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The Graph Theoretical Approach to Bankruptcy Prediction

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Abstract: This paper examines the applicability of the graph theoretical approach to bankruptcy prediction. Various statistical techniques have been used to predict bankruptcy including univariate analysis, multivariate discrimination analysis, logit model, probit model, and neural networks. This paper employs the graph theoretical approach to bankruptcy prediction. The empirical findings confirms the validity of the proposed method for predicting bankruptcy. The proposed method in this paper provides an insight into the development of a new approach to the assessment of financial solvency of a company. This paper contributes to the literature by introducing a new approach to bankruptcy prediction.

Keywords: Financial Solvency Matrix; Permanent Function

JEL Classification: G33

1. Introduction

The current financial crisis caused by COVID-19 has significantly affected the global economy especially retail and service industries and caused many companies to declare their bankruptcy. Nowadays, it is very critical for financial statements' users to accurately predict the bankruptcy of companies in these two sectors to manage their investments, take corrective actions and streamline their future relations with companies.

Various statistical techniques have been used to predict business failure starting by Fitzpatrick (1932) who analyzed the ratios of 20 failed firms, followed by Beaver (1966) who used univariate analysis, Altman (1968) with multivariate discrimination analysis, Ohlson (1980) with logit model, Zmijewski (1984) with probit model, and Aziz and Dar (2006) with neural networks. One of the most renowned bankruptcy prediction models is Altman's Z-score formula which was derived from discriminant analysis. Altman's Z-score model has received a wide acceptance by researchers and practitioners since its inception by demonstrating its value in assessing the likelihood that a firm may go bankrupt within two years.

Many studies have used different models in bankruptcy prediction. However, the graph theoretical approach has not been employed in prediction bankruptcy in spite of the popularity of the graph

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theoretical applications in other fields. The purpose of this study is to develop a bankruptcy prediction model based on the graph theoretical methods. To achieve this goal, we used a sample of 84 companies with 42 companies in each of two groups – one group of bankrupt companies and another group of non-bankrupt companies. Graph and matrix methods were employed to construct financial solvency matrices of the sample companies. Permanent matrix functions were computed as financial solvency indices and those indices were applied to predicting solvency and insolvency of companies in two groups. Finally, the predicting accuracy of Permanent matrix indices and Z-scores were compared.

The remainder of the paper is structured as follows. Section 2 gives an account of the survey of the literature. Data and methodology are covered in section 3. Empirical results are reported in section 4. The study concludes with section 5.

2. Literature Review

2.1. Bankruptcy Prediction

The extant research has identified the financial ratios as the critical component in all bankruptcy prediction models. For instance, Beaver (1966) used 30 financial ratios and assumed that one ratio can predict distress in firms, which might create inconsistency in predication by using one single ratio. Altman (1968) used 22 ratios in multivariate discriminant analysis to measure the company's liquidity, profitability, and leverage. Similarly, Olson (1980) used 6 ratios and Zmijweski (1984) used three ratios to measure the companies' profitability, liquidity, and leverage. Notably, there are no specific ratios can be used in all cases to predict company's failure (Barnes, 1987; Altman, 1993; Mohamed, Li & Sanda, 2001).

Although scholars tried to measure the company's liquidity, leverage, and profitability, different researchers selected different financial ratios based on their popularity and predictive ability (Beaver, 1966; Altman, 1968; Ohlson, 1980; Casey and Bartczak, 1984; Lifschuts and Jacobi, 2010; Wu, Gaunt & Gray, 2010). In this vein, we find it relevant to use several financial ratios that suitably measure the firm's liquidity, profitability, leverage, and efficiency, whether the firm operates in retail or service industry.

2.2. Key Financial Ratios Used in Bankruptcy Prediction

2.2.1. Measurement of Liquidity

The liquidity of retail and service companies are very essential to meet their short-term solvency and predict their bankruptcy; thus, it is important to assess the firm's liquidity by using one of the ratios that is quite relevant to retail & service companies. (Babalola & Abiola, 2013). The current ratio which divides current assets by current liabilities is the most relevant ratio that can assess the firm's liquidity (Zmijewski, 1984; Olson, 1980; Klepáč & Hampel, 2016; Prabowo, 2019). The higher the liquidity, the more the guarantee over the company's debts (Rose & Giroux, 1984; Zmijewski, 1984). Creditors and suppliers prefer a high current ratio to reduce their risk; nonetheless, shareholders prefer a lower current ratio to grow their return on investment (Mohammed & Kim-Soon, 2012). In retail and service cyclical industries, firms may maintain a high current ratio in order to remain solvent during downturns (Mohammed & Kim-Soon, 2012). Accordingly, we find it suitable to use the current ratio to measure the company's liquidity.

2.2.2. Measurement of Profitability

Measuring the business profitability is quite essential to assess its ability to maximize the shareholders' wealth. The most popular financial ratio that can be used to assess the profitability is the Return on Assets (ROA), which has been used by DuPont Company in 1919. ROA is found to be the third most frequently presented ratio in business textbooks after current ratio and inventory turnover ratio (Jewell and Mankin, 2011). ROA has been extensively used in bankruptcy prediction (Altman, 1968; Ohlson, 1980; Lennox, 1999; Agarwal and Taffler, 2008; Wu et al., 2010; Klepáč & Hampel, 2016). Beaver (1966) used ROA as one of the six ratios used to predict business failure. Altman (1968) Z-Score included ROA as one of its five factors used to predict business failure. Hossari and Rahman (2005) ranked the popularity of all financial ratios used in studies predicting business failures and concluded that ROA is one of the most common ratio in all the failure prediction studies. Hence, we find it relevant to include ROA in our research by dividing the earnings before interest and tax by total assets.

The EBITDAM is a measure of corporate performance in the terms of its profitability. The company's ability to generate higher EBITDAM indicates its better performance, usage of debt (Nalurita, 2017), and more prediction of future cash flows (Mey & Lamprecht, 2020). This ratio indicates the management's success in covering all the fixed payments and staying away from bankruptcy. Alternatively, the decrease in EBITDAM is an indicator for company's bankruptcy (Holtemöller & Muradoglu, 2020). Retail and service firms tend to have much lower profitability on sales and much shorter operating cycles than manufacturing firms (Gombola & Ketz, 1983). Therefore, we find it relevant to include the EBITDAM in our study by dividing EBITDAM over sales.

2.2.3. Measurement of Leverage

The company's ability to meet its long-term obligations is quite essential for creditors and owners, and can be measured by the leverage ratio. The leverage ratio presents the company's dependence on debt and the debt extent in a company's capital structure (Lin et al., 2014; Son et al., 2019). High debentures is considered as one of the main cause that leads a company to bankruptcy. Prior scholars used different ratios to measure the company's leverage (Beaver, 1966; Deakin, 1972; Ohlson, 1980; Zmijewski, 1984; Rashid & Abbas, 2011; Lin et al., 2014; Son et al., 2019), where some of them used the market value of total equity over total debt (Rashid & Abbas, 2011; Lin et al., 2014), and other used the book value of total equity over total debt (Son et al., 2019). We used the latter ratio due to the absence of the market value in many retail and service companies because they are not listed. With the increased demand in on-line sales, this measure gained the retailers' focus to leverage in-store sales with e-commerce. As the percentage of sales shift to e-commerce, the ability for a retailer to fund its debts from equity becomes a financial measure of risk (Choe and Neureuther, 2020). To conclude, we used the book value of total equity to total liability to measure the firm's leverage.

2.2.4. Measurement of Activity

The activity of companies can be measured by operational ratios such as total assets turnover which divides net operating revenues by total assets. This ratio is more relevant than inventory turnover ratio due to the absence of inventory in service companies. Activity or efficiency is a basic measure of firm's creditworthiness because it demonstrates management's ability to maximize its use of resource (Mohamed et al., 2001). Total assets turnover is a strategic measure that demonstrates the firm's efficiency in using its assets to generate revenues (Lin et al., 2011). Total assets turnover ratio gives an insight about the internal management to the different stakeholders. A low asset turnover ratio indicates excess production or mediocre management of inventory or poor practices, which is a main cause for business failure (Choe & Neureuther, 2020). Hence, we used the total assets turnover as a measurement of company's efficiency.

2.3. Graph and Matrix Methods

The graph theory explores the topological properties of a graph designed as a model representing pairwise relations between objects. The usefulness of graph theory lies in that many types of relations and processes in science and technology can be modeled by graphs and the unified formalism of graphs enables the solution of many problems by examining the topological properties of graphs (Choe & Zhou 2015; Choe & Neureuther, 2020).

Since the inception of Graph theory developed by Euler in tackling with Konigsberg Bridge Problem in 1736, Graph theory has been applied in a wide range of fields including computer science, linguistics, chemistry, physics, biology, mathematics, and sociology. Recently, the application of Graph theory has extended in the fields of quality management (Grover, Agrawal and Khan, 2004; Kulkarni 2005), supply chain management (Talib, Rahman & Qureshi, 2011; Thakkar, Kanda & Deshmukh, 2008), and transportation network (Choe, 2017).

Graph and matrix methods have been applied to the analysis of the failure causes of a machine tool (Rao and Gandhi, 2002) and the evaluation of supply chain collaboration (R. Anbanandam et. Al. 2009). This paper employs graph and matrix method for the analysis of financial solvency of companies to explore the applicability of the graph theoretical method to bankruptcy prediction.

3. Methodology

The methodology for the proposed analysis consists of the following steps:

Step 1. Select key financial performance indicators as measures of liquidity, profitability, financial leverage, and activity that contribute to financial solvency of a corporation. Five financial ratios are derived from literature review.

Step 2. Develop financial solvency graph considering five financial ratios selected in Step 1 and their associations over periods. A financial solvency graph is composed of five nodes representing five financial ratios and directed edges showing the relations among financial ratios. Then, convert a financial solvency graph into a financial solvency matrix. The matrix is a 5 x 5 adjacent matrix with off-diagonal elements representing the causality among five financial ratios.

Step 3. Obtain the financial solvency index by computing the permanent function of a financial solvency matrix.

Constructing a financial solvency matrix begins with measuring the associations among financial ratios. In general, the Pearson's correlation coefficient is used to measure the linear association of two variables.

$$\rho_{x,y} = \frac{cov(X,Y)}{\sigma_x \sigma_y} = \frac{\sum_{i=1}^{n}(X_i-\bar{X})(Y_i-\bar{Y})}{\sqrt{\sum_{i=1}^{n}((X_i-\bar{Y})^2}\sqrt{\sum_{i=1}^{n}(Y_i-\bar{Y})^2}} \qquad (1)$$

However, the Pearson's correlation coefficient may not reflect the "pure" association of two variables if the association of two variables is effected by the third variable. We argue that the Partial correlation coefficient can be a better measure of the association of two variables by removing the presumable impact of the third variable on the association.

$$\rho_{x,y:z} = \frac{\rho_{x,y} - \rho_{x,z}\rho_{y,z}}{\sqrt{1-\rho_{x,z}^2}\sqrt{1-\rho_{y,z}^2}} \tag{2}$$

As shown in Equation (2), $\rho_{x,y,:z}$ is the partial correlation coefficient between two ratios, X and Y, adjusted for Z, presuming that X and Y are linearly associated with Z.

The associations among financial ratios can be represented in an adjacency matrix as shown in Figure 1. The matrix is a 5 x 5 square matrix where the off-diagonal elements, R_{ij} represents the causality relations between financial ratios and the strength of the causality relations are used as weights for the analysis of digraphs.

$$\begin{array}{c c}
\text{Ratios} & \begin{array}{ccccc} \text{R1} & \text{R2} & \text{R3} & \text{R4} & \text{R5} \end{array} \\
\begin{array}{c} \text{R1} \\ \text{R2} \\ \text{R3} \\ \text{R4} \\ \text{R5} \end{array} & \begin{bmatrix} 0 & R_{12} & R_{13} & R_{14} & R_{15} \\ R_{21} & 0 & R_{23} & R_{24} & R_{25} \\ R_{31} & R_{32} & 0 & R_{34} & R_{35} \\ R_{41} & R_{42} & R_{43} & 0 & R_{45} \\ R_{51} & R_{52} & R_{53} & R_{54} & 0 \end{bmatrix}
\end{array}$$

Figure 1. Financial Solvency Matrix

The permanent function of the financial solvency matrix, Per (A) can be used as a measure of financial

$$\operatorname{perm}(A) = \sum_{\sigma \in S_n} \prod_{i=1}^{n} a_{i,\sigma(i)}.$$

solvency index, which can be written as:

4. Empirical Results

4.1. Sample Selection

A total of 84 companies was selected for analysis. These companies were selected from two groups. The first group (G1) was composed of 42 companies that filed a bankruptcy petition under Chapter 11 during the period 1997-2020. The second group (G2) was composed of a paired sample of 42 companies that were stratified by industry and size in an effort to reduce heterogeneity caused by industry and size differences.

The average size of total asset of bankrupt companies was \$3.89 billion ranging from \$34.7 million and \$25.8 billion. The majority of bankrupt companies were selected from the service sector, mainly from the retailing industry. Over 75% of bankrupt companies filed a petition for bankruptcy in the past two years (2018-2020).

The average size of total asset of non-bankrupt companies was \$3.96 billion ranging from \$27.8 million and \$58.7 billion. Financial data of sample companies were retrieved from Bloomberg Database. Financial ratios for bankrupt companies were derived from those companies' four years (16 quarters) financial statements prior to bankruptcy. Financial data for non-bankrupt companies were obtained from financial statements of their most recent four year that they reported.

4.2. Constructing Matrices and Computing Permanent Indices

Financial ratio data of the sample companies were collected from Bloomberg Database. One particular thing to point out is that while Altman Z-score formula assess a corporation's financial solvency as of a specified point in time, the proposed method assess a corporation's financial solvency for a period of time as the construction of the financial solvency matrix requires examining the causality relations among financial ratios over periods, at least eight periods to get statistically meaningful results. Each quarter's financial solvency matrix was constructed by examining the causality relations among financial ratios over the past eight quarters. A total of twenty quarters' financial ratios were examined to construct twelve financial solvency matrices per company. For example, Akron Inc. filed a petition for bankruptcy in May 2020. The company's financial data of the past 20 quarters (Q2 2015 – Q1 2020) were collected. The financial solvency matrix of the most recent quarter prior to bankruptcy (Q1 2020) was constructed by using financial data of the past eight periods (Q2 2018-Q1 2020). Twelve financial solvency matrices of Akron Inc. were constructed for analysis. This procedure was repeated until 1,008 financial solvency matrices of 84 sample companies were obtained.

Permanent functions of financial solvency matrices were calculated to get financial solvency indices. These indices, Per(A), were applied to the prediction of bankruptcy. Tables 1 and 2 show comparisons between Z-scores and Per(A) indices in predicting bankruptcy.

Both Z-scores and Per(A) indices were applied to bankrupt companies to assess their bankruptcy prediction power as demonstrated in Table 1. Z-score model accurately predicted 28 bankrupt companies and inaccurately predicted 5 bankrupt companies one year (Z YR1) prior to actual bankruptcy. Two years (Z YR2) prior to actual bankruptcy, Z-score model accurately predicted 24 bankrupt companies and inaccurately predicted for 7 companies.

Per(A) indices predicted 25 bankrupt companies accurately and inaccurately predicted three bankrupt companies one year (P YR1) prior to the actual bankruptcy. Per(A) indices predicted 23 bankrupt companies accurately and 6 bankrupt companies inaccurately two years (P YR2) prior to the actual bankruptcy.

Table 1. Applying Z-Scores & Per(A) Indices to Bankrupt Companies

#	G	COMPANY	Z YR1	CLASS	ERROR	Z YR2	CLASS	ERROR	P YR1	CLASS	ERROR	P YR2	CLASS	ERROR
1	B	AAC Holdings Inc	1.404	B		2.216	Q		3.004	B		3.068	Q	
2	B	Akorn Inc	-0.356	B		2.697	Q		3.336	NB	E	3.006	B	
3	B	Ames Dept Stores Inc	2.376	Q		3.221	NB	E	3.036	Q		3.103	Q	
4	B	API	1.140	B		1.103	B		3.003	B		3.005	B	
5	B	Approach Resources Inc	0.124	B		-0.003	B		2.998	B		3.005	B	
6	B	Bon-Ton Stores Inc	1.867	Q		1.899	Q		2.998	B		3.034	Q	
7	B	Carbo Ceramics Inc	1.267	B		1.126	B		3.001	B		3.068	Q	
8	B	Chesapeake Energy Corp	-0.406	B		-0.741	B		3.000	B		3.026	Q	
9	B	Dean Foods Co	3.477	NB	E	4.290	NB	E	3.081	Q		3.000	B	
10	B	Destination Maternity Corp	2.128	Q		2.197	Q		3.012	Q		3.000	B	
11	B	Diamond Offshore	0.891	B		1.344	B		3.073	Q		3.314	NB	E
12	B	EP Energy Corp	-1.652	B		-0.489	B		3.159	NB	E	3.013	Q	
13	B	Factory 2-U Stores Inc	2.841	Q		4.828	NB	E	3.012	Q		2.997	B	
14	B	Fairchild Corp	1.134	B		0.761	B		3.000	B		3.002	B	
15	B	Finlay Enterprises Inc	1.382	B		2.206	Q		2.999	B		3.071	Q	
16	B	Foresight Energy LP	0.666	B		0.484	B		3.000	B		2.998	B	
17	B	Fred's Inc	3.330	NB	E	4.395	NB	E	3.059	Q		3.004	B	
18	B	Frontier Communications Corp	0.404	B		-0.016	B		3.002	B		3.033	Q	
19	B	GNC Holdings Inc	2.925	Q		2.830	Q		3.002	B		3.001	B	
20	B	Hancock Fabrics Inc	3.162	NB	E	3.332	NB	E	3.001	B		3.027	Q	
21	B	Hechinger Co	2.406	Q		2.627	Q		3.009	B		3.000	B	
22	B	Hertz Global Holdings Inc	0.566	B		0.578	B		3.000	B		3.000	B	
23	B	Hornbeck Offshore Services Inc	0.381	B		0.359	B		3.003	B		3.025	Q	
24	B	Hytera Communications Corp	2.691	Q		2.694	Q		3.011	Q		2.999	B	
25	B	Intelsat SA	-0.515	B		-0.437	B		3.030	Q		3.176	NB	E
26	B	J.C. Penney Co Inc	1.233	B		1.260	B		3.006	B		3.023	Q	
27	B	Jason Industries Inc	-0.280	B		1.669	B		3.002	B		3.001	B	
28	B	Latam Airlines Group SA	0.805	B		1.123	B		3.001	B		3.191	NB	E
29	B	Lechters Inc	2.678	Q		2.666	Q		3.004	B		3.001	B	
30	B	Levitz Furniture Inc	-0.755	B		-0.039	B		3.019	Q		3.242	NB	E
31	B	Lilis Energy Inc	-0.689	B		-2.245	B		3.149	Q		3.001	B	
32	B	Maxcom Telecomunicaciones	0.117	B		0.707	B		3.005	B		2.999	B	
33	B	McClatchy Co	-4.609	B		-3.918	B		3.076	Q		3.000	B	
34	B	McDermott International Inc	-0.718	B		2.362	Q		3.061	Q		3.001	B	
35	B	Pier 1 Imports Inc	3.407	NB	E	4.573	NB	E	3.249	NB	E	3.004	B	
36	B	Pioneer	0.292	B		0.297	B		3.093	Q		3.693	NB	E
37	B	Pyxus International Inc	1.395	B		1.474	B		3.000	B		3.007	B	
38	B	Sanchez Energy Corp	-0.354	B		-0.655	B		3.002	B		3.002	B	
39	B	Sears Holdings Corp	0.852	B		1.301	B		3.001	B		3.111	Q	
40	B	Stage Stores Inc	2.012	Q		1.813	Q		3.003	B		3.250	NB	E
41	B	Walking Co Holdings Inc	3.268	NB	E	3.906	NB	E	3.127	Q		3.105	Q	
42	B	Whiting Petroleum Corp	0.412	B		-0.298	B		3.008	B		2.999	B	

Table 2 shows the application of Z-scores and Per(A) indices to non-bankrupt companies. The number of accurately predicted companies by Z-scores is 20 while the number of inaccurately predicted companies is 11 one year (Z YR1) prior to the reporting period. Two years prior to the reporting period (Z YR2), Z-scores accurately predicted 18 companies while inaccurately predicted 5 companies. Per(A) indices classified 18 non-bankrupt accurately but 10 companies inaccurately one year (P YR1) prior to the reporting period. Two years (P YR2) prior to the reporting period, Per(A) indices accurately classified 13 companies while inaccurately classified 7 companies.

Table 2. Applying Z-Scores and Per(A) Indices to Non-Bankrupt Companies

#	G	COMPANY	Z YR1	Z YR1	ERROR	Z YR2	Z YR2	ERROR	P YR1	P YR1	ERROR	P YR2	P YR2	ERROR
1	NB	Aaron's Inc	5.587	NB		6.522	NB		3.025	Q		3.001	B	E
2	NB	Abercrombie & FI	2.812	Q		4.650	NB		3.040	Q		3.000	B	E
3	NB	American Eagle Outfitters Inc	3.863	NB		8.908	NB		3.247	NB		3.015	Q	
4	NB	ARO Liquidation Inc	2.418	Q		2.258	Q		2.999	B	E	3.040	Q	
5	NB	Ascena Retail Group	2.254	Q		0.919	B	E	3.154	NB		3.182	NB	
6	NB	BMC	4.828	NB		5.063	NB		3.184	NB		3.199	NB	
7	NB	Boot Barn Holding	4.757	NB		3.887	NB		3.408	NB		3.033	Q	
8	NB	Builders	4.519	NB		4.477	NB		3.200	NB		3.013	Q	
9	NB	Caleres Inc	2.190	Q		5.014	NB		3.153	NB		3.283	NB	
10	NB	Cato Corp	2.828	Q		4.302	NB		3.180	NB		3.173	NB	
11	NB	Christopher & Banks Corp	1.007	B	E	3.467	NB		3.002	B	E	3.180	NB	
12	NB	Conn's Inc	2.181	Q		2.358	Q		3.000	B	E	3.026	Q	
13	NB	Container Store Group	1.005	B	E	0.745	B	E	3.023	Q		3.000	B	E
14	NB	Cool Holdings	-6.965	B	E	-0.875	B	E	3.008	B	E	3.010	Q	
15	NB	CVS	4.546	NB		5.165	NB		3.410	NB		3.047	Q	
16	NB	Designer Brands	0.641	B	E	1.267	B	E	3.031	Q		3.152	NB	
17	NB	Dick's Sporting Goods Inc	4.444	NB		4.394	NB		3.019	Q		3.004	B	E
18	NB	Duluth Holdings Inc	1.699	B	E	4.482	NB		3.598	NB		3.027	Q	
19	NB	GameStop	2.662	Q		5.317	NB		3.957	NB		3.033	Q	
20	NB	Haverty Furniture Cos Inc	7.123	NB		7.645	NB		3.129	Q		3.419	NB	
21	NB	Home Depot	0.706	B	E	0.950	B	E	3.000	B	E	3.056	Q	
22	NB	Hudson's Bay	1.320	B	E	3.270	NB		3.039	Q		2.999	B	E
23	NB	Natural Grocers	4.645	NB		3.995	NB		3.276	NB		3.027	Q	
24	NB	Ollie	10.660	NB		16.395	NB		3.010	B	E	3.244	NB	
25	NB	RH	1.657	B	E	3.025	NB		3.001	B	E	3.027	Q	
26	NB	Rite Aid	2.698	Q		2.673	Q		3.002	B	E	3.027	Q	
27	NB	Ross Stores	6.595	NB		10.657	NB		3.666	NB		3.090	Q	
28	NB	Stein Mart	3.303	NB		2.786	Q		3.000	B	E	3.030	Q	
29	NB	Travelcenter	1.916	Q		4.110	NB		3.005	B	E	3.031	Q	
30	NB	Village Super	6.075	NB		6.219	NB		3.015	Q		3.003	B	E
31	NB	William's	3.271	NB		4.904	NB		3.010	Q		3.000	B	E
32	NB	Centric Brand	1.458	B	E	2.037	Q		3.014	Q		3.300	NB	
33	NB	Citi Trends Inc	3.292	NB		6.419	NB		3.190	NB		3.016	Q	
34	NB	Destination XL Group Inc	2.565	Q		4.487	NB		3.021	Q		3.203	NB	
35	NB	Dillard's Inc	3.188	NB		5.993	NB		3.162	NB		3.052	Q	
36	NB	Express Inc	3.078	NB		6.032	NB		3.288	NB		3.019	Q	
37	NB	Foot Locker	3.730	NB		6.312	NB		3.220	NB		3.025	Q	
38	NB	Goody's	2.509	Q		5.065	NB		3.036	Q		3.151	NB	
39	NB	Nordstrome	1.718	B	E	2.775	Q		3.045	Q		3.421	NB	
40	NB	Rent-A-Car	3.419	NB		3.430	NB		3.229	NB		3.086	Q	
41	NB	Sprouts	3.622	NB		7.190	NB		3.194	NB		3.018	Q	
42	NB	Tile Shop Holding	1.661	B	E	2.979	Q		3.099	Q		3.161	NB	

The assessment of accuracy of both models in predicting bankruptcy one year (YR1) and two years (YR2) prior to the actual bankruptcy or the reporting period is presented in Table 3.

Table 3. Comparison of Bankruptcy Prediction Accuracy

		ONE YEAR PRIOR		TWO YEARS PRIOR	
		Z-Score	Per(A)	Z-Score	Per(A)
# OF ERROR	G1 (Bankrupt)	5	3	7	6
	G2 (Non-bankrupt)	11	10	5	7
	ALL	16	13	12	13
ACCURACY RATE	G1 (Bankrupt)	84.85%	89.29%	77.42%	79.31%
	G2 (Non-bankrupt)	64.52%	64.29%	85.71%	65.00%
	ALL	75.00%	76.79%	81.82%	73.47%

Z-score model predicted bankrupt companies' actual bankruptcy one year prior to bankruptcy with 84.85% accuracy rate while the proposed model's accuracy rate in predicting bankrupt companies' actual bankruptcy one year prior to bankruptcy was 89.29%. Two years prior to the actual bankruptcy of bankrupt companies, the proposed model predicted more accurately (79.31%) than Z-score model (77.42%). However, the accuracy rate of the proposed model in predicting bankruptcy and non-bankruptcy is lower than that of Z-score model when both models were applied to non-bankrupt companies.

5. Conclusions

This paper examines the applicability of a graph theoretic approach to bankruptcy prediction. Bankruptcy prediction model has been one of the most attractive research fields in finance and accounting. The assessment of the likelihood that a firm may go bankrupt provides valuable information to current and prospective investors. Numerous statistical tools have been applied to the development of bankruptcy prediction models, and one of the most renowned bankruptcy prediction models is Altman's Z-score model which was derived from discriminant analysis. Altman's Z-score model has received a wide acceptance by researchers and practitioners since its inception by demonstrating its value in assessing the likelihood that a firm may go bankrupt within two years.

The uniqueness of this paper is that it is the first to propose the graph theoretical method for bankruptcy prediction. The proposed method employs undirected graph and matrix methods to predict bankruptcy. Five financial ratios that are considered good indicators of the financial solvency are identified from a literature review. The interactions of these financial ratios are converted into an adjacency matrix and the matrix permanent is calculated as an index of a firm's financial solvency. We argue that by considering the collective strength of five financial performance indicators along with the causality relations among those financial performance indicators, the proposed method can develop more inclusive measurement of financial solvency of a firm.

The proposed method is different from Altman Z-score formula in two aspects. Firstly, Altman Z-score formula is a linear combination of five key financial performance indicators. The proposed method is a combination of five key financial performance indicators and also interactions among those financial performance indicators. By incorporating the dynamic interactions among key financial performance indicators, the proposed method can develop more inclusive measurement of financial solvency of a firm. Another difference is that Altman Z-score formula assess the status of a firm's financial performance as of a specified point in time while the proposed method assess a firm's financial performance for a period of time as the construction of a firm's financial solvency adjacency matrix requires examination of the interactions among key financial performance indicators for a period of time.

The proposed model and Altman's Z-Score model were applied to two sample groups and the accuracy of these two models in predicting financial solvency of a firm was compared as demonstrated in Table 3. The comparison of two models does not prove the superiority of one model over the other. Nevertheless, this research proves the value of the proposed model in assessing the financial solvency of a firm and the likelihood that a firm may go bankrupt in the near future.

The value of this study is that it is the first research to apply the graph theoretical approach to bankruptcy prediction and the applicability of the graph theoretical approach to bankruptcy prediction is proven in this study. The proposed method in this paper provides an insight into the development of a more comprehensive approach to the assessment of financial solvency of a company.

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