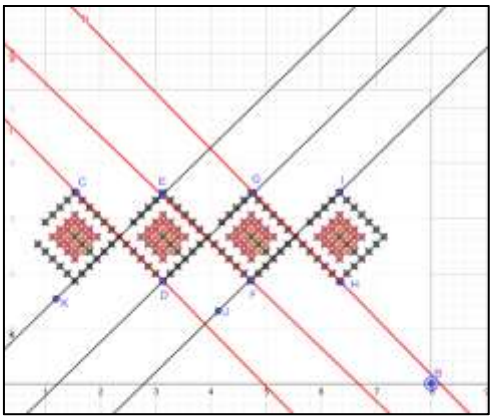

Figure 7. Motives in Traditional Romanian art – Maria in GeoGebra Project

Many objectives have been pursued in recent years to incorporate the software and platform into the official curriculum in Romania. However, until that happens, various applications created by individual educators have grown and are being used in secondary schools and universities. Looking ahead, Europe is expected to have seven million job openings in the entire STEM field by 2025, with significant growth in the aerospace industry. Despite the abundant career opportunities, the aerospace sector faces a substantial occupation shortage, necessitating comprehensive actions by all stakeholders, as acknowledged in the study by WIA-EUROPE Space Business Innovation Centre (SBIC) in the Netherlands[1]. In line with this aspiration, a simple GeoGebra project could be proposed to teach future engineering specialists about 3-D space orientation. Points in the context of three Cartesian coordinates in R^3 space can be introduced through the command line, for example, <A=(2,0,0)> and <B=(2,2,0)>. The segments that connect two points in space can also be represented in a perspective design or projections using the command line <Segment(A,B)> (Figure 8). This project will help students gain a better understanding of 3-D Cartesian axes and their orientation in space.

[1] STEM education in Europe - www.wia-europe.org The Netherland.

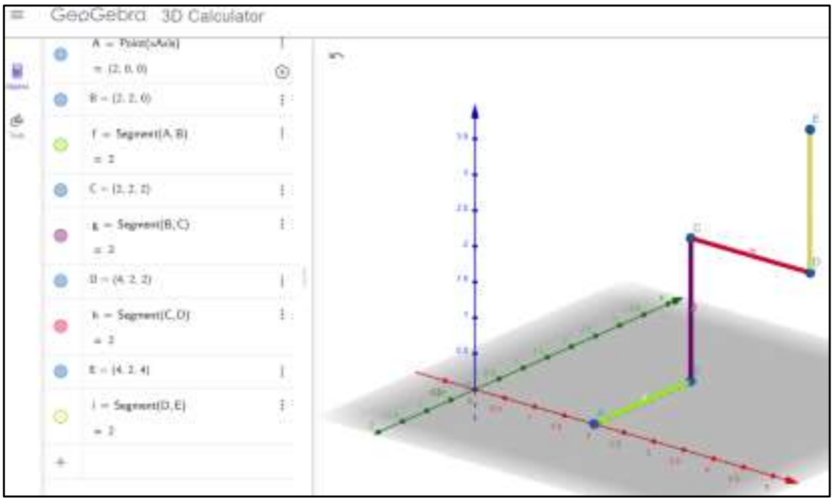

Figure 8. 3-D Space Euclidean Orientation in the GeoGebra Project

4. Third Stage - GeoGebra Including and Being Included in Socialization Platforms and Media Channels

School mathematics curricula worldwide give significant importance to both geometry and algebra, particularly at the secondary school level. Some countries offer entirely separate courses for geometry and algebra, while others alternate topics from various components of mathematics.

The emergence of GeoGebra as an increasingly popular mathematics education resource can be attributed to several key characteristics, such as its open-source status, multiple language translations, versatility (supporting both algebraic and geometric representations), and an online wiki and user forum. Authors have been providing a brief history and description of the GeoGebra software on various occasions while discussing the nascent International GeoGebra Institute (IGI) and its vision, along with related international developments periodically (Markus Hohenwarter & All, 2009). Although this speech about GeoGebra software was written in 2009, it remains relevant today. The mode of communication between people has evolved, becoming more specific and friendlier, evident in YouTube channels and visually engaging Facebook sites. Dedicated channels of communication have incorporated all present and prospective software developments to keep pace with the ever-changing flow of information and the new "multilevel marketing in science." Environmental policies aim to preserve a clean environment for future generations, yet the human component of a clean environment is often overlooked. Similarly, little attention is given to approaches centred on a clean informational environment that can positively impact human nature. The cleanliness or degradation of this environment, as well as control over it, may be designated as the ecology of the IT surrounding environment (ITSE) (Valerian Antohe, 2010). This concept, predicted in 2010, remains more relevant than ever today. One of the most important aspects of GeoGebra Platform it is that all the accepted materials are conform with this rule of a friendly and clean IT environment. Others different sites that utilize GeoGebra's rules and visual solutions includes could be exemplified and here is one of these, the "Photomath platform," where contributors use visual solutions projected on GeoGebra with visual instructions included in Photomath (Figure 9). Various educational blogs by educators cover not only mathematics but also subjects like Physics, Chemistry, Geography, Music, or Piano models using GeoGebra. By using GeoGebra on smartphones, educators can have access to a comprehensive science library in a visual and interactive format, which can be used in the classroom or anywhere they teach.

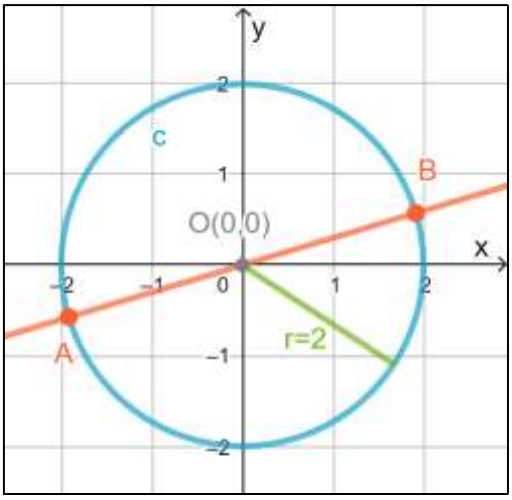

Figure 9. Photomath Visual Solution Task Realised as a GeoGebra Project.

Generally, GeoGebra offers several advantages in the fields of algebra, calculus, physics, and linear programming, among many other fields and subfields. It has continuously helped improve the performance, capabilities, and understanding of students due to its features, which allow learners to practice and visualize rather dense and complex topics. Currently, GeoGebra has a very high potential for this generation of learners as well as their learning and mental processes. Integrating

GeoGebra into the current curriculum and teaching approaches has proven highly effective in various studies in different mathematical and STEM-oriented fields (Ziatdinov, R.; Valles, J.R., Jr., 2022).

5. Perspectives in New Digital Era

The initiation of the GeoGebra Educational Platform in 2010 marked a crucial moment. It paved the way for virtual education and gave rise to phrases like "Let's learn together with the aid of the computer!" "Give me the model of that structure and rotate it in any direction on the screen!" "Open the virtual classroom to participate in this research project!" and more. The present digital era has no boundaries for the numerous actors involved, not only in searching for information but also in sharing their own developed projects with people worldwide. The GeoGebra educational platform forms a dynamic triangle, where learning involves interactions between the teacher, the student, and the computer, diverging from the traditional linear communication, such as Alexander Graham Bell's invention, the phone, where communicators were confined to a line segment. Today, this structure becomes an infinite polyhedron with interconnected vertices representing educators and learners. Information circulates freely within this structure, flowing across teachers and scientists located at various points. This development contrasts the new ChatGPT-OpenAI, which relies on written information on the web, while GeoGebra's platform fosters real-time connections and interactions among its participants.

In the context of science education, students can effectively use programs from the platform to successfully tackle linear programming problems, obtain solutions to linear systems, visualize maximum or minimum points of functions, perform linear regression, construct spatial structures, and solve ordinary differential equations (ODEs). The platform serves as a valuable tool for decision-making problems in economic technology.

Participation in the educational processes spans various levels, from pre-primary schools to academic stages, where educators, regardless of their IT expertise, can explore the platform's structures, use relevant keywords, and courageously accept contributions from others, knowing that their own projects may be used by others tomorrow.

6. Conclusion

Educational software alone does not encompass the entire universe of learning. The submergence of the 2D or 3D Euclidean Space, isomorphic with R^2 and R^3 in a Q^2 or Q^3 space, represented by the computer screen using rational coordinate points, may lead to erroneous interpretations of certain geometric figures or graphs of real function elements. This could be an inconvenient for any software that have a visual imagine via computer screen, Thus, messages about potential errors must be displayed on the screen, maybe in the future. Although the software may not currently address this issue, educators play a pivotal role in guiding students to understand mathematical concepts beyond the computer screen. The student should engage in mathematical analysis with pen and paper, employing differential calculus or real function analysis to perceive the real context. GeoGebra offers infinite possibilities for the development and application of mathematics. Its growth and potential to address real problems in mathematics, physics, chemistry, biology, and other science, in general, are boundless. The software's development relies on a collective set of ideas, and teachers of science should be at the forefront, using the software freely and creatively.

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