An Application of The Markowitz's Mean-Variance Framework in Constructing Optimal Portfolios using the Johannesburg Securities Exchange Tradeable Indices

Sally Huni[1], Athenia Bongani Sibindi[2]

Abstract: The aim of this study was to assess the feasibility of constructing optimal portfolios using the Johannesburg Securities Exchange (JSE) tradable sector indices. Three indices were employed, namely Financials, Industrials and Resources and these were benchmarked against the JSE All Share Index for the period January 2007 to December 2017. The period was split into three, namely before the 2007-2009 global financial crises, during the global financial crises and after the global financial crises. The Markowitz's mean-variance optimisation framework was employed for the construction of global mean variance portfolios. The results of this study demonstrated that it was feasible to construct mean-variance efficient portfolios using the tradable sector indices from the JSE. It was also established that, on the other hand, global mean variance portfolios constructed in this study, outperformed the benchmark index in a bullish market in terms of the risk-return combinations. On the other hand, in bear markets, the global mean variance portfolios were observed to perform better than the benchmark index in terms of risk. Further, the results of the study showed that portfolios constructed from the three tradable indices yielded diversification benefits despite their positive correlation with each other. The results of the study corroborate the findings by other scholars that the mean-variance optimisation framework is effective in the construction of optimal portfolios using the Johannesburg Securities Exchange. The study also demonstrated that Markowitz's mean-variance framework could be applied by investors faced with a plethora of investment constraints and choices to construct efficient portfolios utilising the JSE tradable sector indices in order to realise returns commensurate with their risk preferences.

Keywords: Optimal portfolios; Mean-variance optimisation; Markowitz; Johannesburg Securities Exchange; return

JEL Classification: G01; G110

1. Introduction

Portfolio optimisation is one of the most important considerations and has been a mainstay in finance (Clarke, De Silva & Thorley, 2006, p. 10). The concept was originally articulated by Markowitz (1952) and it addresses the problem of investment choice, which derives to asset allocation and portfolio construction (Markowitz, 1952, p. 77). An efficient portfolio is a structured collection of financial vehicles held by an individual or group of investors with the aim of maximising returns corresponding to certain levels of risk (Popina & Martyniuk, 2016, p. 160). The portfolio may include a pool of investment tools such as shares, obligations, savings certificates, bonds and gold, practically any asset traded for determination of future returns. Constructing an efficient portfolio is practically balancing a basket of risky securities with an admissible level of volatility and a desirable level of risk

[1] PhD Student, Department of Finance, Risk Management and Banking, University of South Africa, South Africa, Address: P.O. Box 392 UNISA 0003, South Africa, E-mail: salyhuni@yahoo.com.

[2] Associate Professor, PhD, University of South Africa, South Africa, Address: P.O. Box 392 UNISA 0003, South Africa, Tel.: +27124293757, Corresponding author email: sibinab@unisa.ac.za.

(Popina & Martyniuk, 2016, p. 160).

The volatility of financial markets in both developed and developing economies is a major concern to stock market investors, scholars and finance field practitioners. The South African market is not immune to such volatilities. The South Africa market is highly integrated with other global markets and is influenced by micro and macro-economic factors such as inflation, inflation expectations, oil prices, exchange rates and real activities (Szczygielski & Chipeta, 2015, p. 49). Such financial market integration, globalisation and technology advances have increased the importance of portfolio risk management.

With the high volatility of the South African market as measured by the South African Volatility Index (SAVI) for the period 2007 to 2016 ranged from as low as 12 to 60 (Cairns, 2016, p. 1), efficient portfolio optimisation becomes a critical subject for both investors and fund managers. Portfolios constructed based on the underpinnings of the Modern Portfolio Theory (MPT) have been found to maintain stability in risk minimisation and perform better than their benchmarks internationally (Giri, 2016, p. 84). However, with a vast number of securities trading daily on the JSE, it becomes a challenging exercise for investors to make choices and effectively pick stocks that could yield expected maximum returns for their desired levels of risk. As the MPT model have been proven to be an effective tool for stock selection and portfolio construction, it is ideal to investigate its strength using the South African tradable sector indices. To the best knowledge of the authors, very limited studies have focused on sector index portfolios construction based on MPT on the JSE market. Most studies that employed the MPT on the JSE focused on the JSE ALSI, mutual funds, ETFs, and individual assets; with little attention being paid to the tradable sector indices.

With the progression of the markets driven by technology and changing investor sentiments, it would be ideal to investigate the traditional MPT based on the mean-variance optimisation (MVO) technique to determine the feasibility of its application on the JSE. The central research question that this study sought to address was, ‘Based on the JSE tradable sector, can an investor that applies Markowitz’s mean-variance MPT achieve maximum returns?’ The objectives guiding the present study were: (1) to establish whether the JSE tradable sector index ($INDI_{25}$, $FINI_{15}$, $RESI_{10}$) represents an optimal portfolio; (2) to establish whether global mean variance portfolios could be constructed using the JSE tradable sector index ($INDI_{25}$, $FINI_{15}$, $RESI_{10}$) and (3) to determine whether there are any diversification benefits associated with selecting a portfolio constructed from the JSE tradable sector index ($INDI_{25}$, $FINI_{15}$, $RESI_{10}$).

The current study deviates from the previous studies in that it employs the traditional MPT model to optimise a portfolio based on the three main South African tradable sector indices. The study is also unique from others in that it covered different phases of the market cycle by analysing data for the period before, during and after the global financial crises to investigate the performance of global minimum variance portfolios (GMVPs).

The rest of the article is organised as follows: Section 2 reviews the related literature. Section 3 describes the research methodology employed in the study. Section 4 presents the empirical findings and Section 5 concludes the article.

2. Review of Related Literature

2.1. Theoretical Literature Review

The MPT is a passive portfolio management approach, constituting three portfolio theories, which are commonly based on the portfolio risk-return profile for portfolio selection and construction (Garaba, 2005, p. 9; Vukovic & Bjerknes, 2017, p. 14). The three theories are the mean-variance analysis (MVA) by Markowitz (1952), the CAPM independently developed by Treynor (1961), Sharpe (1964), Lintner (1965) and Mossin (1966) as well as the arbitrage pricing theory (APT) by Ross (1976). The portfolio optimisation ground-breaking research by Markowitz (1952), qualified him to be the father of modern portfolio theory (Darko, 2012, p. 6).

The MPT theory hypothesises on diversification benefits (Fabozzi *et al.*, 2002, p. 8). By formalising the concept of diversification, Markowitz (1952:78) proposes that investors should consider portfolios based on their collective risk - return characteristics rather than focusing on individual securities without considering how they will perform collectively as a portfolio (Brown, 2015, p. 24). The collective performance of assets in a portfolio can be estimated by utilising the historical returns of the individual assets, the standard deviation and their covariance to calculate the portfolio risk and return. Since the return and risk (mean and variance) relationship are being considered, the model was also referred to as the "mean- variance portfolio model", (Joshipura & Joshipura, 2015, p. 140). Markowitz was the first to show clearly how portfolio variance can be reduced as a result of diversification (Olsen, 2014, p. 9).

Markowitz (1952, pp. 78-80) suggests that the value of a security is best evaluated by its mean, variance, and its correlation to other securities in a portfolio. Within the infinite number of possible alternatives that an investor has to construct a portfolio, balancing the risk and return features of the portfolio can yield optimal results. Portfolio return refers to the anticipated earnings generated from the invested securities or assets (Markowitz, 1999, p. 5). On the other hand, portfolio variance (risk) is a measure of how returns of a set of securities constituting a portfolio fluctuate and deviate from the expected rate of return (Markowitz, 1999, p. 5). In other words, it is the chance of unfavourable events happening.

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Diversification is a method of reducing portfolio risk by apportioning investment among various financial instruments, industries or asset classes (Fragkiskos, 2014, p. 9-12). The level of portfolio risk can be reduced due to diversification (Markowitz, 1952:87-89). There are two types of risk, systematic and unsystematic risk (Rutterford & Sotiropoulos, 2016, p. 2). Systematic risk is macro in nature and is related to an economy as a whole, for example, interest rates and inflation (Rutterford & Sotiropoulos, 2016, p. 2). Investors cannot do anything to lessen systematic risk. Unsystematic risk, on the other hand, is firm - specific and is also known as diversifiable risk (Fragkiskos, 2014, p. 9). Diversification can be used to eliminate or reduce unsystematic risk (Markowitz, 1952, p. 89). MPT suggests that, as the number of securities in a portfolio increase, the level of portfolio risk will be decreasing (Yahaya, Abubakar & Garba, 2011, p. 102). This can be diagrammatically presented as in Figure 1.

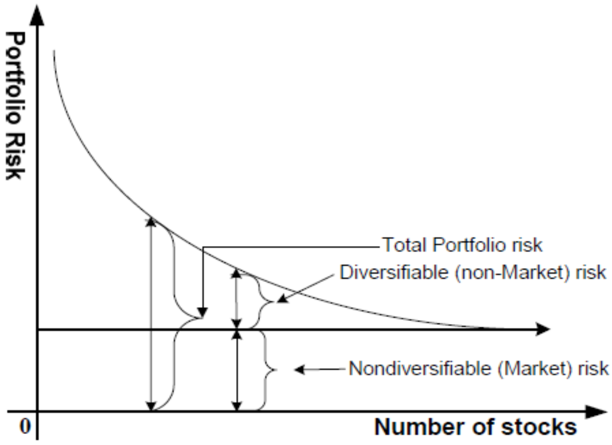

Figure 1. Systematic and Unsystematic risk

Source: Yahaya, et al. (2011, p. 104)

For diversification to be effective, the investment vehicles or securities must have a different reaction to certain market events, which Markowitz termed "the correlation". Superior diversification benefits can be obtained by selecting assets from different industries and asset classes that are uncorrelated (Popina & Martyniuk, 2016, p. 162). Different asset classes such as bonds and stocks react differently to hostile negative market events. As a result, the sensitivity of the entire portfolio will be reduced as the unpleasant movement in one asset class will be offset by a favourable movement in the other class or industry. The more uncorrelated the stocks are, the less the portfolio risk (Popina & Martyniuk, 2016, p. 162).

The law of large numbers states that an investor can diversify among many several assets at the same time maximising returns whereby the actual return of the portfolio will be almost the same as the expected return (Markowitz 1952, p. 79). In other words, the rule says there is a portfolio which gives a maximum return at the same time having a minimum variance (Markowitz, 1952, p. 79). However, Markowitz (1952) disregarded the rule based on the fact that the portfolio with the maximum return is not necessarily the one with the lowest variance. In addition, diversification does not eliminate portfolio risk exclusively since there is always systematic risk which cannot be diversified away (Markowitz, 1952, p. 79).

Markowitz (1952, pp. 78-80) suggests that the value of a security is best evaluated by its mean, variance, and its correlation to other securities in a portfolio. Within the infinite number of possible alternatives that an investor has to construct a portfolio, balancing the risk and return features of the portfolio can yield optimal results. Portfolio return refers to the anticipated earnings generated from the invested securities or assets (Markowitz, 1999, p. 5). Its computation comprises finding the weighted average return of securities included in a portfolio by multiplying individual securities by their respective weights (Kisaka *et al.*, 2015, p. 9). Brown (2012, pp. 9-10) specifies a formula for calculating portfolio returns as follows:

$E(rp) = \sum_{i=1}^{n} w_i \, E(r_i)$ Equation (1)

where:

$\sum_{i=1}^{n} w_i$	1.0;
N	the number of securities;
w_i	the proportion of the funds invested in security *i*;
r_i, r_P	the return on *i*th security and portfolio *p*; and
$E(r_p)$	the expected portfolio returns.

The portfolio variance (risk) is a measure of how returns of a set of securities constituting a portfolio fluctuate and deviate from the expected rate of return (Markowitz, 1999, p. 5). In other words, it is the chance of unfavourable events happening. To calculate the portfolio variance, standard deviations and correlations of each individual security in a portfolio are used. On the other hand, covariance is a measure of how the assets in a portfolio can move in relation to each other (Markowitz, 1959, pp. 96-101). According to Chen, Chung, Ho and Hsu (2010, p. 5), portfolio variance can be expressed as in Equation 2.

$\sigma_p^2 = (w_A \sigma_A)^2 + (w_B \sigma_B)^2 + (w_3 \sigma_3)^2 + 2w_A w_B \rho_{AB} \sigma_A \sigma_B + 2w_A w_C \rho_{AC} \sigma_A \sigma_C + 2w_B w_C \rho_{BC} \sigma_B \sigma_C$
Equation (2)

where:

$\rho_{AB}, \rho_{AC}, \rho_{BC}$ = correlation coefficient between the returns on assets

AB, AC and BC.

$\sigma_A \sigma_B \sigma_C$ = standard deviations of returns of assets A, B and C

$W_A W_B W_C$ = the weight of each asset

Using the computing power technology, the magnitude of all feasible portfolios can be derived by a critical line as depicted in Figure 2.

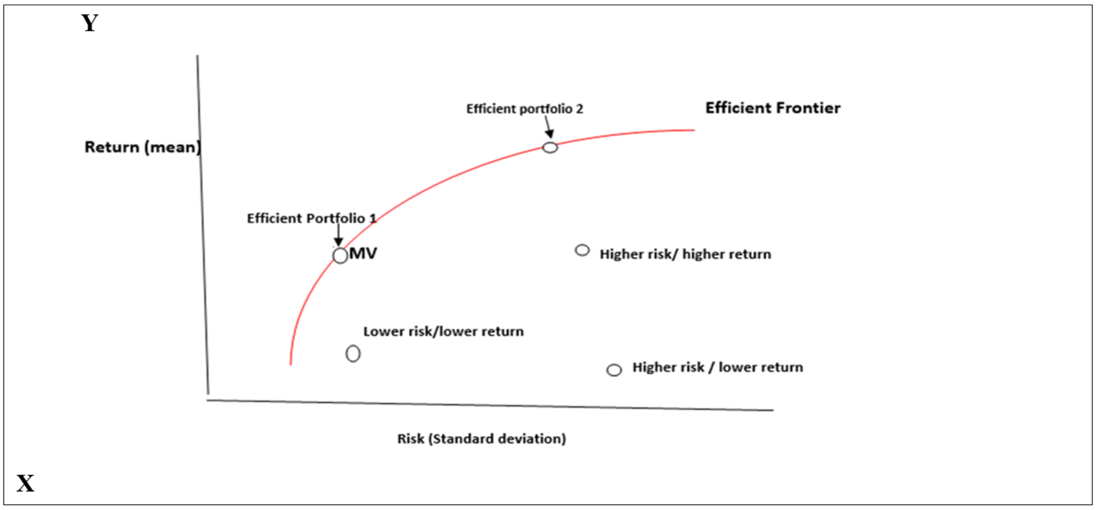

Figure 2. Minimum Variance Portfolio
Source: Ayodeji and Ingram (2015, p. 44)

The set of portfolios constructed in this optimal manner conform to what Markowitz (1952, p. 82) called the “efficient combinations frontier” (the critical line), which is a hyperbolic line that optimum portfolios lies considering the rationality of investors. The efficient frontier is a graph constructed with expected return on y-axis while the risk is on the x-axis (see Figure 2). The most efficient portfolio is one that gives the highest return for a given level of portfolio risk. Any point above the efficient frontier is unattainable and portfolios below the frontier are inefficient and would require rebalancing of asset classes for it to move closer to the efficient frontier. Markowitz concluded that an investor should select the optimum portfolio, which is the one that lies on or the one that is tangent to the efficient frontier (Engels, 2004, p. 12). For a portfolio to be tangent to the efficient frontier, there is need for portfolio weighting and balancing (Markowitz 1952, pp. 82-87). An investor cannot just pick one asset and expect it to be tangent to the efficient frontier. Several assets should be picked and combined in certain weights to form portfolios that will be examined using the model to determine whether the portfolios can be tangential to the efficient line (Markowitz 1952, pp. 82-87).

By optimising portfolios, the idea is to come up with optimal solutions corresponding to certain constraints depending on the investor’s risk - return tastes and preferences (Ayodeji & Ingram 2015, p. 43). The basic assumption is that when investors are given a set of investments with the same reward but different risks, they will choose the minimum risk asset (Rutterford & Sotiropoulos, 2016, p. 1). In other words, investors are assumed to be risk-averse. Risk-averse investors have minimal risk tolerance; hence they desire to hold portfolios with the least risk even if they expect not necessarily the highest levels of return. As the efficient frontier depicts optimum portfolios, such an investor will pick a portfolio with the least variance, which will be located on the lowest point of the efficient frontier (Ayodeji & Ingram 2015, p. 44). The optimum portfolio clearly shows the proportion of capital to be invested in each stock and it signifies a maximum return for investors at a lowest possible variance (Ramanathan & Jahnavi, 2014, p. 123). According to Ayodeji and Ingram (2015, p. 39), such a portfolio is also called the minimum variance portfolio (MVP).

2.2. Empirical Literature Review

Several research studies have empirically tested the feasibility of Markowitz’s MVO model as a portfolio optimisation strategy. Haugen and Baker (1991) tested the MPT model on the United States (US) market based on the Wilshire 5000. From 1972 to 1989, the authors constructed the minimum variance portfolios (MVPs) with a restriction of short selling at the beginning of each quarter. Portfolios constructed were observed to persistently outperform the benchmark (Wilshire 5000) in terms of returns and volatility. On average, the MVPs constructed yielded a return, which was 22% higher and a risk 21% lower than the benchmark. Clarke, et al., (2006) examined the performance of MVPs in the US equity market from 1968 to 2005. More recent portfolio construction techniques were used to construct MVPs using the 1000 largest US stocks. Clarke, et al., (2006) used the S&P500 as a benchmark of the study. Confirming the study of Haugen and Baker (1991), Clarke, et al., (2006) concluded that MVPs achieved an approximate 25% lower volatility than the benchmark without lowering returns.

Bausys (2009) tested the effectiveness of the MVP model on the Baltic stock market using the euro-denominated market capitalisation weighted index OMX Baltic Benchmark (OMXBB) as the index for the analysis from 2001 to 2008. The OMXBBCAPPI (the OMX available as a weight-capped price index) was used as a benchmark against the constructed portfolio. The performance of the constructed

MVP fluctuated during the period. The MVP was outperformed by its benchmark in a market uptrend from the year 2005 until when the market dropped in 2007. Bausys (2009) observed that the MVP performed better than its benchmark in bearish periods, demonstrating a superior performance during the 2007-2008 period of the global crisis and maintained its outperformance thereafter. In another study Darko (2012) investigated the MVP model on the Ghana Stock Exchange using the three selected indices (ALSI, financial and non-financial), from 2007 to 2010. A positive correlation was noted among the indices although it was with a smaller percentage between the financial and the non-financial indices, which meant a difficulty in yielding diversification benefits. As a result, the study confirmed that for an optimum portfolio, an investor ought to allocate 83.44% of the funds in the non-financial index and 16.56% in the financial index to yield optimum returns.

Jiang (2013) investigated the applicability of MPT on the Stock Exchange of Thailand (SET100) from 2010 to 2011. The efficiency of the MPT was tested in an index (SET100) to determine the performance of MPT when a short-term investment horizon is considered. An efficient set of portfolios was computed and constructed and the total return for the portfolio was found to be 32%. Tests were run using the Sharpe ratio, the Treynor ratio (see Hübner, 2005), the Jensen ratio (see Hübner, 2005) and the information ratio (see Goodwin, 1998) to determine the volatility of the portfolio. Jiang (2013) confirmed that the constructed efficient MVP outperformed the SET100 index by a significant margin in terms of both return and risk. Further, Thirimanna, Tilakartane, Mahakalanda and Pathirathne (2013) carried out a study on the Colombo Stock Exchange (Sri Lanka) to assess the performance of the MPT. The study stretched from 2009 to 2011. Two portfolio construction techniques were used, namely the MPT and the cointegration approach (see Wahab & Lashgari, 1993, pp. 716-717). Sector portfolios were created and compared against each other to find a better strategy (Thirimanna, et al., 2013). The portfolios were also compared against the benchmark index, the ASPI (i.e. the Sri Lanka Colombo All Share Price Index). The constructed portfolios were found to perform better than the market in terms of return.

Roopanand (2001) investigated the effectiveness of the MV model on the JSE using the ALSI based on component indices like the industrial and gold index locally as well as using the Dow Jones internationally. Using the period February 1983 to March 1999, historical annual returns for each index were calculated and the betas and covariances computed for a period of 182 months. The results of the study signified that a market portfolio emulating the ALSI only was mean variance inefficient (Roopanand, 2001). Furthermore, it was concluded that domestic returns by SA investors would be maximised by holding the Gold index, which was mean-variance efficient.

Garaba (2005) investigated the power of MPT as a security evaluation portfolio management technique compared to other traditional tools such as fundamental analysis (FA, see Abarbanell & Bushee, 1997), technical analysis (TA, see Lo, Mamaysky & Wang, 2000) and behavioural finance (BF, see Subrahmanyam, 2008) theory on the JSE. A survey was done using a sample of 110 out of the 322 asset management companies listed by the Financial Markets Directory (FMD) as at September 2003. From the questionnaire responses by asset managers, it was concluded that the MPT is not being employed mainly by asset managers for portfolio management and security evaluation. The reason for the low usage of MPT might be its heavy reliance on complex mathematics, which might be a challenge to asset managers. Moreover, MPT assumes no transaction costs; hence, it is difficult for asset managers to adjust the model to factor in transaction costs, taxation and other economic fundamentals (Omisore, Yusuf & Christopher, 2011). On the other hand, FA was regarded as the most significant portfolio management tool. However, Garaba (2005) recommended an integrated portfolio management strategy that incorporates both the traditional portfolio theory (FA and TA) as well as the MPT to enhance investor value and protection.

Rocha (2016) investigated the efficiency of the MPT minimum risk portfolios (MRPs) against the post-modern portfolio theory (PMPT) based on the European stock market. From January 1997 to December 2015, Rocha (2016) obtained stock data of 16 stocks from the Euro Stoxx 50 Index. The historical data were analysed, and daily returns for each stock were computed and then annualised. Efficient frontiers were derived after the average variance and covariance had been computed. The research confirmed that MPT and PMPT produced the same MRPs throughout the investment period.

In the context of South Africa, a number of studies have been conducted to assess this phenomenon. Firstly, Du Plessis and Ward (2009) determined the applicability of the MPT on the JSE for the period January 1997 to December 2007 by analysis stocks and constructed four MVPs under different conditions. They then constructed a portfolio based on ex-ante returns for prediction of returns as well as to predict the covariance matrix. Periodic rebalancing was done on the optimal portfolios constructed and the results obtained compared against the benchmark (ALSI 40 index). They established that for the period studied, the MVPs constructed outperformed the ALSI 40 even under the constrained conditions of no short-selling and/ or no more than 10% in any single security (Du Plessis & Ward, 2009).

Secondly, the MPT was also applied in South Africa by Du Plessis (2014) on the Industrial Development Corporation (IDC). The main objective was to determine if the IDC was optimising its capital allocation and to further establish which sectors should be invested more for the IDC to realise maximum returns. The results of the study showed that the capital allocation strategy of the IDC was not being optimised. The sectors being prioritised were electricity, gas, steam and water supply. From 2010 - 2014, 47.2% of capital was allocated to that sector which was only generating about 0.6% of the economic formal jobs in South Africa at the time. By applying the MV theory, Du Plessis (2014) allocated a limit of 20% capital to each prioritised sector. A portfolio was optimised in such a way that maximised the IDC strategic objectives, which included creation of employment and increasing real growth output (Du Plessis, 2014). Generally, Du Plessis (2014) concluded that the IDC could employ the MV framework efficiently as an allocation, decision making and optimising tool to attain favourable returns as well as to achieve both its long and short-term objectives.

Thirdly, Brouwer (2015), conducted a study using the MVO model to find the optimum portfolio of exchange traded funds (ETFs) on the JSE for the period 2009 to 2013. An analysis was done in a bid to determine the interrelatedness between the ETFs as a measure of diversification (Brouwer, 2015). Multiple optimisation runs were done with different risk - return combinations to draw up an efficient frontier of portfolio allocations. The performance of the optimised portfolio was evaluated for the year thereafter. The results of the study confirmed a positive performance of the optimised portfolio. However, the study period was limited to four years only, a period which might be too short to determine a long-term performance of the constructed portfolio.

Fourthly, Oladele and Bradfield (2016) conducted a study on the JSE using seven different techniques to construct low-risk portfolios, including the MVP based on the nine FTSE/JSE sectors. From January 2003 to December 2013, portfolios were created using methodologies targeting low volatility, low beta, maximum diversification and low correlation among others. The techniques used by the authors included the equally- weighted, the equal-weighted low beta, the low volatility single index model, the equal risk contribution, the naïve risk parity (see Grundy & Malkiel, 1996), the maximum diversification portfolio and the MVP. Portfolios were then rebalanced annually and compared against the ALSI. As observed by Oladele and Bradfiel (2016), the low volatility portfolios created outperformed the JSE ALSI in terms of risk and returns.

Fifthly, Contreras et al., (2016) also utilised the JSE market as part of the unit of analysis in their study. They examined the performance of mean-variance optimised (MVO) equity portfolios for retail investors, in 22 countries. For a period of 10 years, from 2005 to 2014, stock price data from the 22 markets were utilised and a back-test of MVO portfolio optimisation was conducted (Contreras et al., 2016). The findings of their study confirmed that most MVO portfolios obtain a higher level of return than their respective benchmark indexes, and in many cases, do so without noticeable increases in their volatility. Furthermore, stability in the outperformance of the MVO portfolios was also noted on the JSE except for the year 2008. Limiting the study to equity portfolios only might have caused the underperformance of the constructed MVOs during 2008. As effective diversification involves different asset class investments, including more asset classes such as bonds and commodities, could have resulted in a different outcome in a period of a market downswing (Markowitz, 1952, p. 89).

In both developed and developing markets, MVPs have proven to perform better than their benchmarks. In terms of both return and volatility, the studies by Haugen and Baker (1991), Clarke et al. (2006), Bower and Wentz (2005), Gupta and Basu (2009), Paudel and Koirala (2006), Bausys (2009), Ahuja (2011), Darko (2012) as well as, Baltes and Dragoe (2015), among others, revealed a common result, namely that the MVPs are better performers than their benchmarks. Having compared the use of the MV model against MAD, Bower and Wentz (2005) also concluded that the MV method is effective for portfolio optimising. The efficiency of the MAD model was almost equal with the MV method, yielding an average return which was 0.0013% higher than that of the MV model. In the context of South Africa, Du Plessis (2014) found that an MVP framework can also be applied successfully outside financial markets.

With the evolution of financial markets driven by technology and evolving investor sentiments, the MVP framework have shown to be a strong and effective portfolio optimising tool in both developed and developing markets (Baker & Haugen 2012; Blitz & Van Vliet 2007; Haugen & Baker, 1991; Jagannathan & Ma, 2003, among others). As developing economies are more volatile than developed markets due to economic and political distress, MVPs could benefit developing market investors as they have proven to perform better than their benchmarks despite the market trend (Bausys, 2009; Darko, 2012; Gupta and Basu 2008; Mbithi, 2014; Razak, et al., 2014; Roopanand, 2001; Thirimanna, et al., 2013, among others). In an efficient market, where all the security information is publicly available, the diversification power of MVPs becomes key to winning, especially when the diversification is done effectively (Markowitz, 1952, pp. 88-89). In South Africa, limited attention has been paid to constructing portfolios based on sector indices. It is imperative to highlight that sector indices allow investors to hold a basket of different securities from different sectors of the economy.

3. Research Methodology

3.1. Sample Description and Data Sources

The target population of relevance to this study was all the tradable indices on the JSE. According to the JSE equity market statistics (JSE, 2018), there are more than 800 potential investments, of which approximately 300 are tradable indices and sub-indices on the JSE. However, not all the tradable indices are active on the JSE. The top JSE indices comprise of the Top 40, JSE ALSI, $INDI_{25}$, $RESI_{10}$ and $FINI_{15}$ indices. The tradable sector indices form an integral part of the financial world on the JSE, as it constitutes assets from all the South African sectors. The tradable sector indices give investors access to hold value of a number of companies from the same South African sector pooled together in one big basket. Investors can monitor these indices for decision making with respect to their portfolios.

The $INDI_{25}$, $RESI_{10}$ and $FINI_{15}$ indices were purposively selected as the sample of this study. The three indices represent different South African market sectors, which facilitates sector diversification onto the constructed portfolio. According to the Industry Classification Benchmark (ICB) of South Africa, the JSE ALSI was sub divided into three South African sectors, which are SA Resources, SA Financials and SA Industrials. SA Resources constitutes 12%, SA Financials 24% while SA Industrials constitutes 64% of the JSE ALSI (JSE Regulatory Report, 2013). The three indices capture the most liquid, tradable instruments in their respective sectors.

Using the Bloomberg financial database, the daily closing prices for each of the three indices were downloaded. The main information of interest was the daily closing price for each index, which was used to compute daily returns and volatilities. Considering the three tradable sector indices to under study, a total of 50 shares were included in the indices. Each of the three tradable sector indices had a base date of 24 June 2002. The data collected from Bloomberg included the historical price records for the three indices, namely $INDI_{25}$, $RESI_{10}$ and $FINI_{15}$. The study period was divided into three periods, starting with the pre GFC period which spanned from 1 January 2007 to 30 June 2007, the GFC and the post GFC period. The GFC period, spanning from 1 July 2007 to 31 August 2009; and the post GFC period which spanned from 1 September 2009 to 31 December 2017. The Excel Solver and Stata version 15 software were used to analyse the data.

3.2. Construction of an MVP

The first step in the construction of the MVP is to calculate the portfolio return. The portfolio return was calculated using Microsoft Excel based on the following equation:

$$E(r) = \sum x_i \times R_i \qquad \text{Equation (3)}$$

Where:

E(r) = the expected portfolio return

x_i = the weight allocated to each of the three sectors ($INDI_{25}$, $RESI_{10}$ and $FINI_{25}$)

R_i = the expected return of each sector calculated based on historical data

Optimal weights of the portfolio were. The next step was calculating the portfolio standard deviation and covariance. The required values for the standard deviation and the covariance were computed from the historic daily return data. The total portfolio risk was obtained by computing the standard deviation of its time series returns. In this case, the 132–month annualised standard deviation for the daily returns was computed. A covariance matrix was then computed by applying the relationship in Equation 4.

$$\sigma_\rho^2 = (w_{INDI_{25}}\,\sigma_{INDI_{25}})^2 + (w_{RESI_{10}}\,\sigma_{RESI_{10}})^2 + (w_{FINI_{15}}\,\sigma_{FINI_{15}})^2 \\ + 2w_{INDI_{25}}w_{RESI_{10}}\rho_{INDI_{25}RESI_{10}}\sigma_{INDI_{25}}\sigma_{RESI_{10}}$$

$$+ 2w_{INDI_{25}} w_{FINI_{15}} \rho_{INDI_{25}FINI_{15}} \sigma_{INDI_{25}} \sigma_{FINI_{15}}$$

$$+ 2w_{RESI_{10}} w_{FINI_{15}} \rho_{RESI_{10}FINI_{15}} \sigma_{RESI_{10}} \sigma_{FINI_{15}} \quad \text{Equation (4)}$$

Where:

ρ = the correlation coefficient between the returns on assets $INDI_{25}$, $RESI_{10}$ and $FINI_{25}$

w = the weight of the asset

σ = the asset standard deviation

When calculating covariance, the objective was to determine how assets within a portfolio move together. When exposed to market volatilities, assets from different asset classes normally react differently, hence assessing how they move jointly will determine the effectiveness of combining them in a portfolio. The sample covariance between assets can be calculated by applying Equation 5.

$$sample\ Cov\,(x, y) = \sum \left(\frac{(X_i - \bar{x})(Y_i - \bar{y})}{n-1} \right) \qquad \text{Equation (5)}$$

Where:

X = the independent variable ($INDI_{25}$, $RESI_{10}$ and $FINI_{25}$ indices)

Y = the dependent variable ($INDI_{25}$, $RESI_{10}$ and $FINI_{25}$ indices)

n = number of data points in the sample

$\bar{x}$ = the mean of the independent variable X

$\bar{y}$ = the mean of the dependent variable Y

The objective for constructing the MVP is to optimise a portfolio, in other words minimising the variance while at the same time maximising the returns. The values obtained from Excel Solver were used to trace the efficient frontier, which is a parabolic line from which efficient portfolios lie.

3.3. Uncapped Minimum Variance Portfolio

The major focus of the study was to construct an efficient MVP, hence portfolio 1 was an uncapped weight portfolio from which the exact weights were determined by the Excel solver. This portfolio is referred to as Global MVP Uncapped (GMVPU). The GMVPU portfolio addresses whether an optimal portfolio could be constructed out of the JSE tradable sector index. It also proves if a global mean variance portfolio could be constructed using the JSE Tradable sector index.

First, some random weights to be invested in each of the three indices were assigned based on the budget constraint. The total weights added up to 1, or 100%. This constraint was then formulated as a quadratic program and imposed into Excel Solver as in Equation 6.

$$X_{PORT} = X_{INDI_{25}} + X_{FINI_{15}} + X_{RESI_{10}} = 1 \qquad \text{Equation (6)}$$

Where:

X_{PORT} = the portfolio weight

$X_{INDI_{25}}$ = the weight of the industrials sector

$X_{FINI_{15}}$= the weight of the financial sector

$X_{RESI_{10}}$= the weight of the resources sector

The MVP construction steps as outlined in Section 3.2 would then implemented. Having obtained the portfolio variables required, that is the weights, returns and standard deviations, Excel Solver would adjust the weights until an optimum MVP was obtained.

4. Empirical Findings

The results of the study documented that an MVP could be constructed out of the JSE tradable sector indices. These could be mapped onto an efficient frontier and an investor could select an MVP that matches their risk preferences. The efficient frontier of a GMVPU is presented in Figure 3.

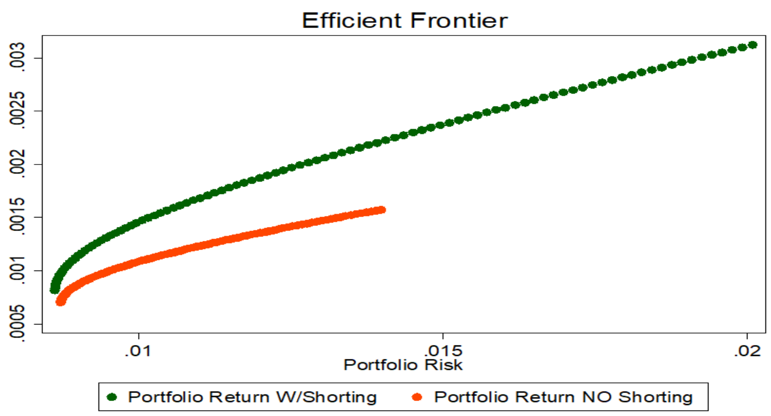

Figure 3. Efficient Frontier for the GMVPU Portfolio

The risk-return profile of the GMVPU is presented in Table 1. This profile is compared to that of JSE ALSI index whose results are presented in Table 2.

Table 1. Portfolio Results for the Period during the Sample Period

		GMVPU
Pre-GFC period	Return	0.0698
	Standard deviation	0.0088
	Asset weights	FINI= 0.00
		RESI= 0.00
		INDI = 100%
GFC period	Return	0.0130%
	Standard deviation	0.0170
	Asset weights	FINI= 0.07
		RESI=0.00
		INDI =0.93
Post-GFC period	Return	0.0643%
	Standard deviation	0.0101
	Asset weights	FINI= 0.21
		RESI=0.04
		INDI =0.75

Table 2. Average Daily Risk-Return Profile of the JSE ALSI Index

	Average return	Standard deviation
Pre-GFC	0.1017%	0.0101
GFC	0.0152%	0.0183
Post-GFC	0.0462%	0.0094

When compared against the benchmark, the constructed GMVPU is observed to be under-performing during the pre-GFC period. The GMVPU yielded an average daily return of 0.0698% which is 0.0319% lower than the JSE ALSI (0.1017%). However, the constructed portfolio had a lower standard deviation of 0.0088 when compared to the 0.0101 of the JSE ALSI. Risk-averse investors could pick the GMVPU, as they will be mainly concerned about minimising risk. During the GFC period, the GMVPU started outperforming the JSE ALSI. The GMVPU daily average return of 0.0131% was higher than the JSE ALSI daily average return for the GFC period (0.0152%) In terms of risk, the GMVPU yielded a lower standard deviation of 0.0170 when compared to 0.0183 of the JSE ALSI. After the GFC period, that is from November 2008 going forward, the GMVP maintained its performance, beating the JSE ALSI by a margin of 0.0182% (0.0644% against 0.0462% of the ALSI), in terms of return. The standard deviation for the GMVPU was 0.0101, which is slightly higher than that of the JSE ALSI (0.0094), hence the GMVPU maximised returns with a little or no significant altering of the level of risk.

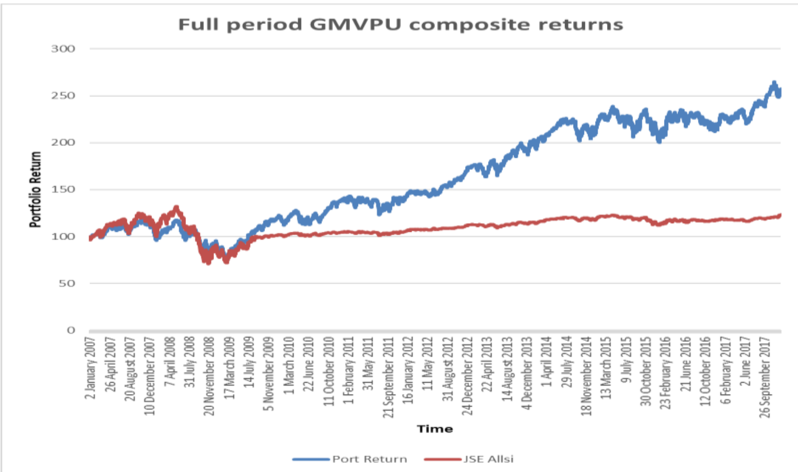

Figure 4. GMVPU Return against JSE ALSI

The risk-return combinations of the GMVPU against the JSE ALSI for the whole period is depicted on Figure 4. This is a holistic indication of what transpired before the GFC, during the GFC and after the GFC. The trends in Figure 4 signifies that the tradable indices on the JSE to a certain extent follow the movement style of the JSE ALSI. When the market goes down, the performance of the GMVPU also declines but with a lesser margin. However, due to the power of sector diversification associated with investing in tradable indices, the GMVPU stays on top of the ALSI and performs better except during the 2007-2008 market downswing.

To determine if there were any diversification benefits that could accrue to an investor who constructed a GMVPU portfolio the variance-covariance and correlation matrices were computed as presented in Table 3.

Table 3. Variance–Covariance and Correlation Matrix

Variance - Covariance				Correlation			
	FINI	**RESI**	**INDI**		**FINI**	**RESI**	**INDI**
FINI	**0.034**	**0.023**	**0.022**	**FINI**	**1**		
RESI	**0.023**	**0.064**	**0.022**	**RESI**	**0.47**	**1**	
INDI	**0.022**	**0.022**	**0.025**	**INDI**	**0.72**	**0.55**	**1**

The asset classes are positively correlated, though not highly correlated. The correlation coefficient between the $FINI_{15}$ and the $RESI_{10}$ indices was observed to be 0.47, while that of the $RESI_{10}$ and the $INDI_{25}$ indices was 0.55. Finally, the correlation coefficient between $FINI_{15}$ and the $INDI_{25}$ was 0.72. Such a positive relationship among the assets means that the indices will be reacting almost in the same way to adverse market conditions. Despite the positive correlation observed among the three indices, the collective risk-return characteristics of the three assets are better off when compared to their individual respective risk-return combinations. In other words, diversifying a portfolio among the three indices is beneficial, as it yielded better returns associated with lower standard deviations compared to the JSE ALSI.

The results of this study are consistent with the findings of Brouwer (2015) on the performance of the Markowitz MVP on JSE. However, the point of departure of this study from Brouwer (2015) is that, the latter focused on ETFs and the sample period of the study spanned only four years. On the other hand, the present study focused on the JSE tradable indices for a period of 10 years. As a result, it can be concluded that the MVP framework can be applied in any asset class successfully. The results of this study corroborate that of Brouwer (2015), as the GMVP constructed in this study was observed to outperform the benchmark index in terms of risk-return combination. Further, the findings of this study resonate with that of Contreras *et al* (2016) on the performance of the JSE. However, the GMVPs was observed to outperform the JSE ALSI even during the GFC period. The difference of the two studies also lies in the fact that Contreras *et al* (2016) focused on equity retail portfolios while this present study utilised the JSE tradable indices. Despite the differences in securities employed, the same conclusion was reached about performance of the MVPs on the JSE after the GFC period.

5. Conclusion

The three objectives that were underpinning this study were to establish whether: (1) the JSE tradable sector index ($INDI_{25}$, $FINI_{15}$, $RESI_{10}$) represented an optimal portfolio; (2) global mean variance portfolios could be constructed using the JSE tradable sector index ($INDI_{25}$, $FINI_{15}$, $RESI_{10}$) and (3) there were any diversification benefits associated with selecting a portfolio constructed from the JSE tradable sector index.

The findings of this study validated all three propositions. MVO has been observed to be instrumental and effective for portfolio optimisation on the JSE. The GMVP constructed was observed to be consistent in minimising risk better than the benchmark JSE ALSI index throughout the period. As such, investors and field practitioners can employ the model for portfolio construction since it is virtuous in risk minimising especially during market downswings. Further, effective diversification can be achieved when different asset classes are used in a portfolio.

Annual or semi-annual rebalancing of the GMVP could have resulted in an improved performance of the portfolios. Rebalancing portfolios have the effect of maintaining the initial optimum asset weights within the portfolio. However, the GMVP constructed was observed to outperform the benchmark even without rebalancing the portfolio.

Further studies could investigate what could have been the effect of including more asset classes such as bonds and commodities into the portfolio during the GFC period.

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